Quantum Algorithms for Open Lattice Field Theories

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Our work: Jay Hubisz, Bharath Sambasivam, Judah Unmuth-Yockey "Quantum Algorithms for Open Lattice Field Theory." ArXiv:2012.05257, Dec. 2020., http://arxiv.org/abs/2012.05257

Erik Gustafson, Michael Hite, Jay Hubisz, Bharath Sambasivam, Judah Unmuth-Yockey "Quantum simulation of open lattice field theories" *in preparation*

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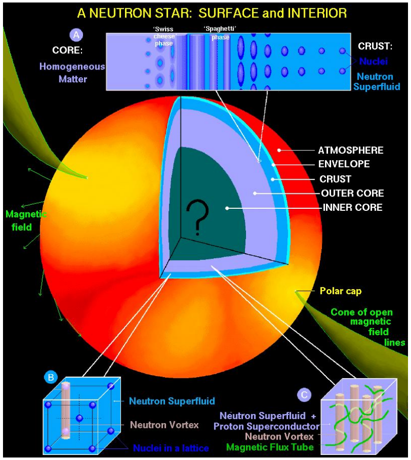
Why open field theories?

- Open quantum systems well described by effective field theories.
- Can at-times be described, or approximated by non-Hermitian Hamiltonians.
- Hard to simulate classically because of the Sign Problem.
- A good example is QCD at finite μ.



- Quantum computers can potentially solve this. However, we are only in the noisy intermediate scale quantum (NISQ) era.
- May be able to simulate a simple, lower dimensional field theories in the near future.
- Probing QFTs at complex couplings could reveal phase structure: Lee-Yang Zeros, Fisher Zeros etc.

Quantum Operations

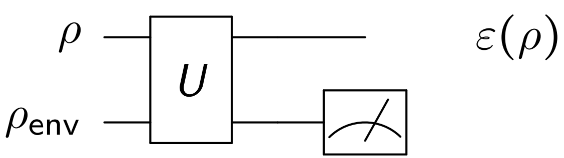
- We present quantum algorithms under the *Quantum Operations* formalism: $\varepsilon : \rho \longrightarrow \rho'$
- Two examples- unitary evolution and measurement

$$\varepsilon(\rho) = \hat{U}\rho\hat{U}^{\dagger}, \quad \text{and} \quad \varepsilon_m(\rho) = \hat{M}_m\rho\hat{M}_m^{\dagger} \tag{1}$$

- Evolution of an open system can be mocked up via a quantum operation on an enlarged, unitary system, followed by measurements on the ancillary qubits.

- Quantum operations can be written down using operator-sum representations:

$$\varepsilon(\rho) = \sum_k \hat{E}_k\rho\hat{E}_k^{\dagger}, \quad \text{where} \quad \hat{E}_k = \langle e_k|\,\hat{U}\,|e_0\rangle$$



are the *Kraus Operators*.

- Trace completeness: $\sum_k \hat{E}_k^{\dagger}\hat{E}_k = \mathbb{1}$

Probabilities and Quantum Jumps

- Consider the general N-dimensional Hamiltonian $\hat{H} = \hat{G} + i\hat{K}$. The aim is to simulate trotterized evolution. $(-\hat{K} > 0$ w.l.o.g)
- Evolution by $\hat{E}_0$ gives the desired dynamics: $p_s = \mathrm{Tr}\left(\hat{E}_0\rho\hat{E}_0^\dagger\right)$
- The other $\hat{E}_i$'s are the inevitable "quantum jumps": $p_{f,i} = \mathrm{Tr}\left(\hat{E}_i\rho\hat{E}_i^\dagger\right)$
- The probability of success, p_s depends on the normalization of $\hat{E}_0$ and is bounded above

$$p_s \leq \mathrm{Tr}\left(e^{2\delta t\,\hat{K}}\rho\right) \tag{2}$$

- Due to this, at very late evolution times, the probability of success is exponentially small.
- However, for small system sizes, simulatable in near-term quantum hardware, interesting physics can be extracted for small evolution times.

Random walk through time

- For a general non-Hermitian Hamiltonian, $\hat{H} = \hat{G} + i\hat{K}$, the Kraus operators for the quantum operation are

$$\hat{E}_{\pm} \simeq \frac{1}{\sqrt{2}} \exp\left(\mp i \hat{G}\, \delta t\right) \exp\left(\pm \hat{K}\, \delta t\right) \tag{3}$$

- The unitary acting on the enlarged space (full system + 1 ancilla) is given by

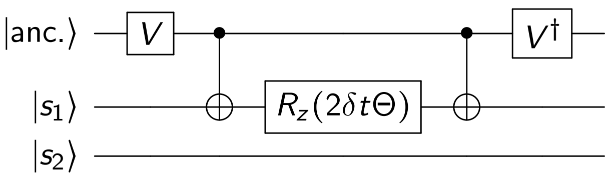
$$\hat{U}^{\mathrm{RW}} \approx \hat{W}\,\hat{T}, \quad \text{where} \tag{4}$$

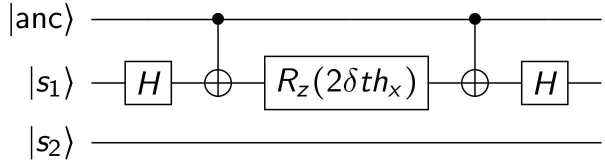
$$\hat{W} \simeq e^{-i\,\delta t\,\hat{\sigma}_z \otimes \hat{G}} + \mathcal{O}\left(\delta t^2\right), \text{ and } \hat{T} \simeq \frac{1}{\sqrt{2}}((\mathbb{1} - i\hat{\sigma}_y) \otimes \mathbb{1}) e^{i\,\delta t\,\hat{\sigma}_y \otimes \hat{K}} + \mathcal{O}\left(\delta t^2\right) \tag{5}$$

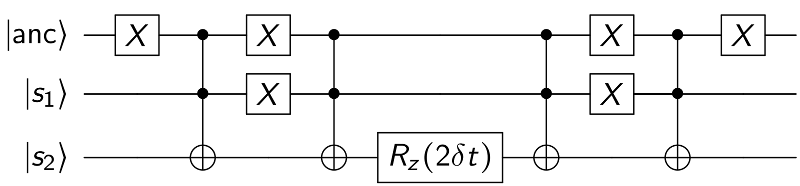
- If $\hat{G}$ and $\hat{K}$ are implementable, then $\hat{U}^{\mathrm{RW}}$ is too.
- This is closest to a fair coin toss, in terms of probabilities. Failure simply sets the system back by one time step, which isn't catastrophic.

Random Walk Circuit for 2 sites

$$\hat{H} = \underbrace{-\lambda \sum_{<i,j>} \hat{\sigma}_i^z \hat{\sigma}_j^z - h_x \sum_i \hat{\sigma}_i^x}_{\hat{G}} \underbrace{+ i\,\theta \sum_i \hat{\sigma}_i^z}_{\hat{K}} . \tag{6}$$







V diagonalizes σ_y

Single qubit anti-hermiticity

- Consider the anti-Hermitian part of the Hamiltonian acting on local degrees of freedom (relevant to Lattice field theories).
- Focus on the single qubit quantum operation $\hat{k} = \Theta(\hat{\sigma}_z - s\,\mathbb{1})$
- The relevant Kraus operator is

$$\hat{E}_0 = \begin{pmatrix} e^{(1-s)\delta t\,\Theta} & 0 \\ 0 & e^{-(1+s)\delta t\,\Theta} \end{pmatrix} \tag{7}$$

- There are many ways to complete the trace of this quantum operation.
- A multi-qubit trotterized anti-Hermitian piece can, in general be decomposed into a single qubit piece and a Unitary entangler.
- Each single qubit anti-Hermitian piece can always be rotated to align with the $\hat{\sigma}_z$ direction.

Damping Channels

- The Kraus operators for the quantum operation are

$$\hat{E}_0^{\mathrm{DC}} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, \quad \hat{E}_1^{\mathrm{DC}} = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}, \tag{8}$$

  where $\gamma = 1 - e^{-4\,\Theta\,\delta t}$
- A controlled y-rotation, with the ancilla as the target can implement this:

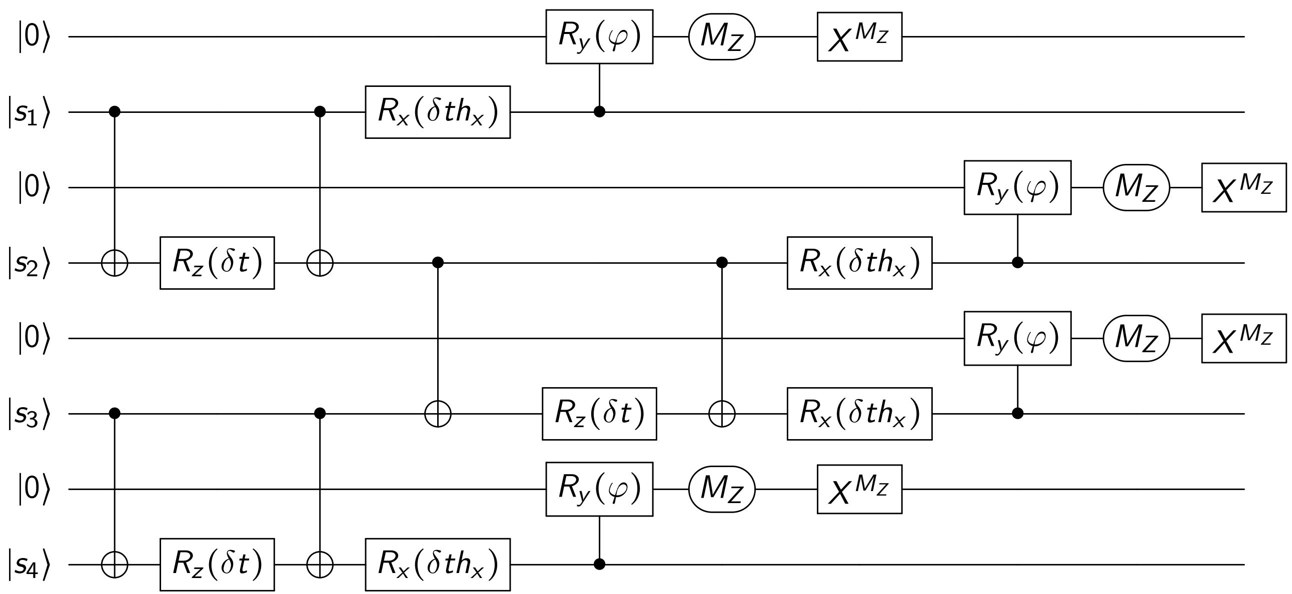
$$|0\rangle\,|\psi\rangle \longrightarrow |0\rangle\,\hat{E}_0^{\mathrm{DC}}\,|\psi\rangle - |1\rangle\,\hat{E}_1^{\mathrm{DC}}\,|\psi\rangle \tag{9}$$

- The probability of success (measuring '0' on the ancilla) is maximal

$$p_s = \mathrm{Tr}\Big(\hat{E}_0 \rho \hat{E}_0^\dagger\Big) = 1 - \frac{\gamma}{2}(1 - r\cos\theta) \tag{10}$$

Damping Circuit for 4 sites

$$\hat{H} = \underbrace{-\lambda \sum_{<i,j>} \hat{\sigma}_i^z \hat{\sigma}_j^z - h_x \sum_i \hat{\sigma}_i^x}_{\hat{G}} + \underbrace{i\,\theta \sum_i \hat{\sigma}_i^z}_{\hat{K}} . \quad (11)$$

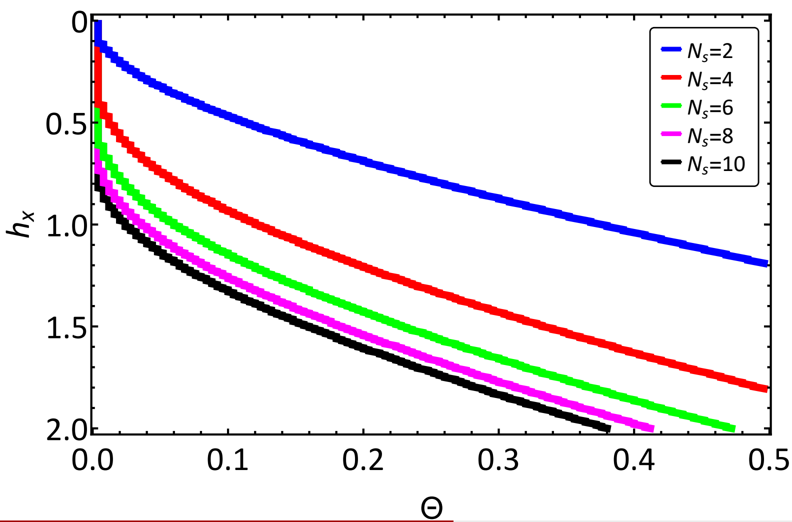


Application to a simple Lattice FT

- We apply this to the 1-dimensional quantum Ising model with an imaginary longitudinal magnetic field described by the Hamiltonian

$$\hat{H} = \underbrace{-\lambda \sum_{<i,j>} \hat{\sigma}_i^z \hat{\sigma}_j^z - h_x \sum_i \hat{\sigma}_i^x}_{\hat{G}} + i\,\theta \underbrace{\sum_i \hat{\sigma}_i^z}_{\hat{K}} \, . \qquad (12)$$

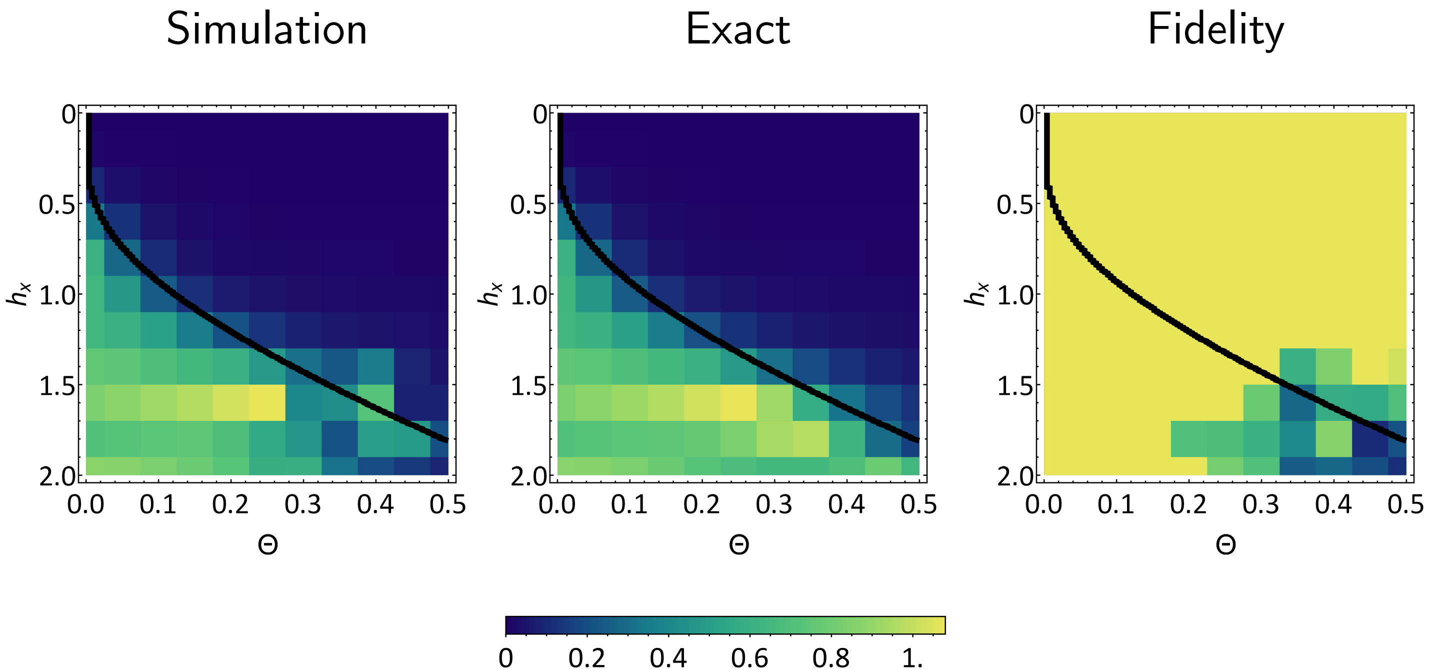
- This is a well studied model, and serves as a good benchmark.



Exceptional lines for different system sizes, which is the Lee-Yang edge in large system limit. Above the lines, the ‘ground state‘ is degenerate with complex conjugate ‘energies‘. On the line, one ground eigenstate is lost.

Rényi entropy plots

Plot of S_2 measured after 350 time-steps, with $N_s = 4$, $\delta t = 0.01$



Rényi entropy is measurable using Parity measurements[1]

[1] S. Johri, D. S. Steiger, and M. Troyer, Phys. Rev. B 96, 195136 (2017)

Conclusion

- Non-Hermitian Hamiltonians can describe some effective theories of physics, many of which suffer from the sign problem, making them hard to simulate classically.
- Quantum computers can potentially solve this problem, but aren't advanced enough to simulate complicated field theories.
- However, simple Field Theories for small system sizes can be simulated in contemporary quantum computers.
- We presented measurement based quantum channels, with quantum jumps to approximate these non-Hermitian field theories and apply it to a 1-D Ising chain with an imaginary longitudinal magnetic field.
- Errors become $\mathcal{O}(1)$ at very long times, however interesting physics of quantum phase structure can be extracted for small system sizes, at small circuit depth which make these channels amenable to NISQ era study.
- In ongoing work, we are studying implementation on current hardware, and generalizations of the QITE algorithm to non-Hermitian Hamiltonians.

Thank you ☺

Jay Hubisz, Bharath Sambasivam, Judah Unmuth-Yockey "Quantum Algorithms for Open Lattice Field Theory." ArXiv:2012.05257, Dec. 2020., http://arxiv.org/abs/2012.05257