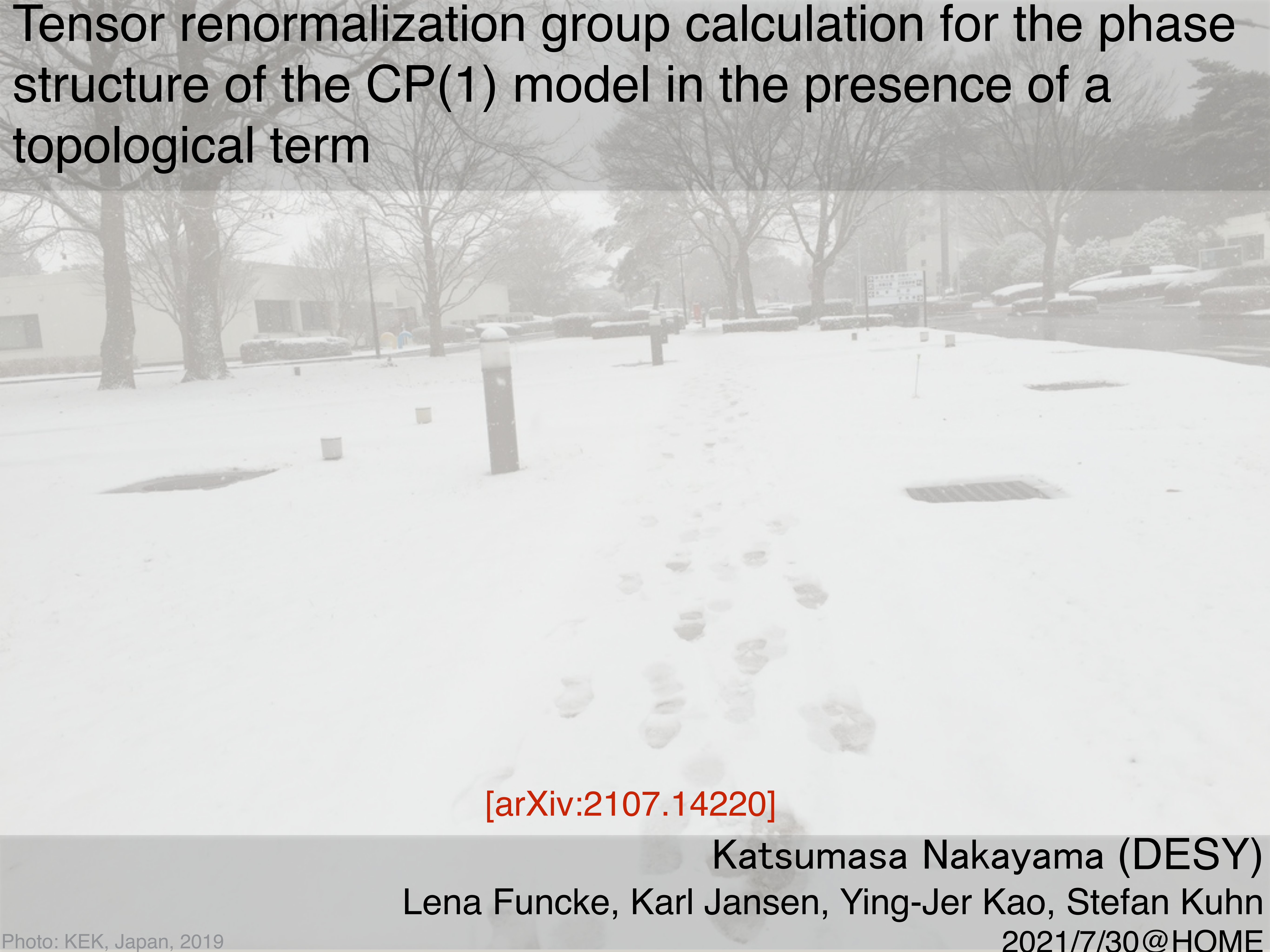
Tensor renormalization group calculation for the phase structure of the CP(1) model in the presence of a topological term



[arXiv:2107.14220]

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Photo: KEK, Japan, 2019

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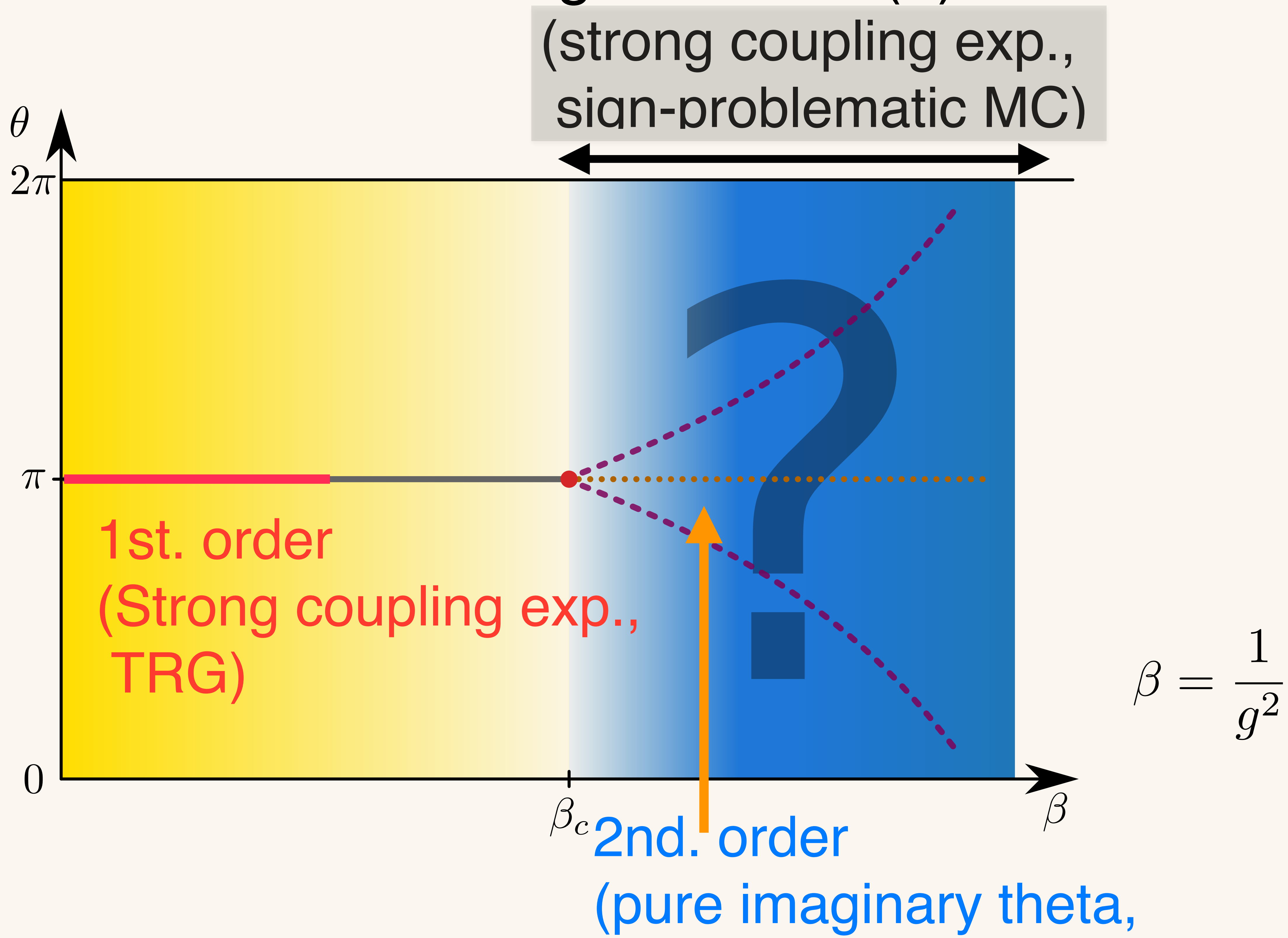
◉ Motivation: Phase diagram of CP(1) model



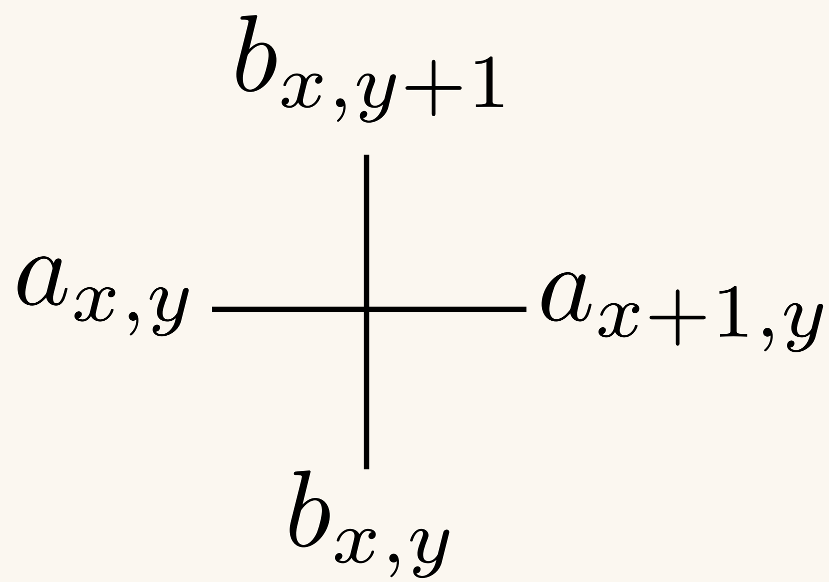
→ Large beta region is not strongly confirmed.

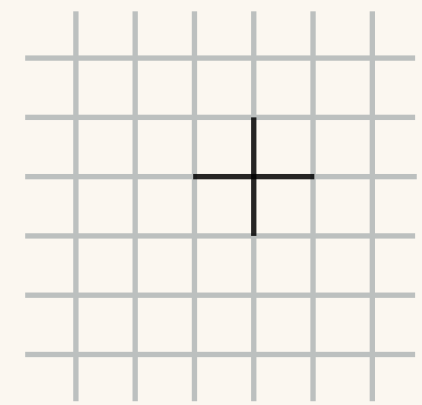
◉ Tensor representation

◇ TRG directly calculate the physical quantity.

$$Z = \sum_{a,b} \prod_{x,y} T_{a_{x,y}, a_{x+1,y}, b_{x,y}, b_{x,y+1}}$$

$$T_{a_{x,y}, a_{x+1,y}, b_{x,y}, b_{x,y+1}} =$$





◇ No sign-problem. (Using truncation, without sampling)

↓

We need to discuss the systematic error from truncation.

$\theta = 0$ case

From Lagrangian to the tensor

$$e^{-S_{\theta=0}} = \exp\left[-2\beta\sum_{x,\mu}\left[z_x^* z_{x+\hat{\mu}} U_\mu + z_x z_{x+\hat{\mu}}^* U_\mu^\dagger\right]\right]$$

◇ Using expansion by orthogonal function $f_{l,m}$

$$\int dz f_{l,m}(z', z) f_{l',m'}^*(z'', z) = \frac{1}{d_{l,m}} \delta_{l,l'} \delta_{m,m'} f_{l,m}(z', z)$$

$l + m \leq k_{\max}$

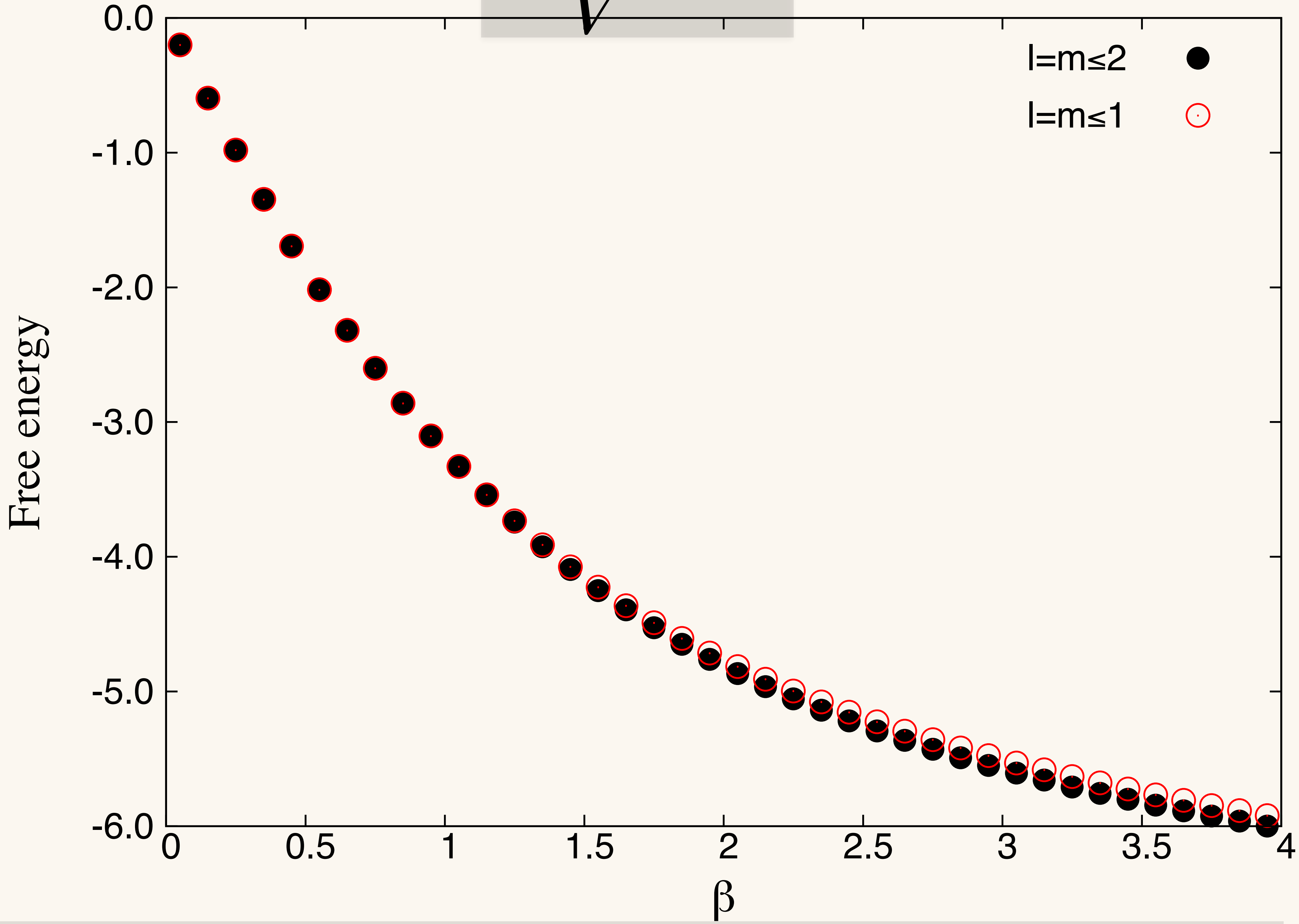
$I_n(x)$: Modified Bessel

$$e^{-S_{\theta=0}} = \prod_{x,\mu} \frac{1}{2\beta} \sum_{l,m=0}^{\infty} I_{l+m+1}(4\beta) d_{l,m} f_{l,m}(z_x, z_{z+\hat{\mu}})$$

We need to truncate the index l, m.

◉ Free energy of CP(1) model with $\theta = 0$

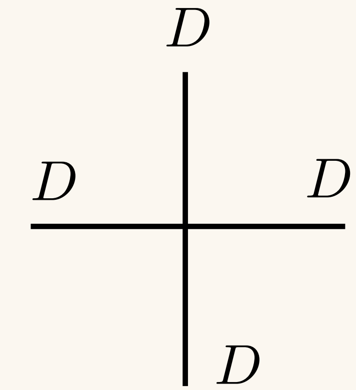
$$-\frac{1}{V}\ln Z$$

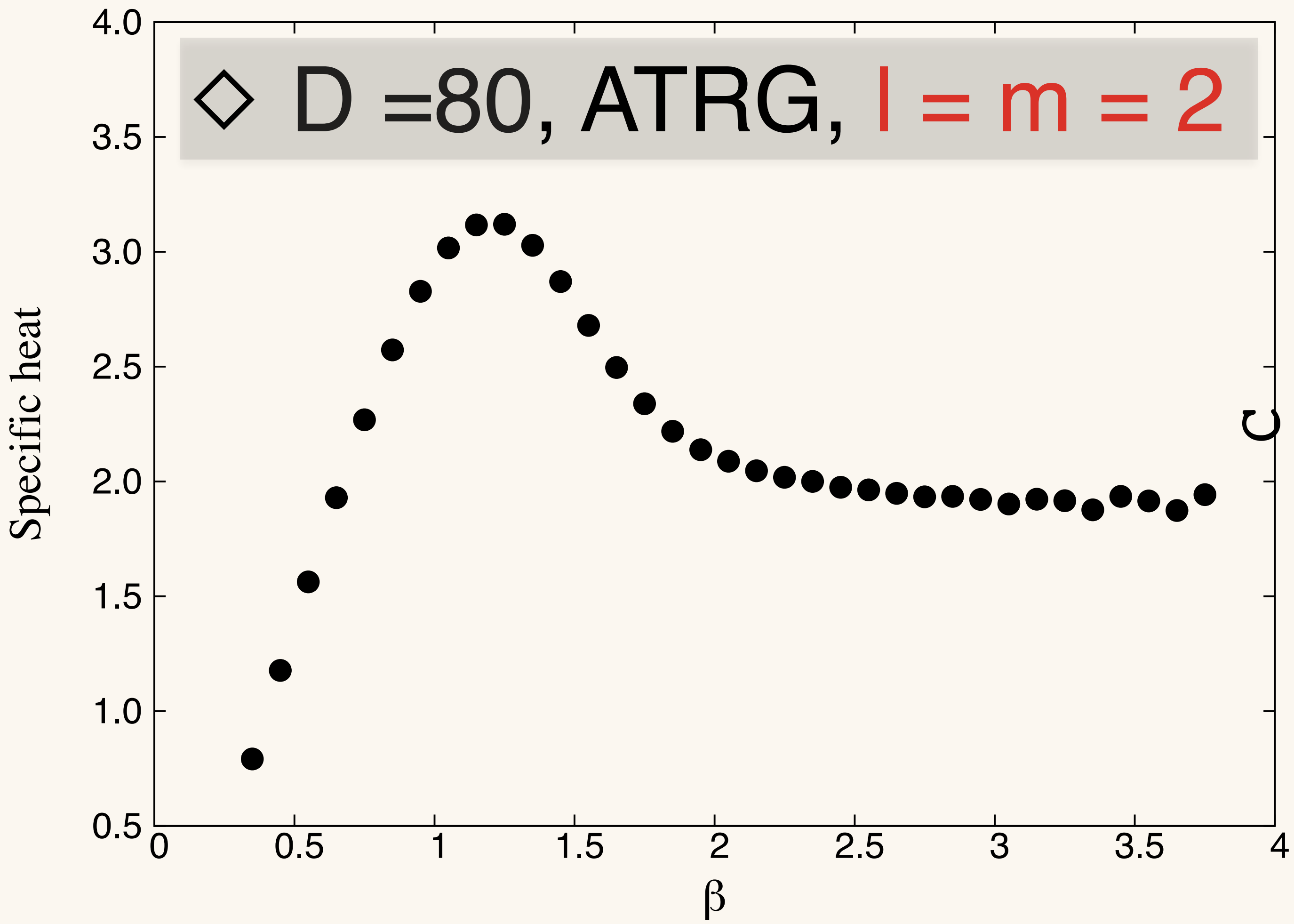


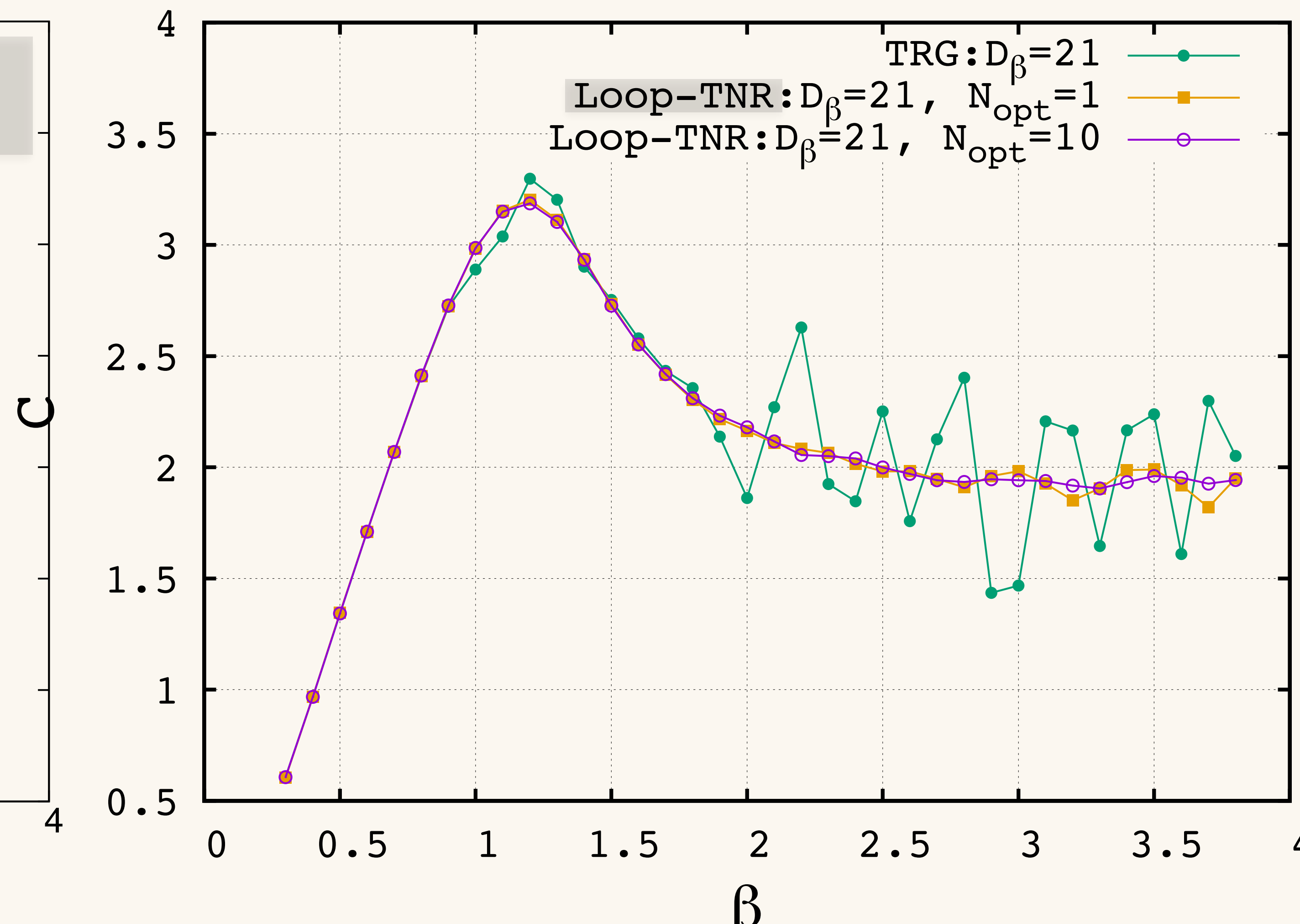
→ Truncation of l, m are small, especially in $\beta < 1.4$.

● Specific heat of CP(1) model with $\theta = 0$.

$$\frac{\beta^2}{V}\frac{\partial^2}{\partial\beta^2}\ln Z$$







[H. Kawauchi, S. Takeda arXiv:1710.09804]

→ Sufficiently large bond size D produces reliable results.

$\theta \neq 0$ case

◉ Character expansion of the theta term

◇ Theta term is also character expanded.

$$e^{i\frac{\theta}{2\pi}q_p} = \sum_{n_p\in\mathbb{Z}} e^{in_p q_p}\frac{2\sin(\pi n_p + \theta/2)}{\theta + 2\pi n_p}$$

$$q_p = A_{x,1} - A_{x+\hat{1}-\hat{2},2} - A_{x-\hat{2},1} + A_{x-\hat{2},2} \mod 2\pi$$

↓

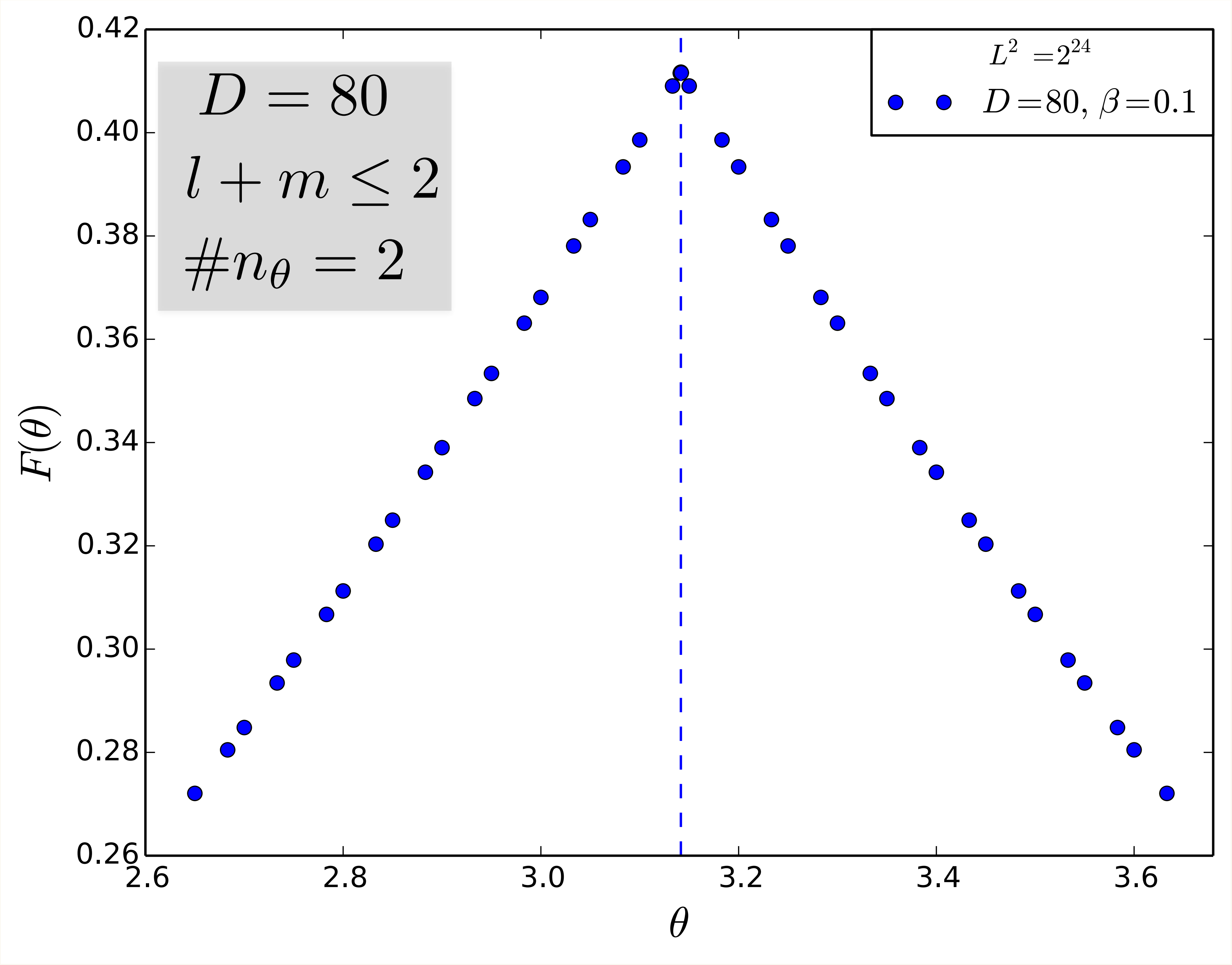
We need to truncate the index n_p.

→ The order of the truncation error is $O(1/n_p)$.

◉ Free energy with theta term

◇ beta = 0.1

Bond-weighted TRG



→ Clear kink structure imply the first-order transition

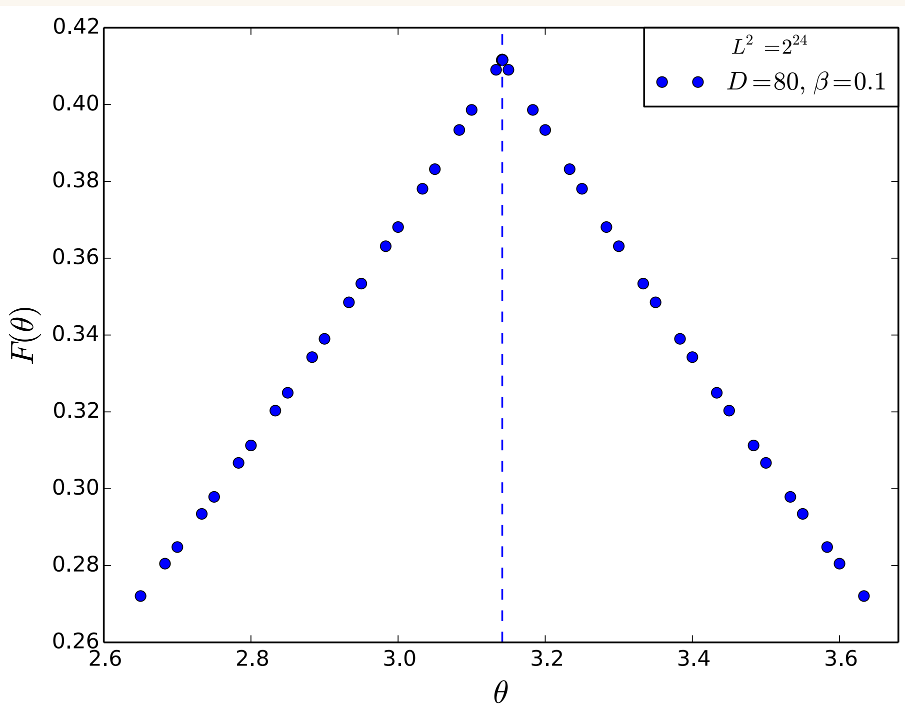
◉ Free energy with theta term

$D = 80$

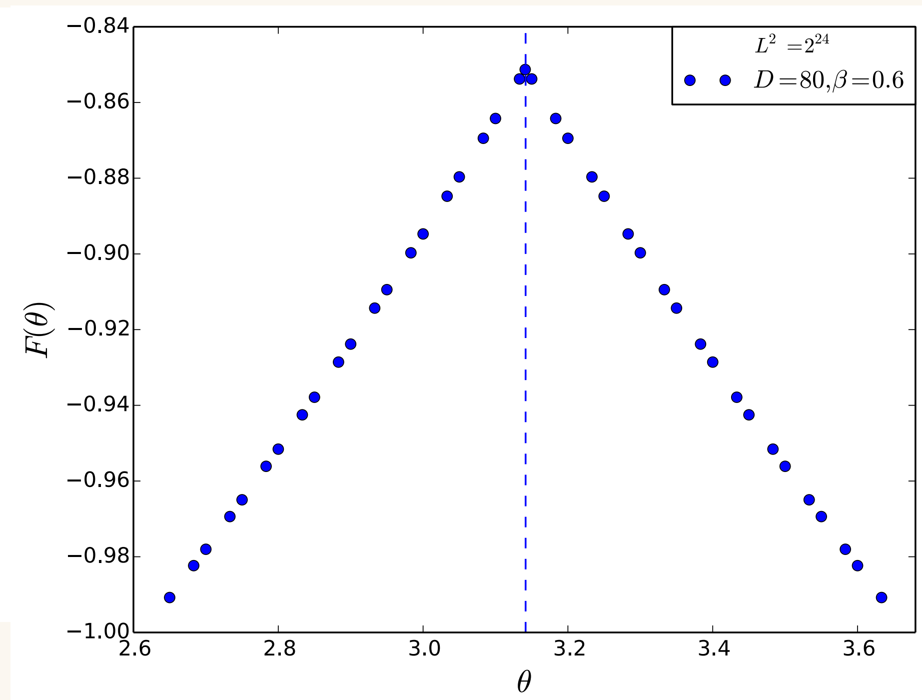
$l + m \leq 2$

$\# n_\theta = 2$

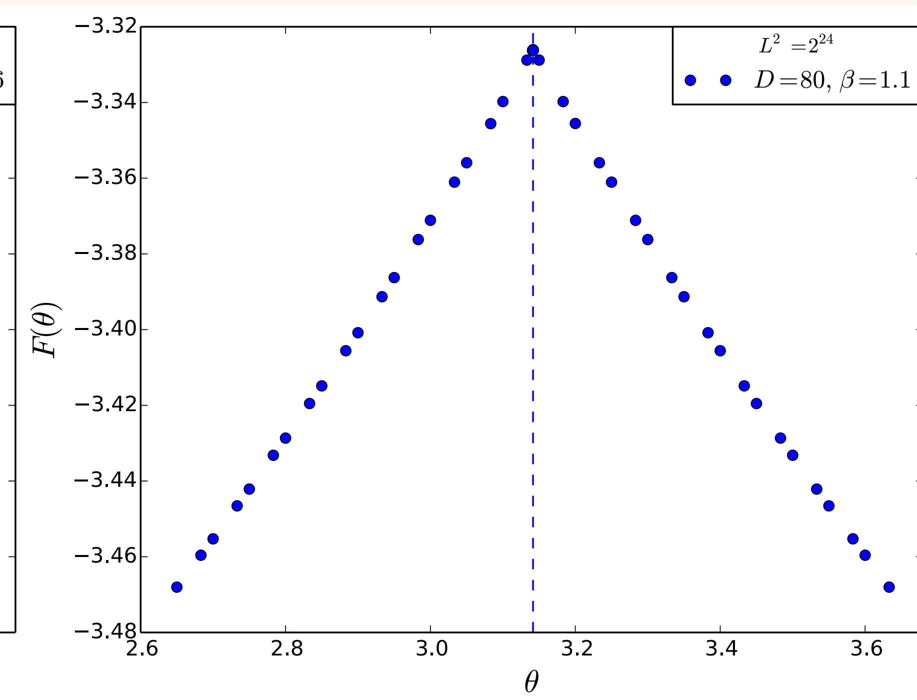
◇ beta = 0.1



◇ beta = 0.6



◇ beta = 1.1



→ Clear kink structure imply the first-order transition

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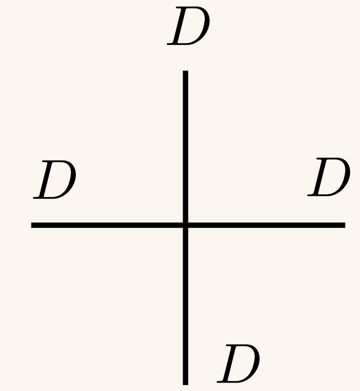
◇ We should estimate the systematic error.

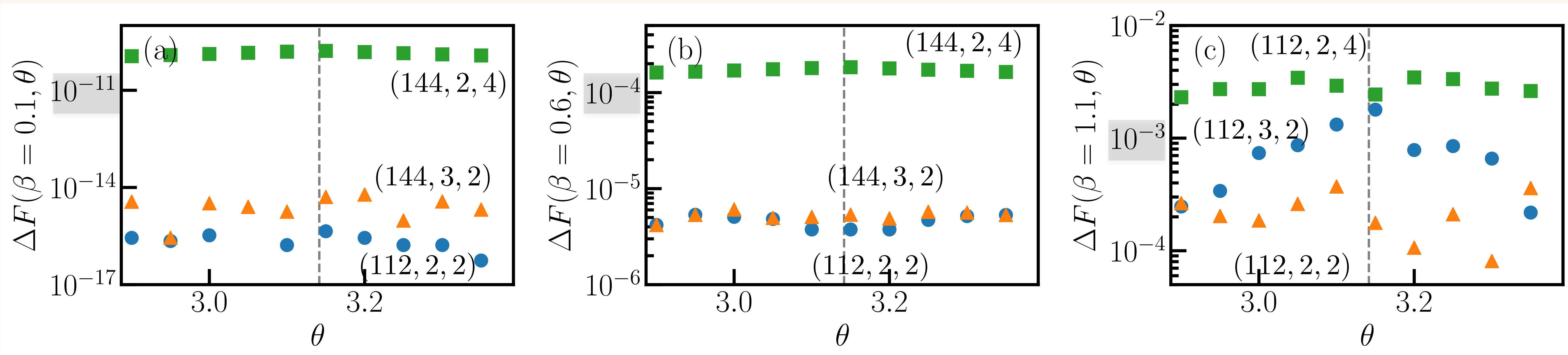
◉ Systematic error

$$l + m \leq k_{\max}$$



$$\Delta F(\beta, \theta) = |F(\beta, \theta)_{(D, k_{\max}, \#n_\theta)} - F(\beta, \theta)_{(80,2,2)}|$$

◇ beta = 0.1 ◇ beta = 0.6 ◇ beta = 1.1

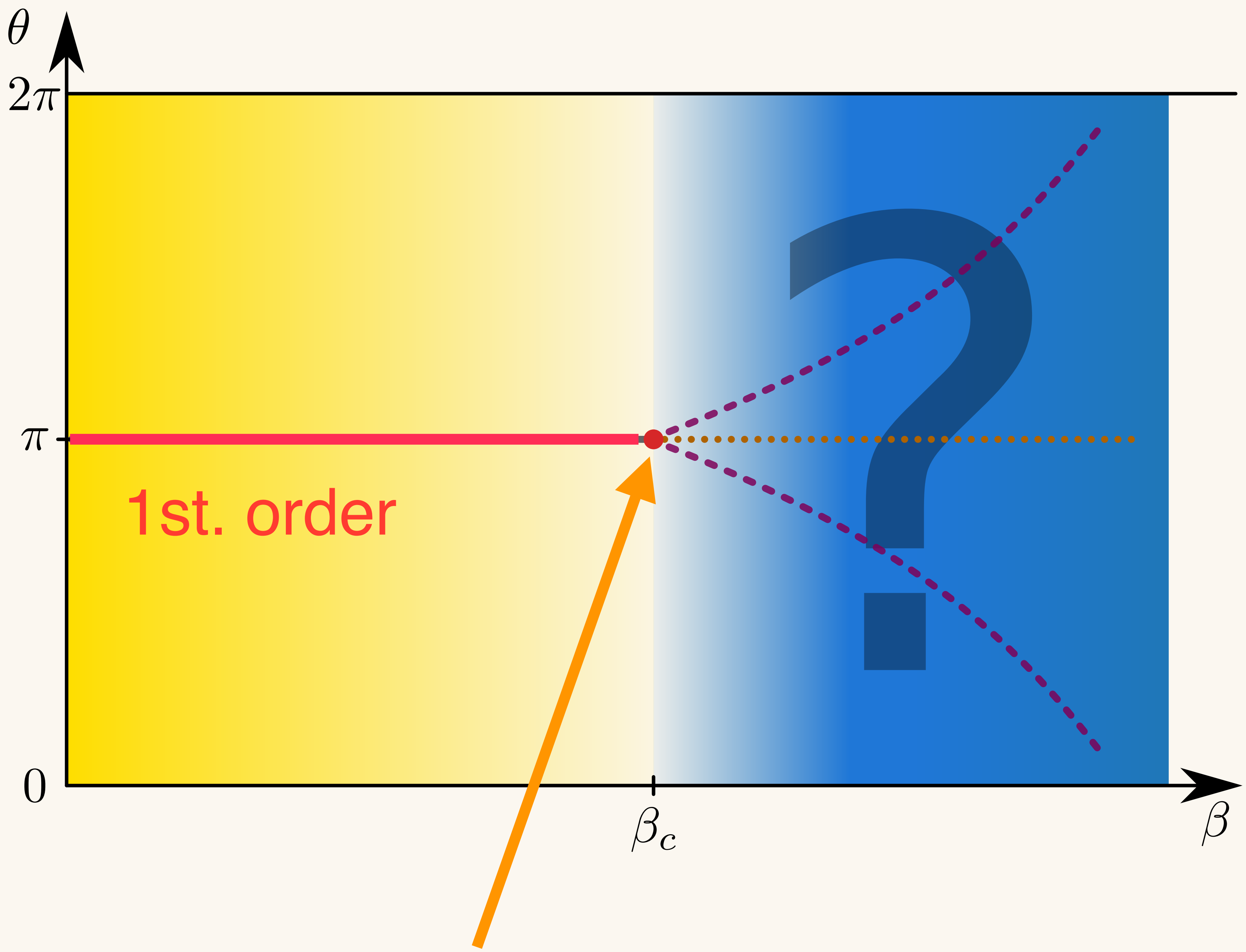


◇ SVD bond size truncation: $|F(\beta, \theta)_{(112,2,2)} - F(\beta, \theta)_{(80,2,2)}|$

◇ #Charactor expansion term: $|F(\beta, \theta)_{(144,3,2)} - F(\beta, \theta)_{(80,2,2)}|$

◇ #topological term, $\#n_\theta$: $|F(\beta, \theta)_{(144,2,4)} - F(\beta, \theta)_{(80,2,2)}|$

◉ Motivation: Phase diagram of CP(1) model



→ In our analysis, $1.1 \leq \beta_c$ is strongly favored.

◉ Summary

◇ We calculate free energy of CP(1) model to study the phase diagrams.

◇ From $\theta = 0$ calculation, sufficiently large bond size D is needed for precise calculation.

◇ Up to $\beta = 1.1$, our calculation shows first order transition at $\theta = \pi$, without any bifurcation.

◇ We estimate the systematic errors, and it is only $O(10^{-2\sim3})$.