HYBRID ANALOG-DIGITAL QUANTUM SIMULATIONS FOR QUANTUM FIELD THEORIES

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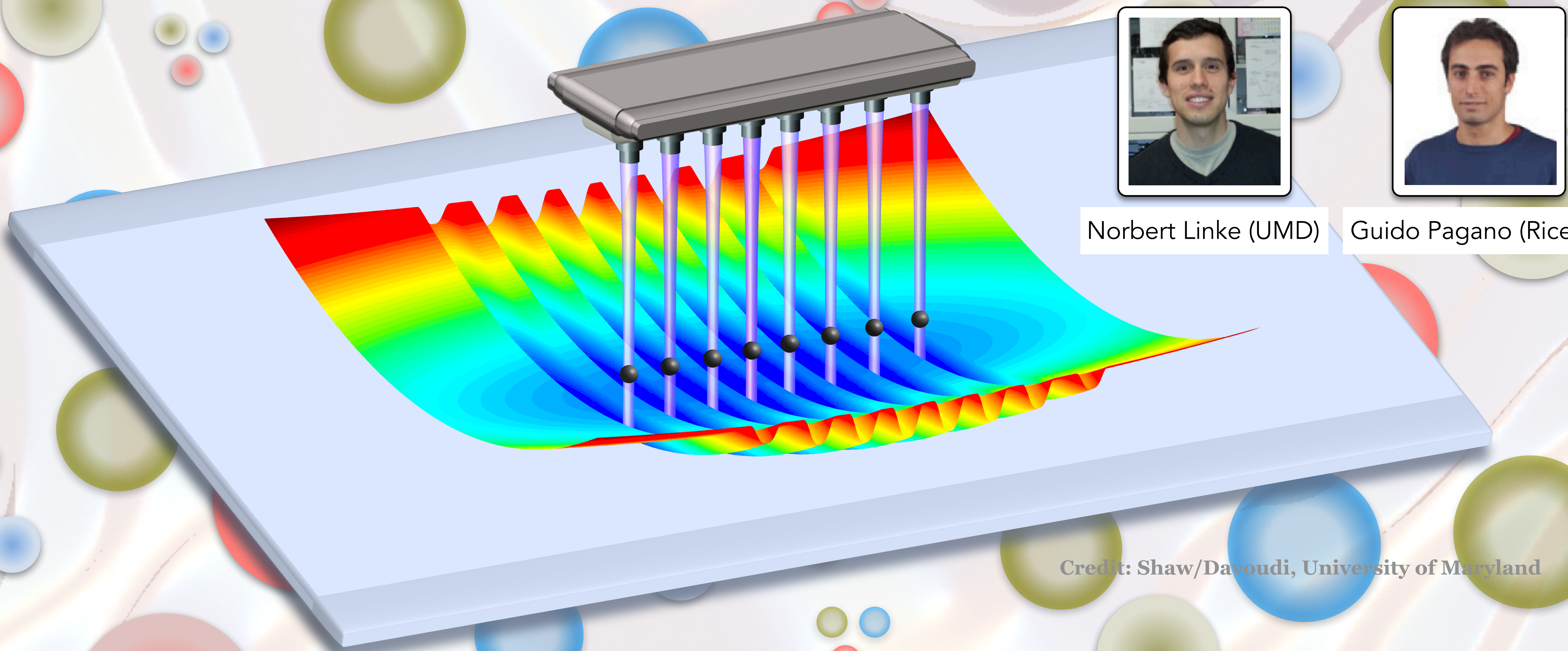


Work in collaboration with:

Norbert Linke (UMD)

Guido Pagano (Rice U)

The 38th International Symposium on Lattice Field Theory

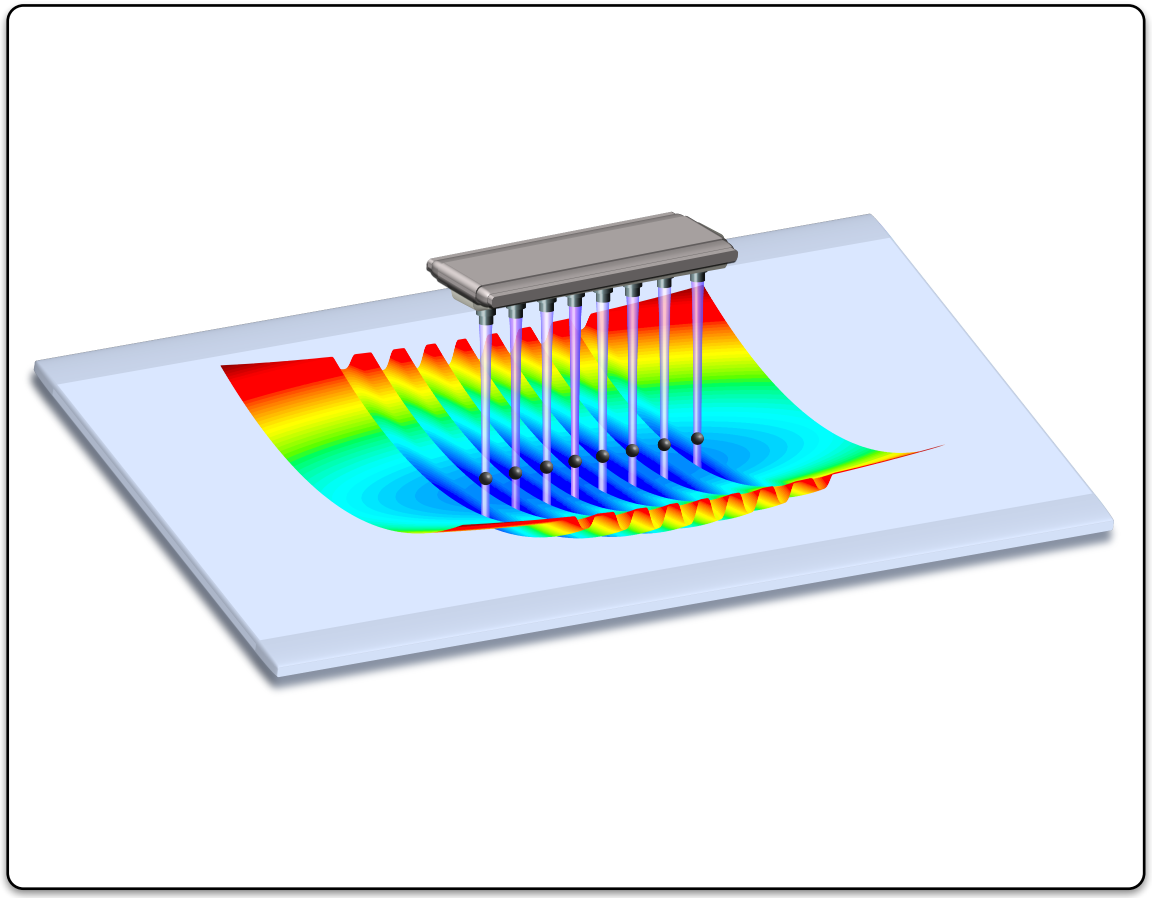
Zoom/Gather@MIT

July 26-30, 2021

TO PUT THINGS IN
THE CONTEXT…

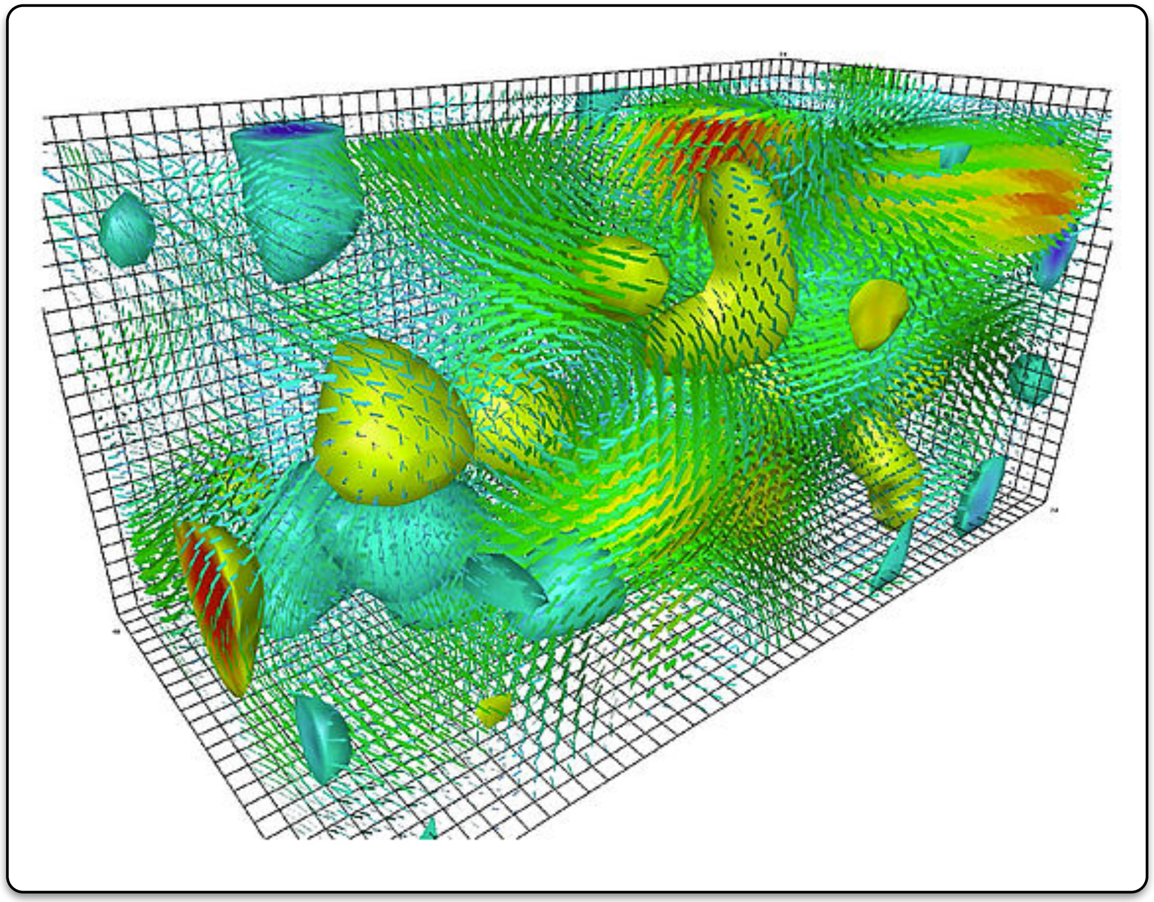
THE END GOAL FOR US: QUANTUM SIMULATION OF QUANTUM CHROMODYNAMICS!

A controlled quantum system



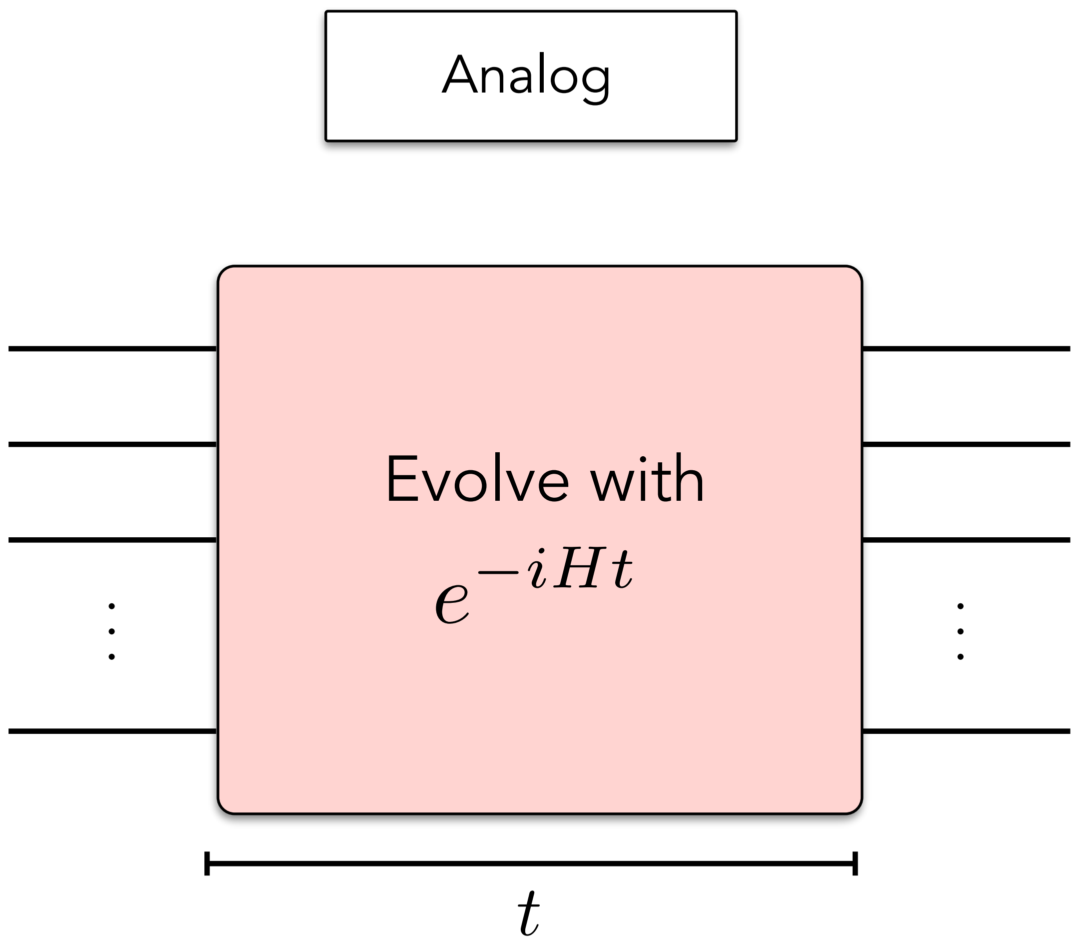
?

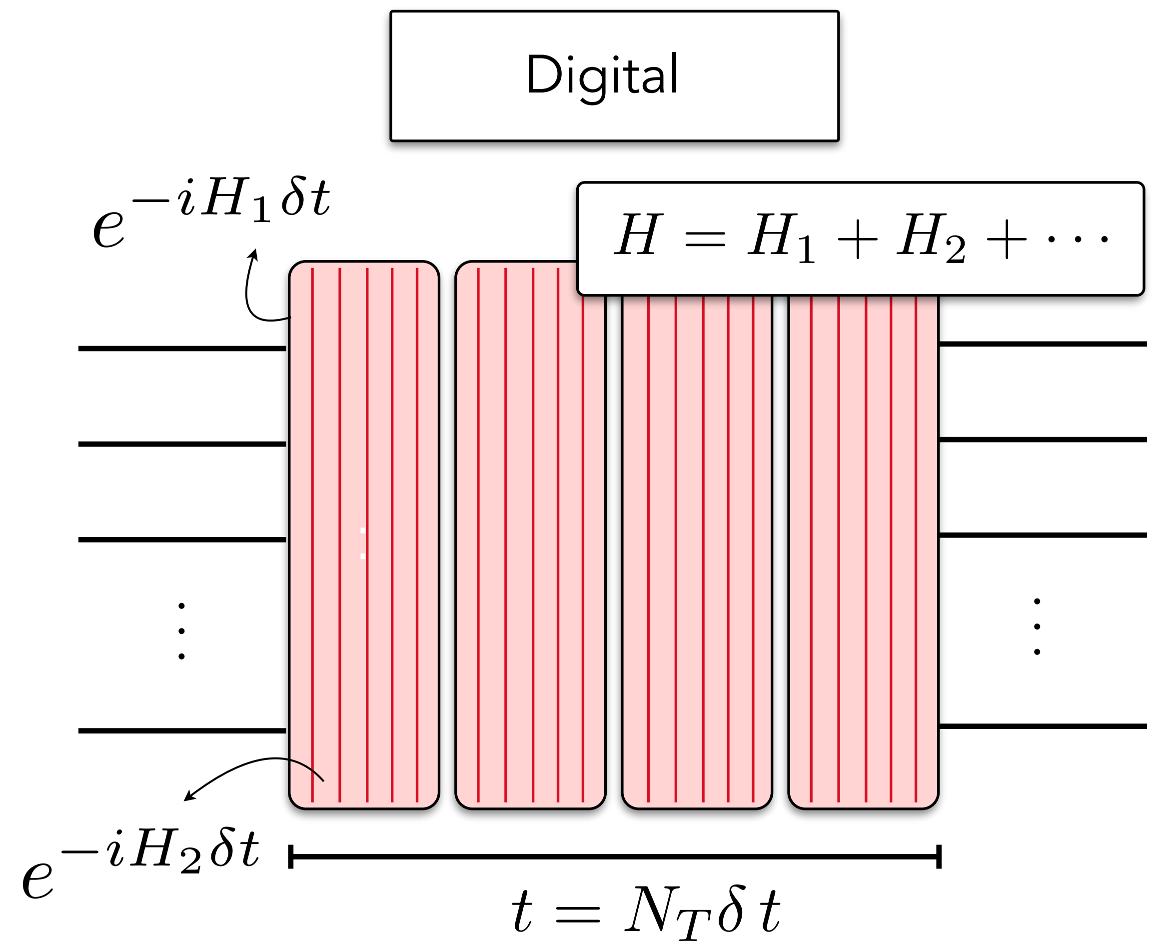
Strong-interaction physics



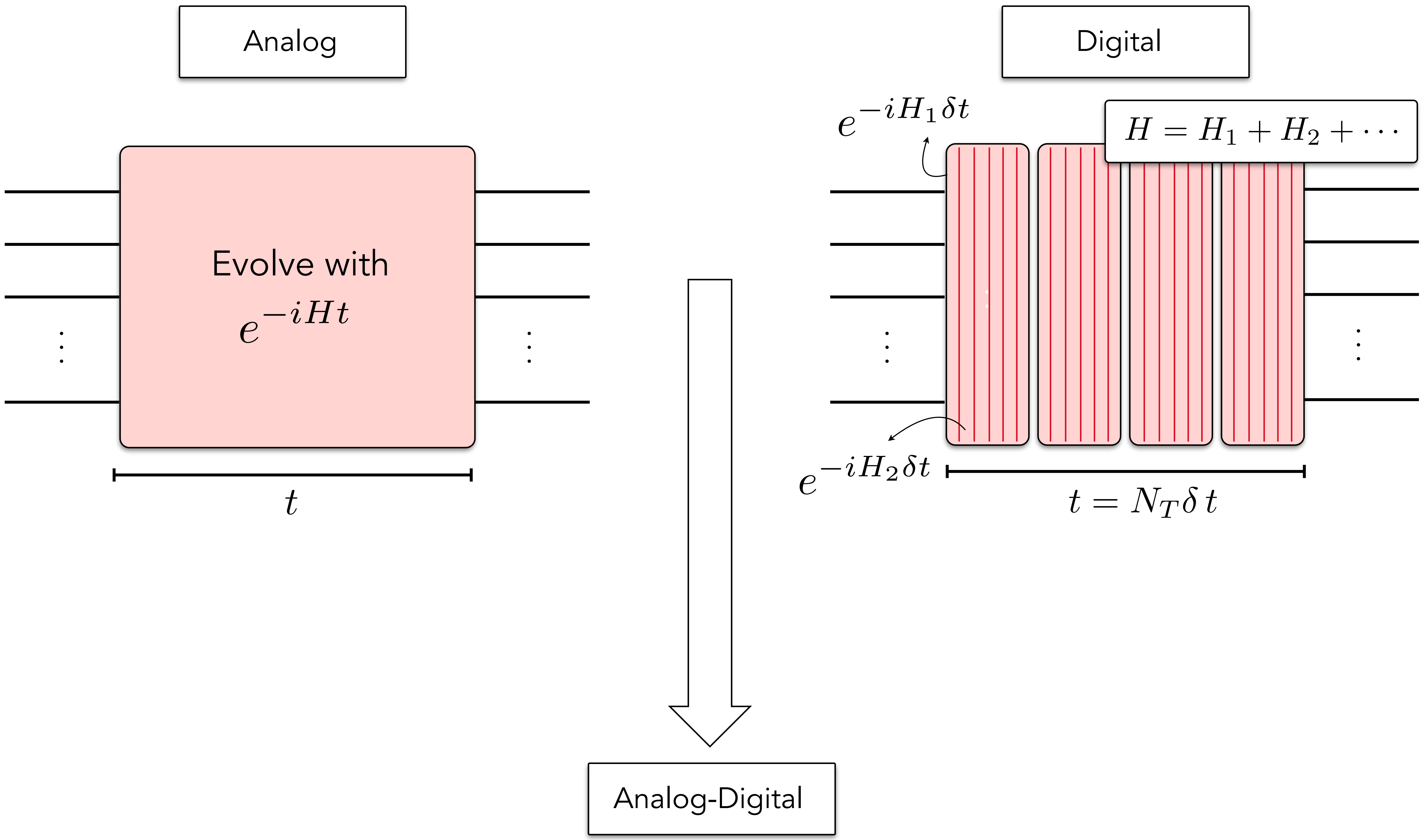
CREDIT: ANDREW SHAW, UNIVERSITY OF MARYLAND

DIFFERENT APPROACHES TO QUANTUM SIMULATION



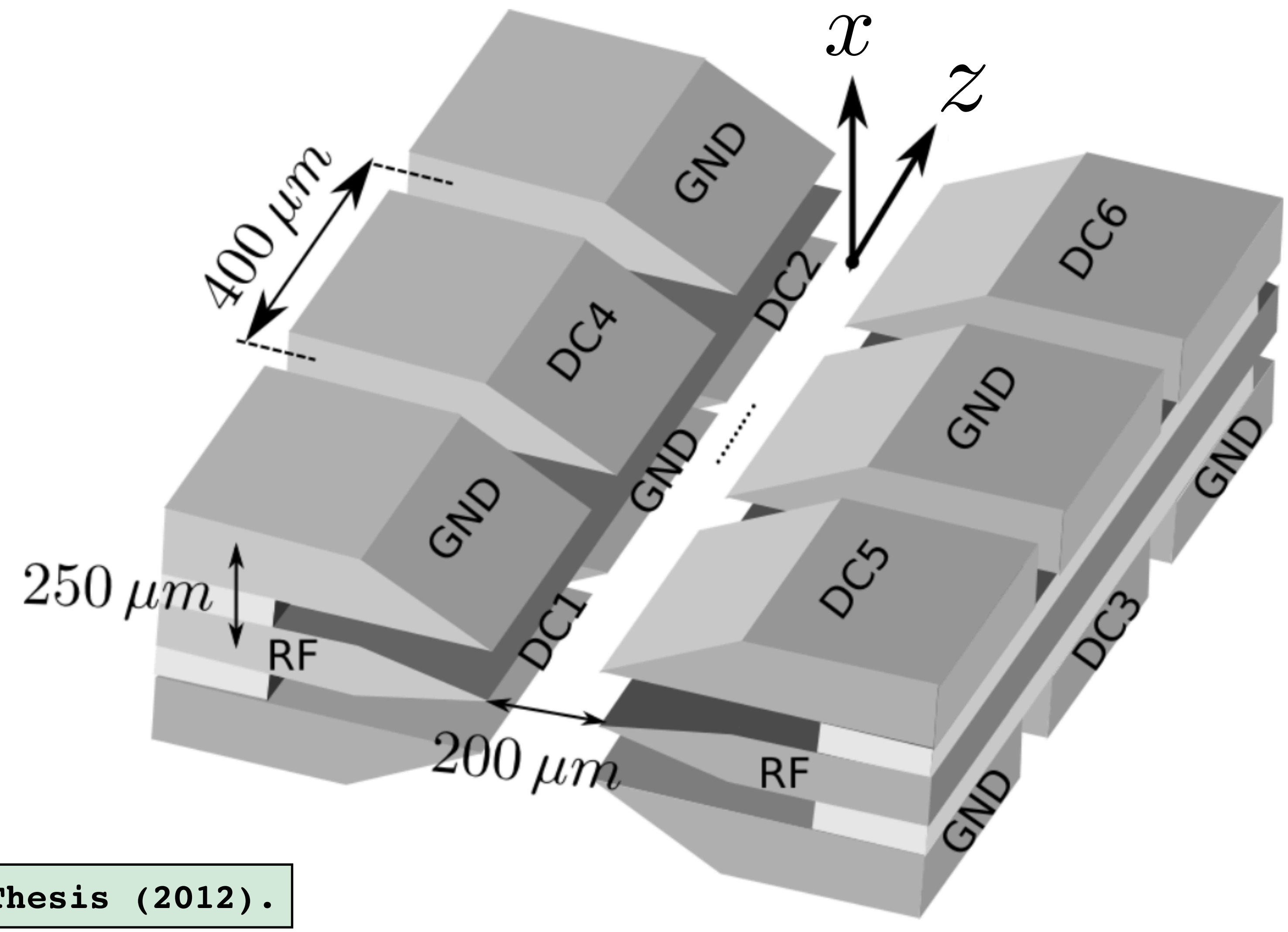


DIFFERENT APPROACHES TO QUANTUM SIMULATION

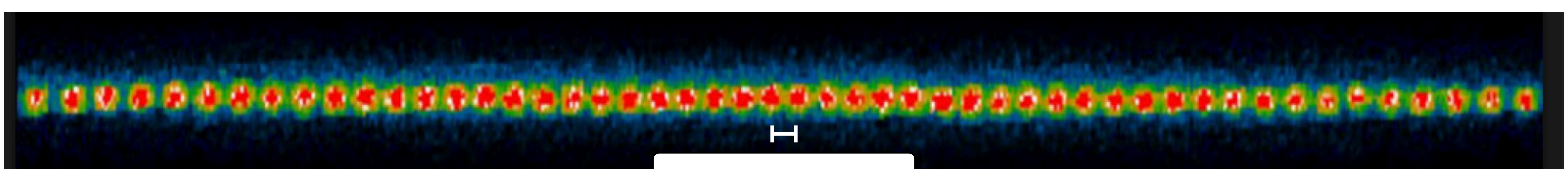


A QUICK TOUR TO THE UNDERLYING PHYSICS OF TRAPPED-ION SIMULATORS

A RADIO-FREQUENCY PAUL TRAP:



Islam, UMD PhD Thesis (2012).



$\sim$ few μm

UMD, Monroe Lab.

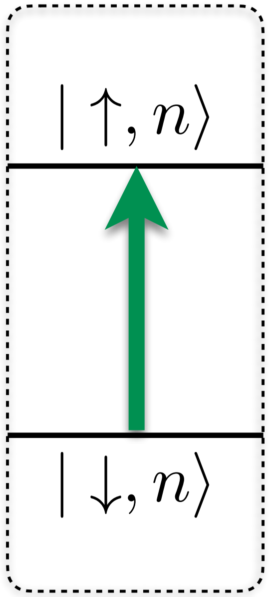
ION-LASER INTERACTIONS

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

$\mathcal{O}(\eta^0)$:

$$H_{carr} = -\frac{\Omega}{2}\left(\sigma^+ e^{-i\phi} + \sigma^- e^{i\phi}\right)$$

One-qubit operations



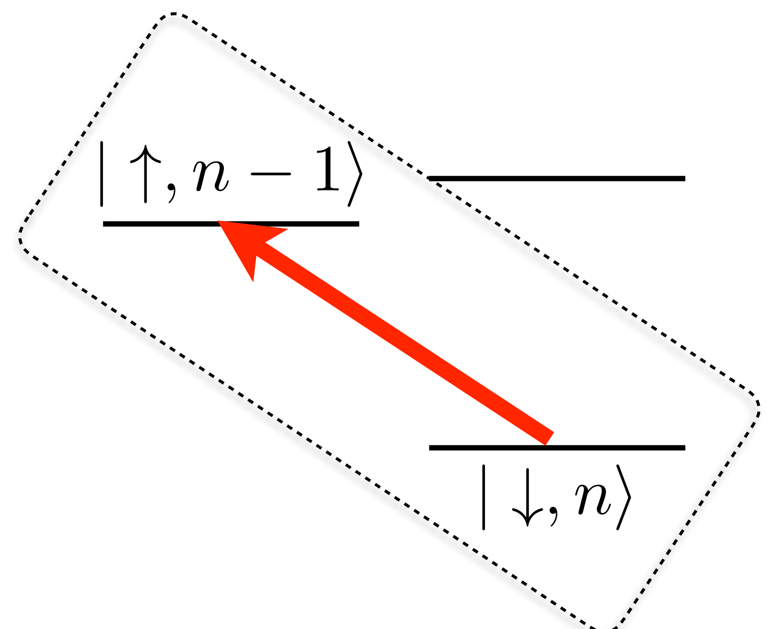
ION-LASER INTERACTIONS

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

$\mathcal{O}(\eta^1):$

$$H_{rsb} \approx \frac{i}{2}\eta\Omega\left[a^\dagger\sigma^- e^{i\phi} - a\sigma^+ e^{-i\phi}\right]$$

Spin-phonon transitions



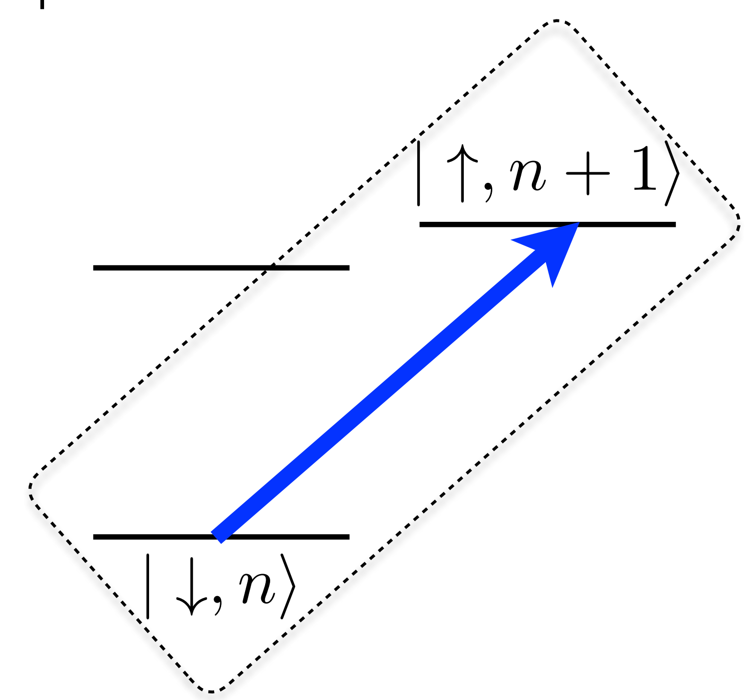
ION-LASER INTERACTIONS

Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012)

$\mathcal{O}(\eta^1)$:

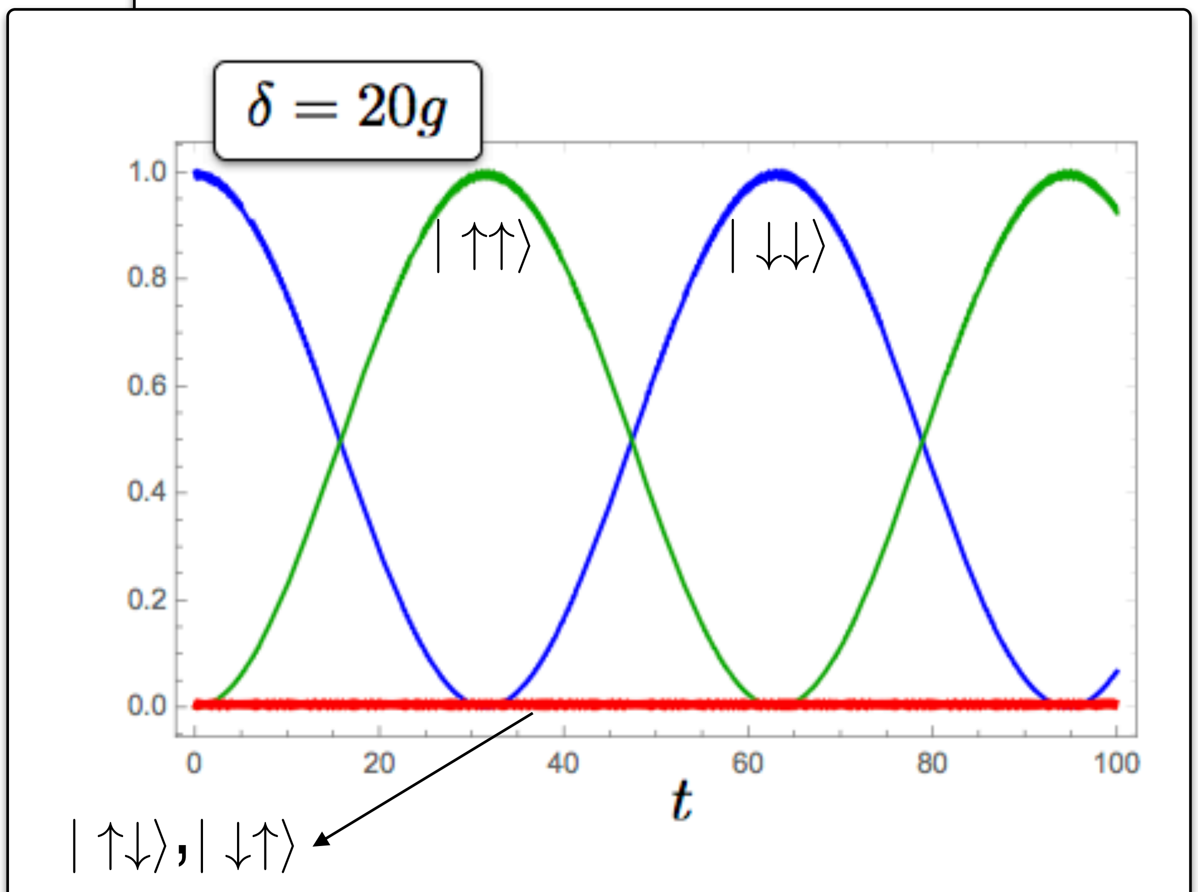
$$H_{bsb} \approx \frac{i}{2}\eta\Omega\left[a\sigma^- e^{i\phi} - a^\dagger\sigma^+ e^{-i\phi}\right]$$

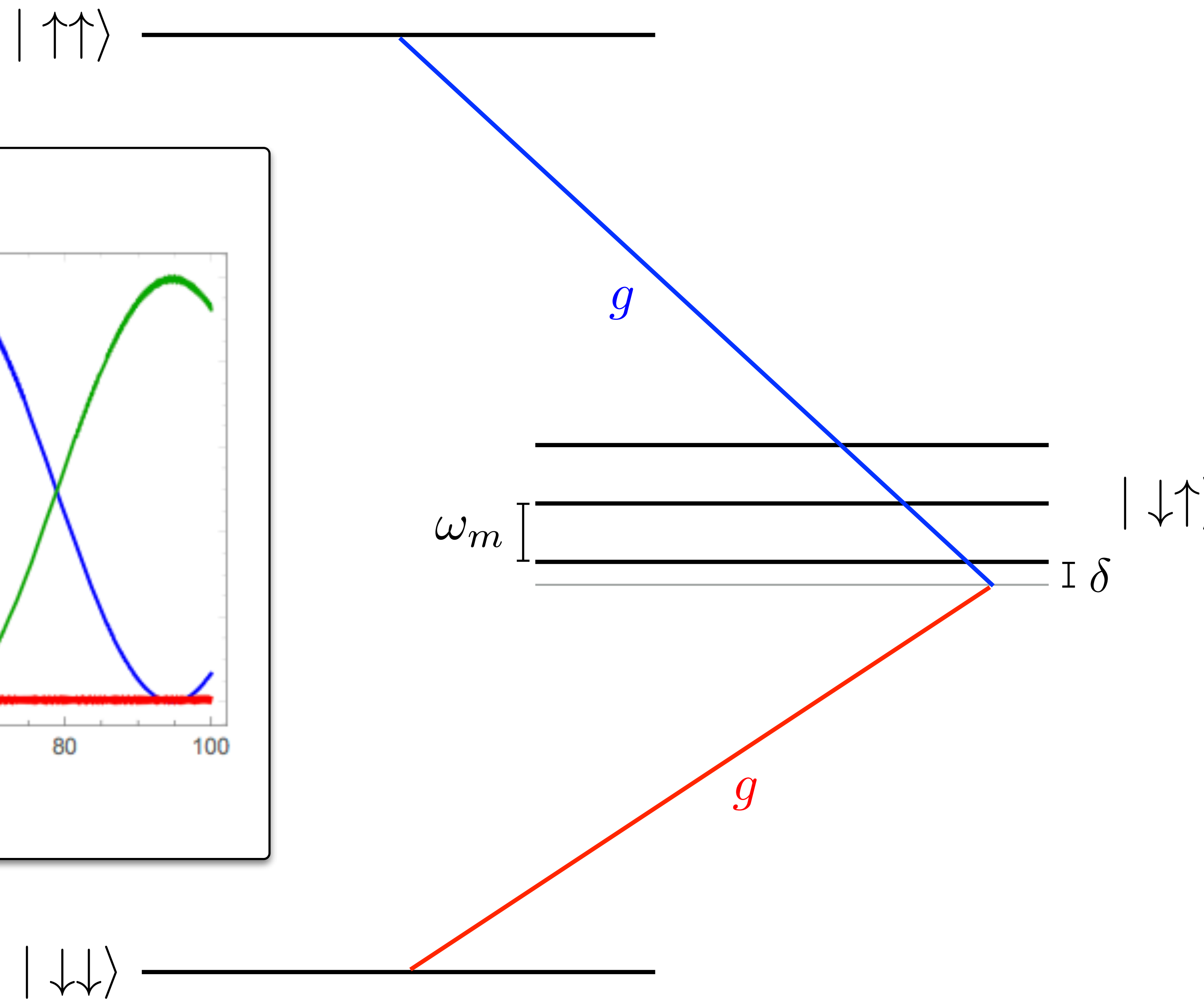
Spin-phonon transitions



TWO-QUBIT ENTANGLING OPERATION

Adiabatic elimination technique and the use of sideband transitions effectively couples two spins





$$H_{\text{eff}} \propto \sigma_x^{(i)} \otimes \sigma_x^{(j)}$$

Cirac and Zoller, Phys.Rev.Lett.74, 4091 (1995),
Sorenson and Molmer, Phys. Rev. A 62, 022311 (2000)

DIGITAL, ANALOG, AND HYBRID MODES OF TRAPPED-ION SIMULATORS

ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph].

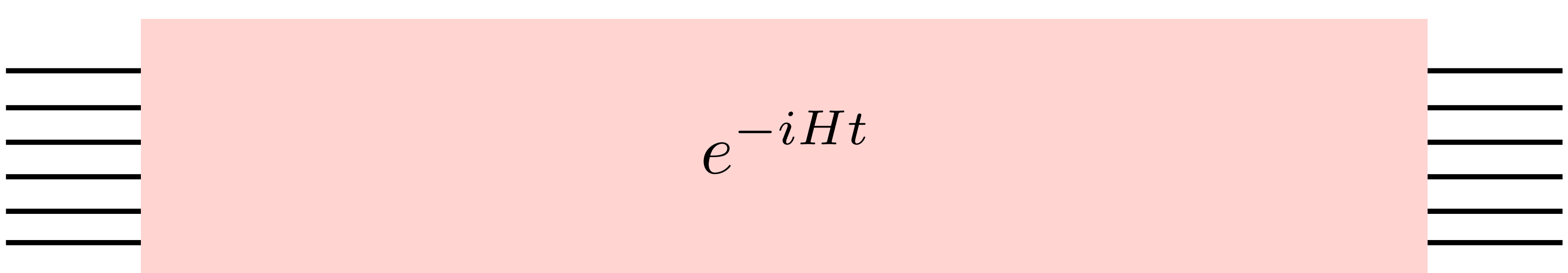
Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv: 2107.xxxx [quant-ph].

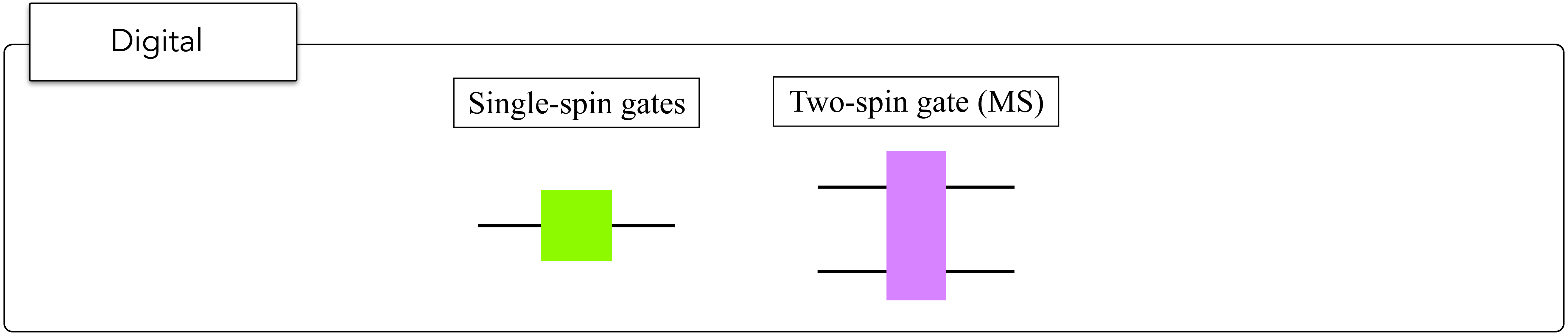
See also: Bermudez et al, Pays.Rev.A79, 060303 R (2009).

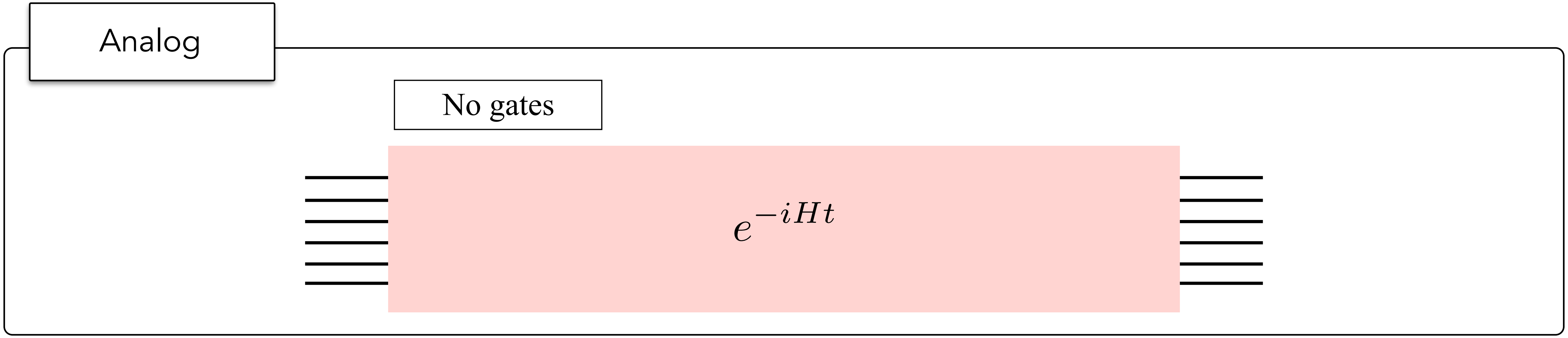
See also interesting work by:
Lamata et al, Phys. Rev. Lett. 98 253005 (2007), Gerritsma et al, Nature 463, 68 (2010), Casanova et al, Phys. Rev. Lett. 107, 260501 (2011), and Bermudez et al, Phys. Rev. X 7, 041012 (2017).

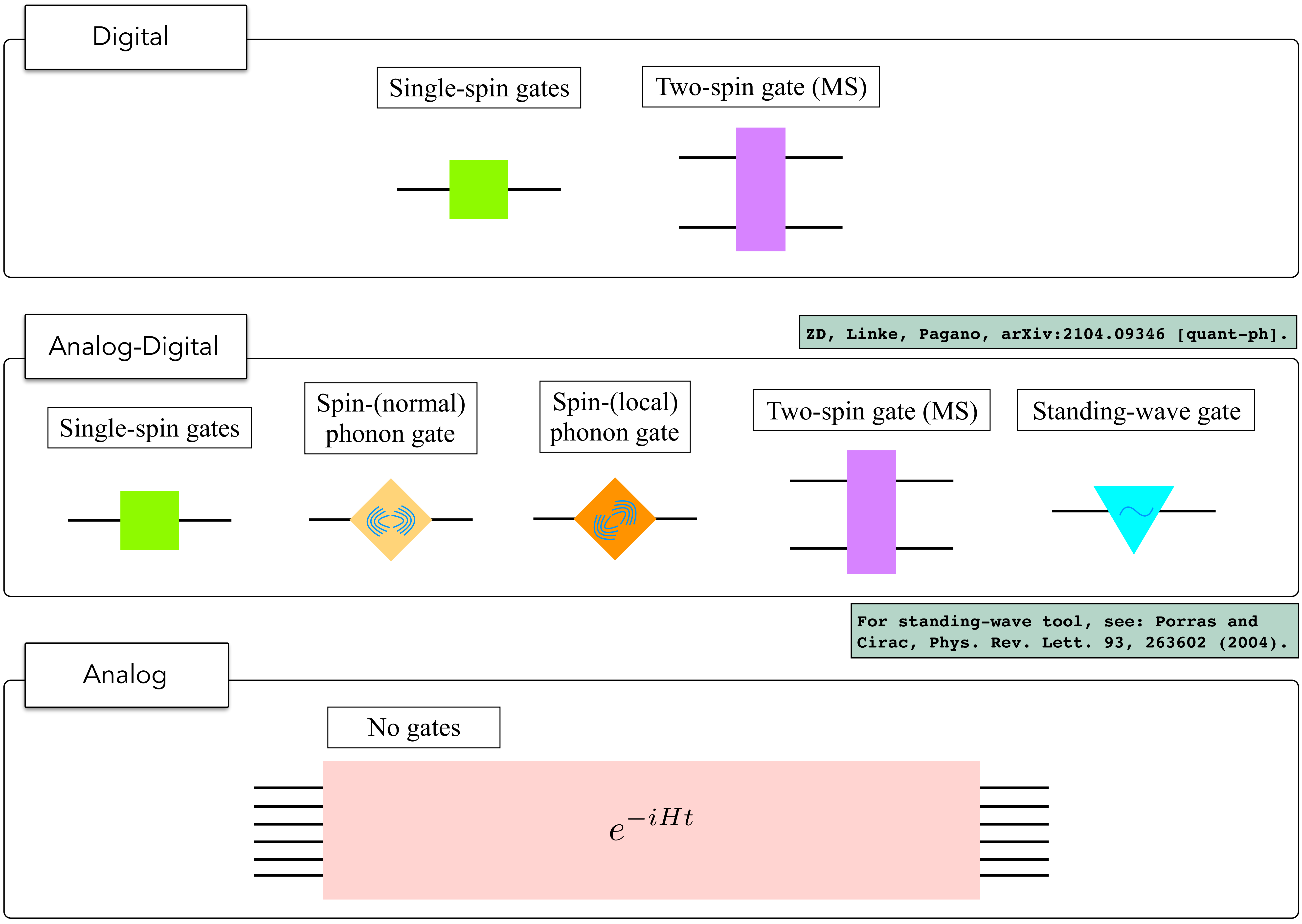
Analog

No gates




Digital
Single-spin gates
Two-spin gate (MS)


Analog
No gates
e^{-iHt}

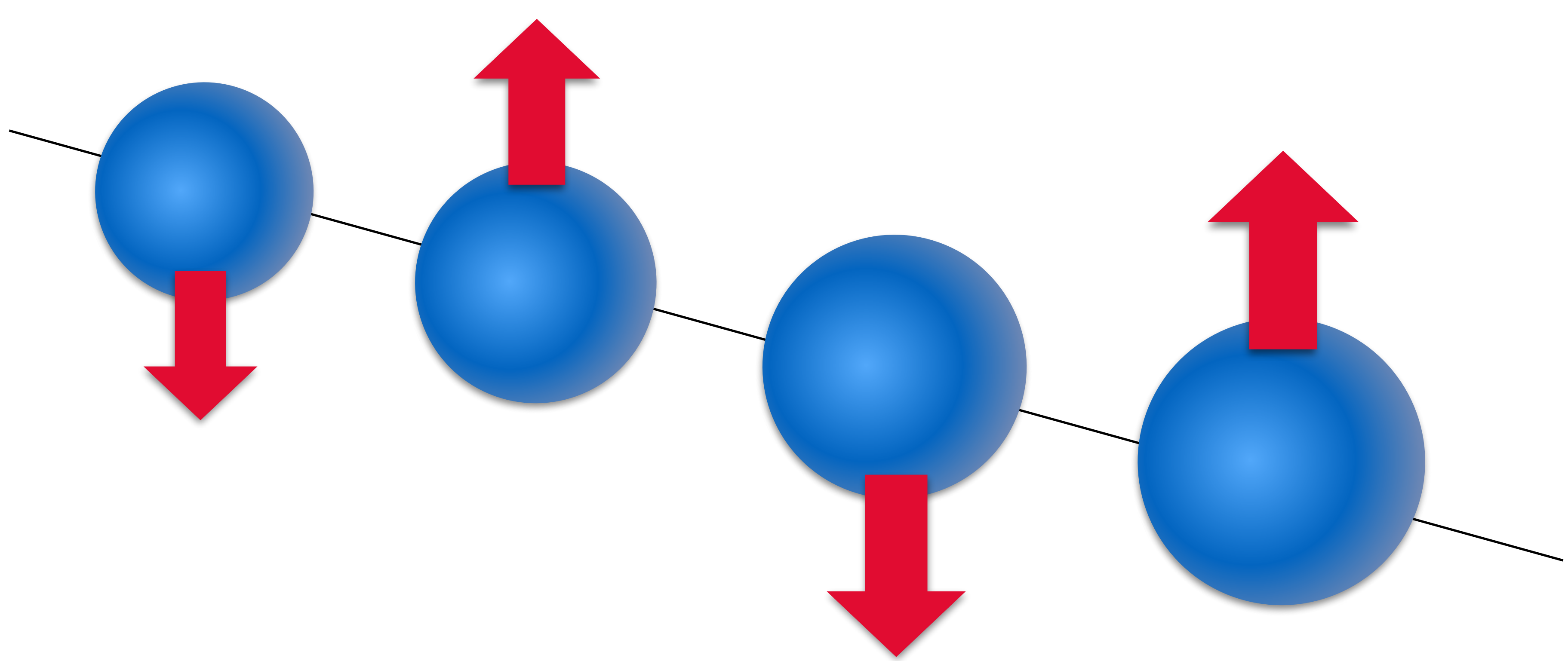


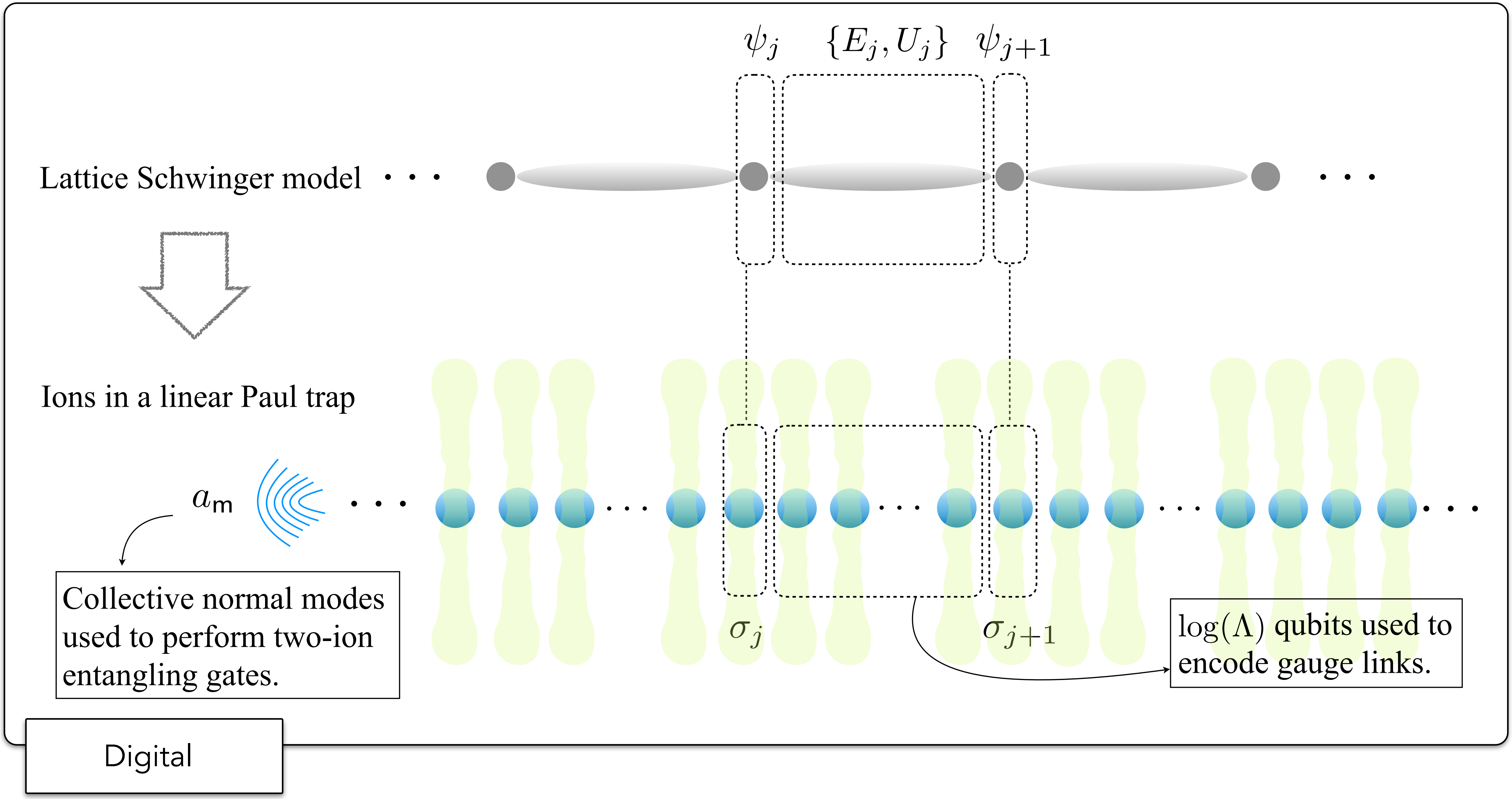
Digital
Single-spin gates
Two-spin gate (MS)
Analog-Digital
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].
Single-spin gates
Spin-(normal)
phonon gate
Spin-(local)
phonon gate
Two-spin gate (MS)
Standing-wave gate
For standing-wave tool, see: Porras and
Cirac, Phys. Rev. Lett. 93, 263602 (2004).
Analog
No gates
e^{-iHt}

A LATTICE GAUGE THEORY EXAMPLE STUDIED WITHIN DIGITAL AND HYBRID MODES OF THE TRAPPED-ION SIMULATOR

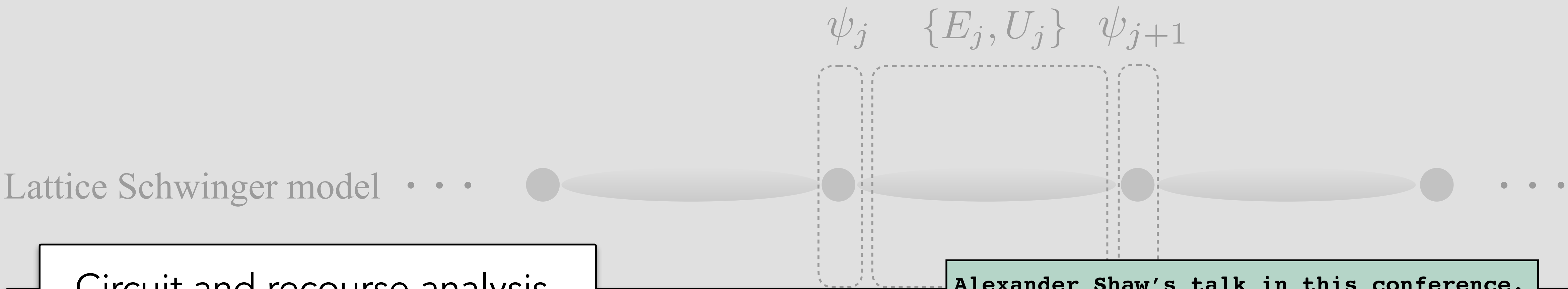
LATTICE SCHWINGER MODEL: A TESTBED FOR QUANTUM SIMULATION OF LATTICE GAUGE THEORIES

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \mathrm{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$





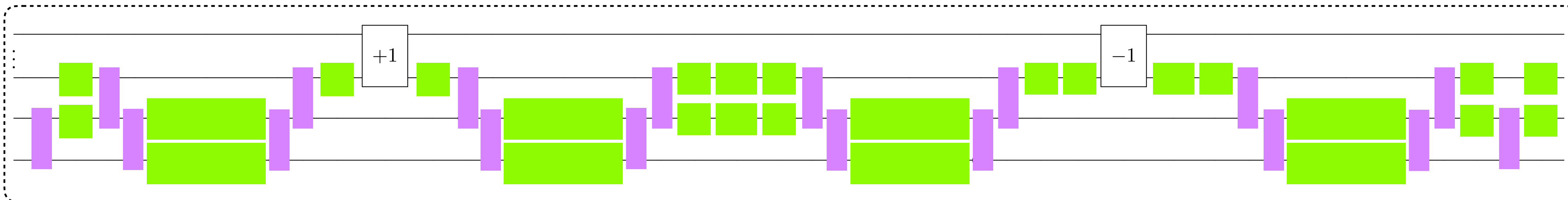
$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$



Circuit and recourse analysis

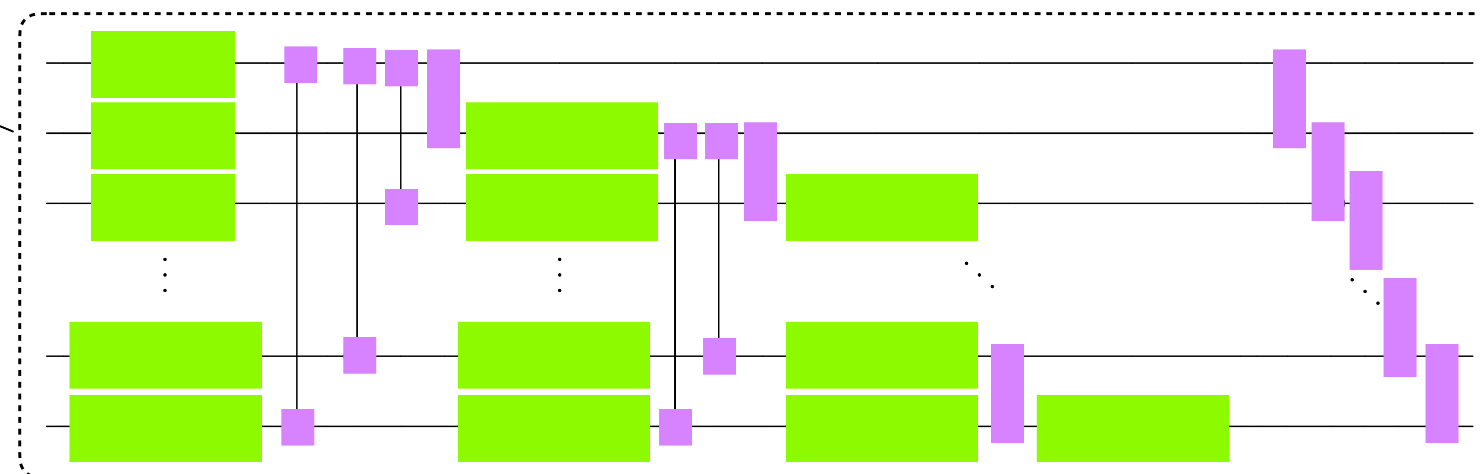
Alexander Shaw's talk in this conference.

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020)



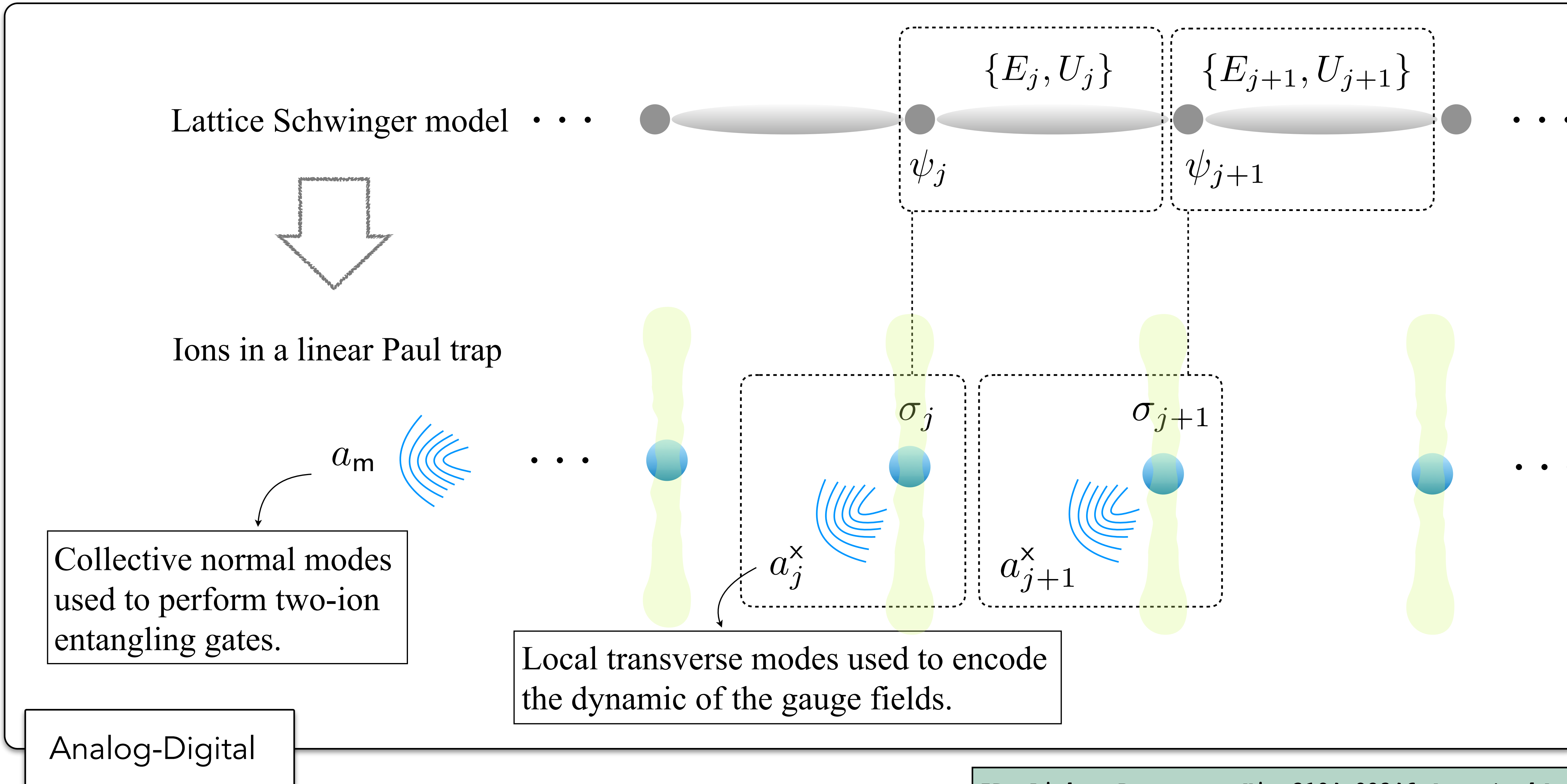
Sample gauge-fermion interaction block

Part of electric field interactions acting on gauge DOF registers



Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT
$x = 10^{-2}$	—	7.3e4	—	1.6e5	—	3.4e5	—	7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$	—	1.6e4	—	3.5e4	—	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
$x = 1$	—	4.6e3	—	9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^2$	—	2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	6.0e4

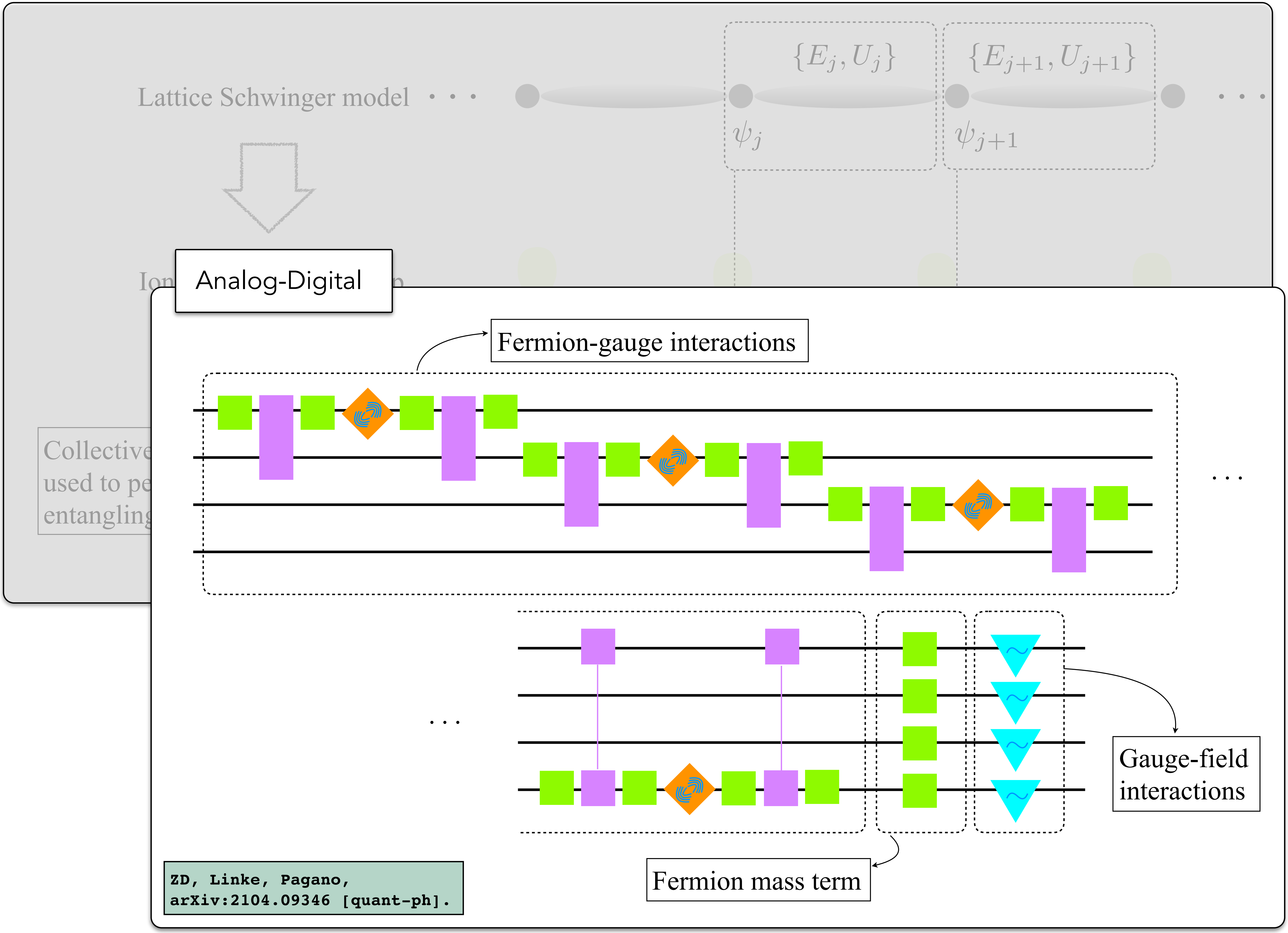


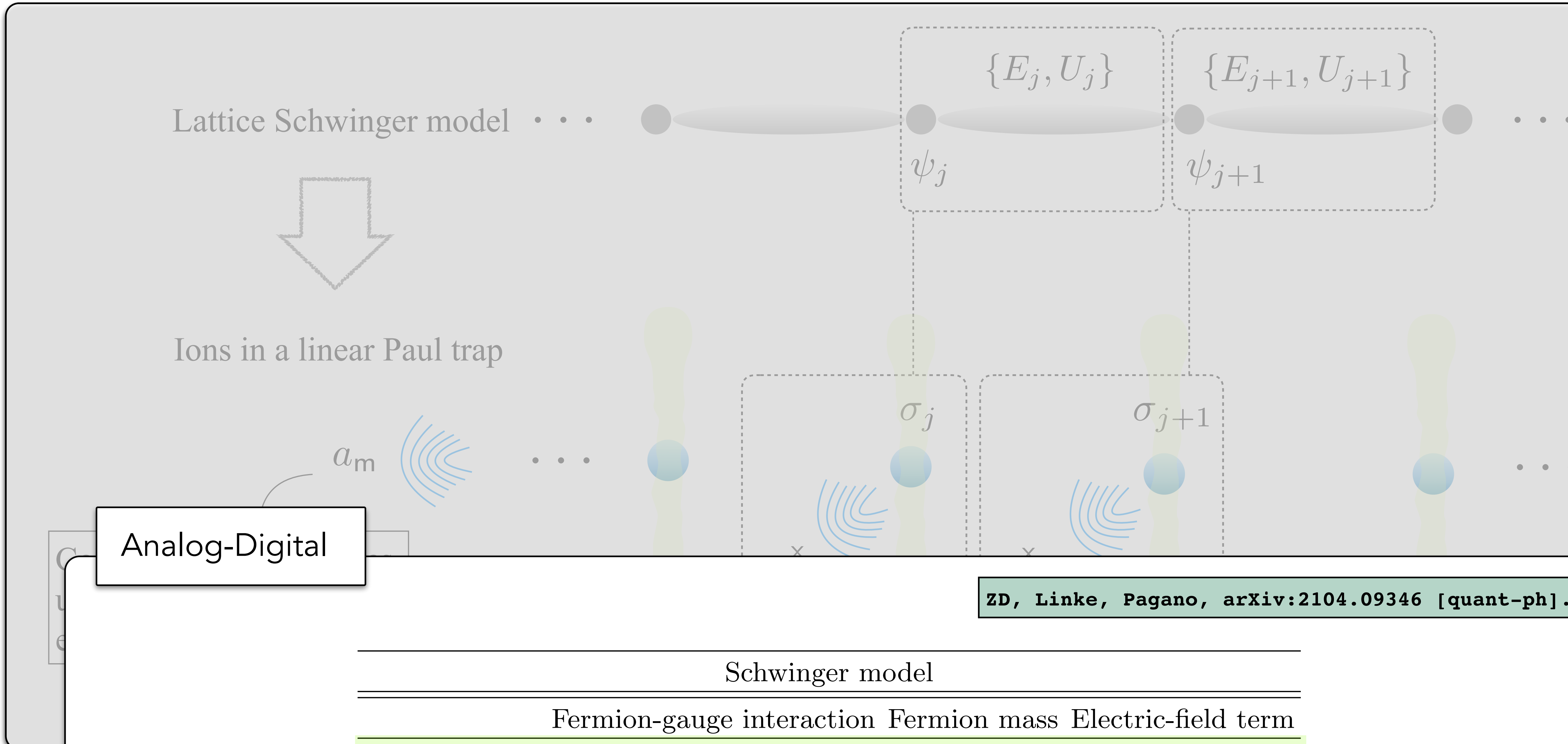
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

See Yang et al, Phys. Rev. A 94, 052321 (2016) for the highly-occupied bosonic model of the Schwinger model.

See also Casanova et al, Phys. Rev. Lett. 108, 190502 (2012), Lamata et al, EPJ Quant. Technol. 1, 9 (2014), and Mezzacapo et al, Phys. Rev. lett. 109, 200501 (2012) for analog-digital approaches to other interacting fermion-boson theories.

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \mathrm{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$


Lattice Schwinger model
$\{E_j, U_j\}$
$\{E_{j+1}, U_{j+1}\}$
ψ_j
ψ_{j+1}
Analog-Digital
Fermion-gauge interactions
Gauge-field interactions
Fermion mass term
ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].



Analog-Digital

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

Schwinger model			
	Fermion-gauge interaction	Fermion mass	Electric-field term
Analog-digital	$\mathcal{O}\left(N\right)$	$\mathcal{O}\left(1\right)$	$\mathcal{O}(N)$
Digital	$\mathcal{O}\left(N^2\left(\log\Lambda\right)^2\right)$	$\mathcal{O}\left(1\right)$	$\mathcal{O}\left(N\left(\log\Lambda\right)^2\right)$

Let us compare the circuit structure of digital and analog-digital cases when gauge DOF are present.

A SCALAR FIELD THEORY COUPLED TO FERMIONS (YUKAWA THEORY) WITH THE HYBRID MODE OF THE SIMULATOR

$$H^{(I)}_{\text{Yukawa}} = \sum_{j=1}^{N} \left[\frac{i}{2b}(\psi_j^\dagger \psi_{j+1} - \psi_{j+1}^\dagger \psi_j) + m_\psi (-1)^j \psi_j^\dagger \psi_j \right]$$

$$H^{(II)}_{\text{Yukawa}} = b \sum_{j=1}^{N} \left[\frac{\Pi_j^2}{2} + \frac{(\nabla \varphi_j)^2}{2} + \frac{m_\varphi^2}{2} \varphi_j^2 \right]$$

$$H^{(III)}_{\text{Yukawa}} = gb \sum_{j=1}^{N} \psi_j^\dagger \varphi_j \psi_j,$$

Model Hamiltonian

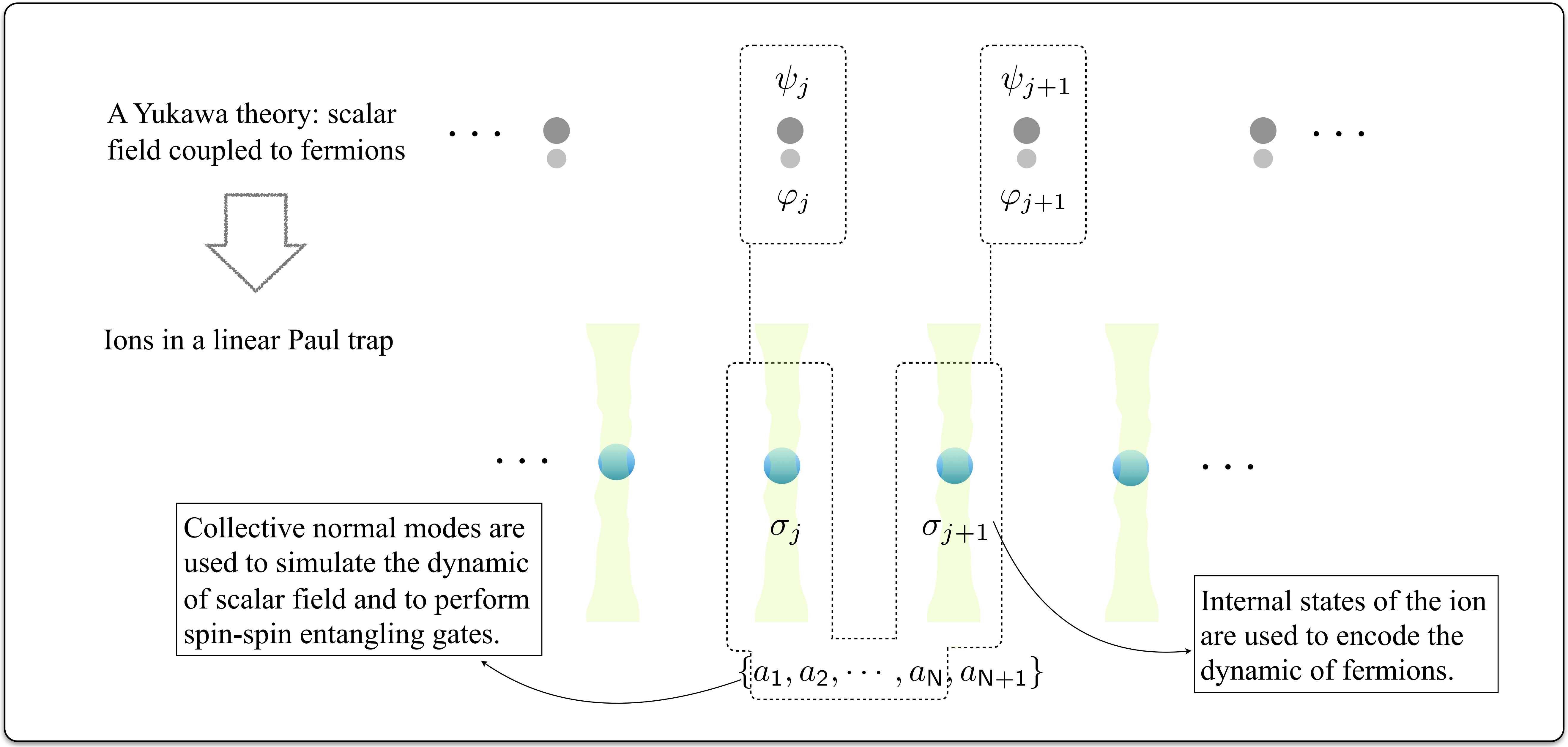
Trapped-ion Hamiltonian

$$H^{(I)'}_{\text{Yukawa}} = \frac{1}{4b} \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x,$$

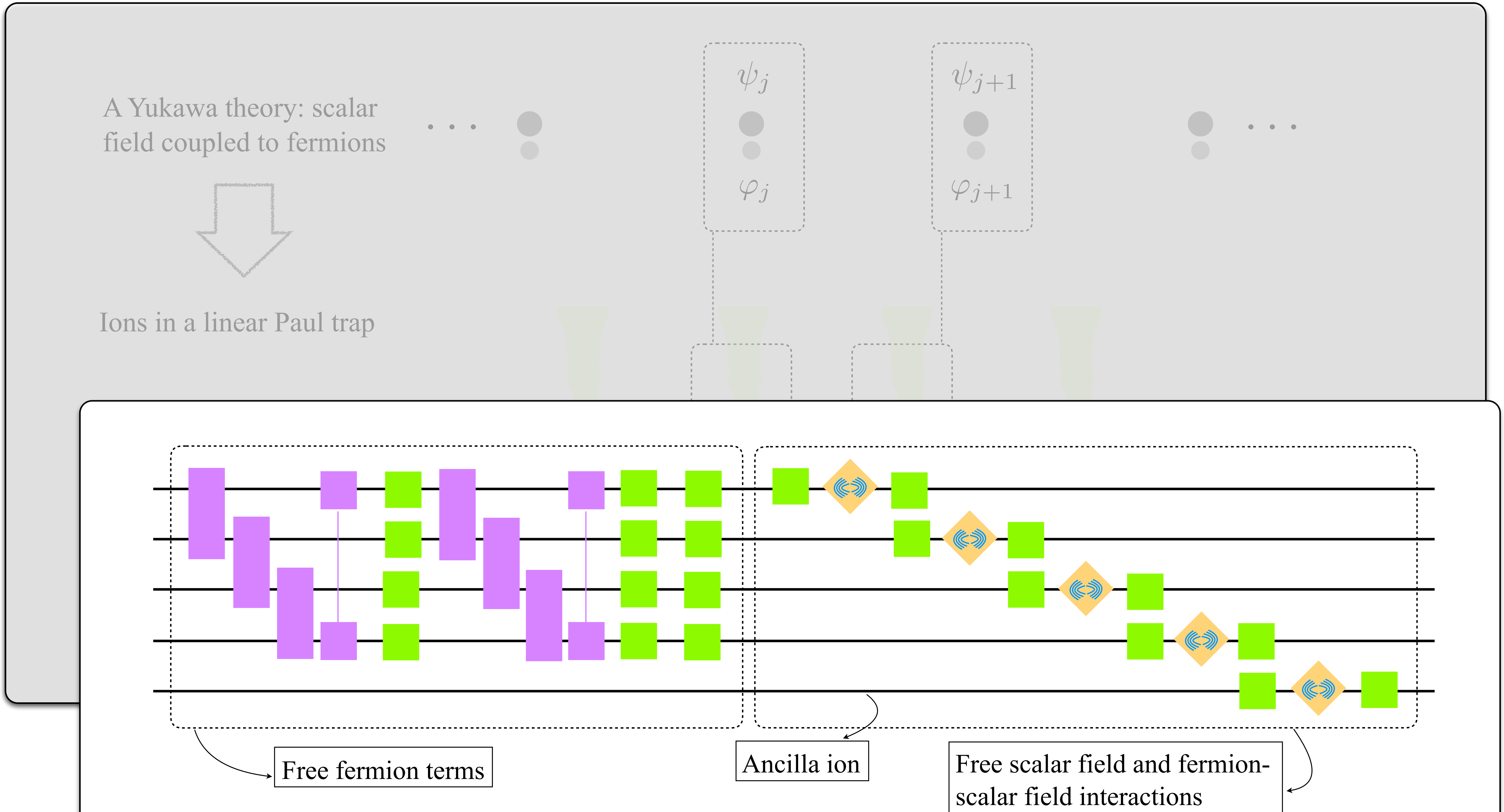
$$H^{(II)'}_{\text{Yukawa}} = \frac{1}{4b} \sum_{j=1}^{N} \sigma_j^y \sigma_{j+1}^y,$$

$$H^{(III)'}_{\text{Yukawa}} = \frac{m_\psi}{2} \sum_{j=1}^{N} (-1)^j \sigma_j^z + \text{const.},$$

$$H^{(IV)'}_{\text{Yukawa}} = \sqrt{\frac{g^2 b}{8N}} \sum_{j=1}^{N} (\mathbb{I}_j + \sigma_j^z) \sum_{\mathsf{m}=1}^{N} \frac{1}{\sqrt{\varepsilon_\mathsf{m}}} \times$$
$$(a_\mathsf{m}^\dagger e^{-i\frac{2\pi j}{N}(\mathsf{m}-\frac{N}{2}-1)} + a_\mathsf{m} e^{i\frac{2\pi j}{N}(\mathsf{m}-\frac{N}{2}-1)}) + \sum_{\mathsf{m}=1}^{N} \varepsilon_\mathsf{m} (a_\mathsf{m}^\dagger a_\mathsf{m} + \frac{1}{2}).$$



ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

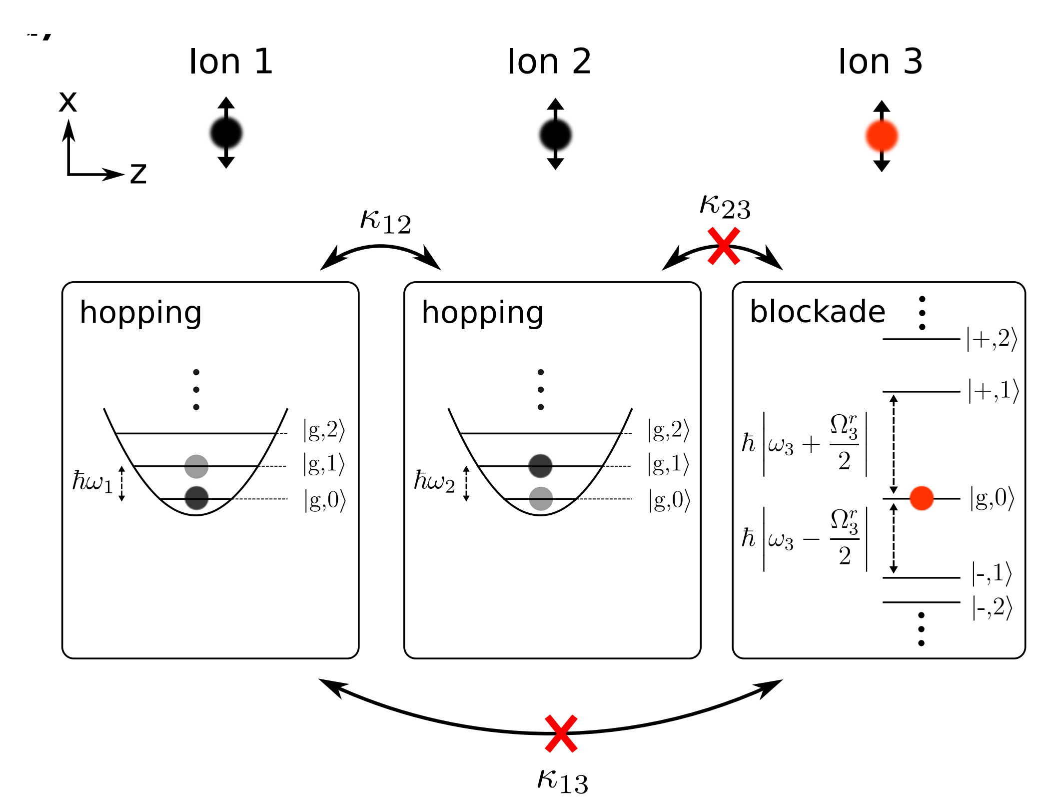


Yukawa theory				
	Fermion hopping	Fermion mass	Free scalar fields	Fermion scalar-field interaction
Analog-digital	$\mathcal{O}(N)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Digital	$\mathcal{O}(N)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}\left(N^2 (\log \Lambda)^2\right)$

ZD, Linke, Pagano, arXiv:2104.09346 [quant-ph].

IS PHONON CONTROL EXPERIMENTALLY FEASIBLE? YES…AT LEAST FOR SMALL SYSTEMS SO FAR!

Phonon hopping and blockade demonstrated in trapped-ion simulators:



Debnath et al, Phys. Rev. Lett. 120, 073001 (2018).

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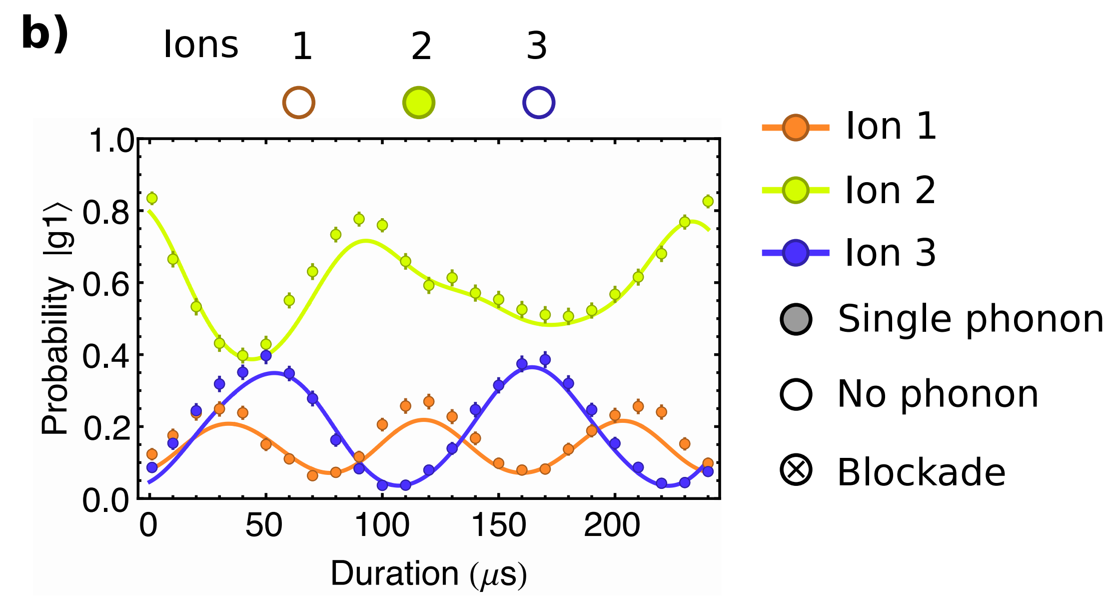
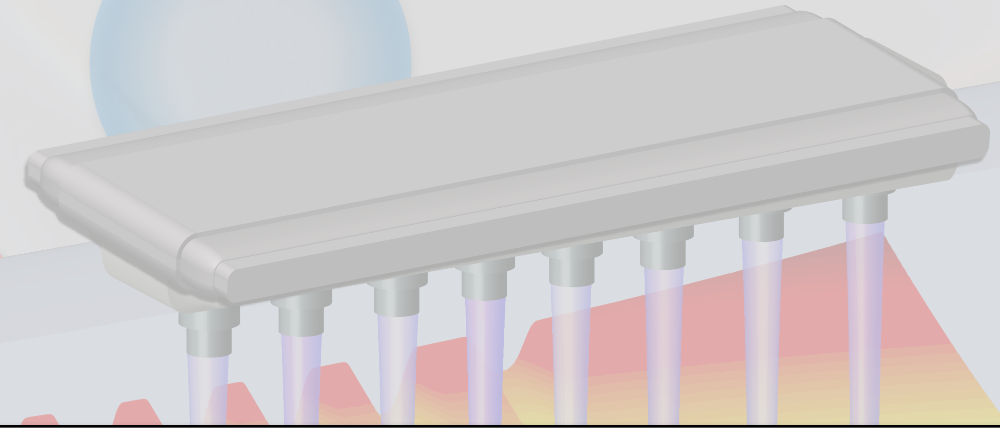
Monroe-Linke Group





SUMMARY

Trapped-ion simulators have become a successful platform for quantum simulation of many-body physics. **Monroe et al, Rev. Mod. Phys. 93, 25001 (2021).**

The first successful implementations of gauge-field theory dynamics on trapped-ion simulators/computers have emerged for small systems. **Martinez et al, Nature 534, 516 EP (2016).**

Phonons can be manipulated and measured in these systems, hence the possibility of using phonons as both virtual and dynamical degrees of freedom is realistic.

Simulating complex dynamics of quantum field theories may benefit from digital, analog, and hybrid implementations depending on the problem at hand.

THANK YOU