Quantum simulation of quantum mechanics with a theta-term for an 't Hooft anomaly

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Simulating Quantum Mechanics with a θ-term and an 't Hooft Anomaly on a Synthetic Dimension

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θ-term: a sign problem

massive charge-n Schwinger model (QED_2):

$$\overline{\psi}(\gamma_\mu(\partial_\mu + inA_\mu) + m)\psi + \frac{1}{4e^2}F_{\mu\nu}F_{\mu\nu} + \frac{i\theta}{4\pi}\epsilon_{\mu\nu}F_{\mu\nu}$$

4d SU(n) Yang-Mills theory:

$$\frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu} + \frac{i\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Quantum simulation?

Spatially dimensionally reducing to a QM problem

Quantum mechanics on a circle: an avatar

$$S_\theta[x] = \int d\tau \left[\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + \lambda(1-\cos(nx)) - \frac{i\theta}{2\pi}\frac{dx}{d\tau}\right]$$

Davide Gaiotto et al., JHEP 2017, 91

Yuta Kikuchi, and Yuya Tanizaki, Prog. Theor. Exp. Phys. (2017) 113B05

$x \sim x + 2\pi$

$\theta \sim \theta + 2\pi$

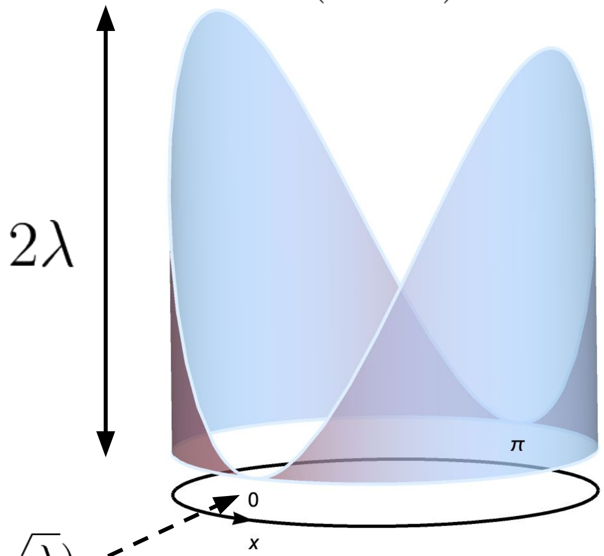
$\mathbb{Z}_n$ symmetry: $x \mapsto x + \frac{2\pi}{n}$

at $\theta = 0$ or π:

$\hat{T}$-symmetry:

$\tau \mapsto -\tau,\ x(\tau) \mapsto x(-\tau),\ \dot{x}(\tau) \mapsto -\dot{x}(-\tau)$

$(n = 2)$



$$\sim \frac{1}{2}\omega^2 x^2\ (\omega = n\sqrt{\lambda})$$

’t Hooft anomaly, and global inconsistency with θ

$(n \text{ even}, \ \theta = \pi)$

global sym. $\mathbb{Z}_n$

global sym. $\hat{T}$

gauging $\mathbb{Z}_n$
(gauge field introduced) ⇒

gauge sym. $\mathbb{Z}_n$

~~global sym. $\hat{T}$~~

implies ⇒

(mixed) ’t Hooft anomaly between $\mathbb{Z}_n$ & $\hat{T}$

realized by ⇓

nontrivial vacuum structure (e.g., spontaneous symmetry breaking: degenerate GS)

In charge-n massive 2d Schwinger model or 4d YM:

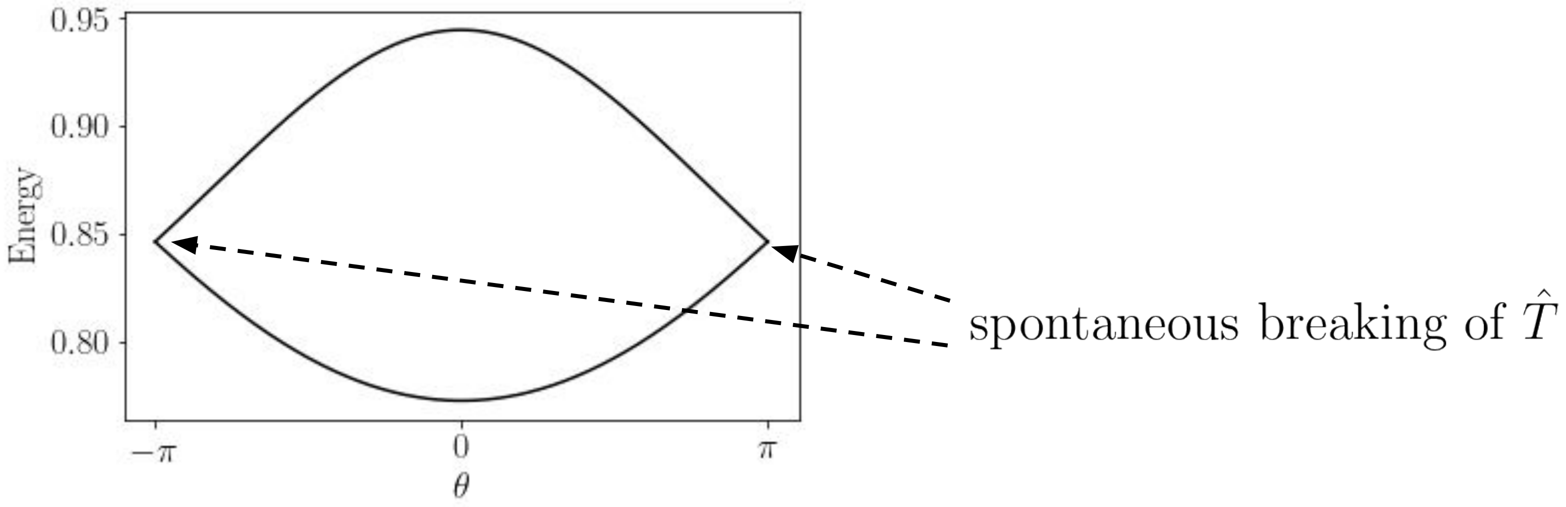
$\mathbb{Z}_n$: generalized global 1-form symmetry

$\hat{T} \sim$ time-reversal symmetry (CP for 4d)

Gaiotto et al., JHEP 2015, 172

Energy spectrum

$$n = 2,\ \omega = 2$$



’t Hooft anomaly, and global inconsistency with θ

n odd

global inconsistency:
midler than ’t Hooft anomaly

some nontrivial
vacuum structure

Tunneling: instanton-anti-instanton interference

one-instanton tunneling amplitude

$$\langle x=\pi|e^{-H\mathcal{T}}|x=0\rangle$$
$$\propto \exp\left(-S_I-\frac{i\theta}{2}\right)+\exp\left(-S_I+\frac{i\theta}{2}\right)$$
$$=2\exp\left(-S_I\right)\cos\left(\frac{\theta}{2}\right)$$

vanishes at $\theta=\pi$

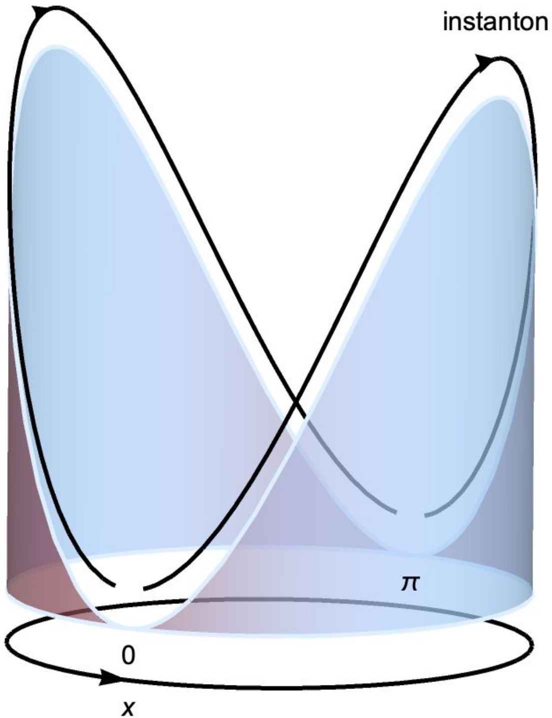
consistent with degenerate GS

$\exp\left(-S_I-\frac{i\theta}{2}\right)$

anti – instanton

$\exp\left(-S_I+\frac{i\theta}{2}\right)$

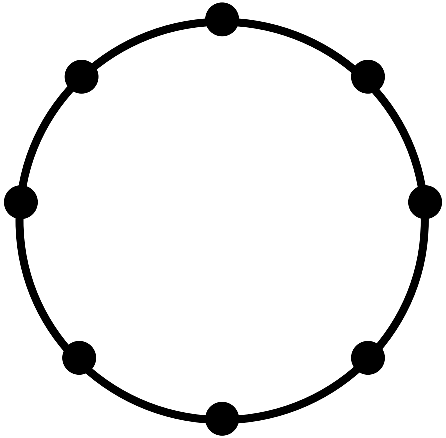
instanton



Discretization

$$S_\theta[x] = \int d\tau \left[\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + \lambda(1 - \cos(nx)) - \frac{i\theta}{2\pi}\frac{dx}{d\tau} \right]$$

$$H = \frac{1}{2}\left(p - \frac{\theta}{2\pi}\right)^2 + \lambda\left(1 - \cos\left(nx\right)\right)$$



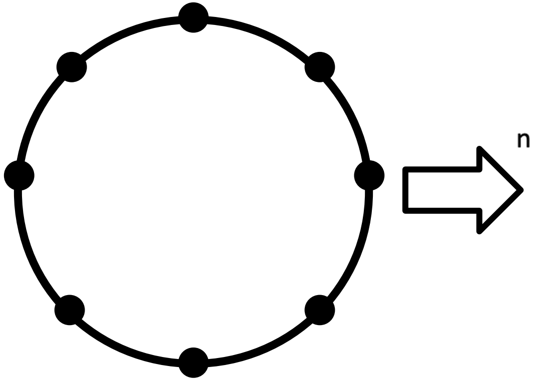
n_s lattice sites
with PBC

$$H = \sum_i \left[w_{i,i+1} b^\dagger_{i+1} b_i + w^*_{i,i+1} b^\dagger_i b_{i+1} + V_i b^\dagger_i b_i \right]$$ (tight-binding model)

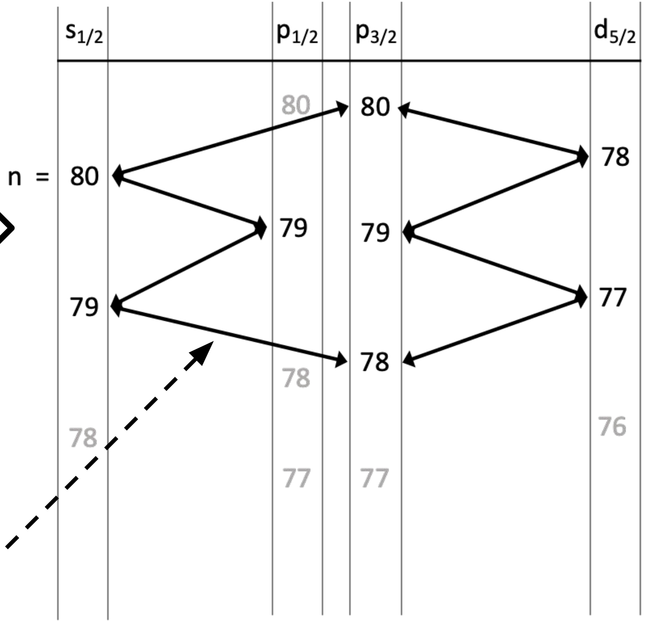
$$w_{i,i+1} \propto \exp\left(\frac{i\theta}{2\pi n_s}\right) \qquad \theta = \arg \prod_i w_{i,i+1} \pmod{2\pi}$$

Rydberg synthetic dimension

“1d” encoded in “0d”



atomic orbitals

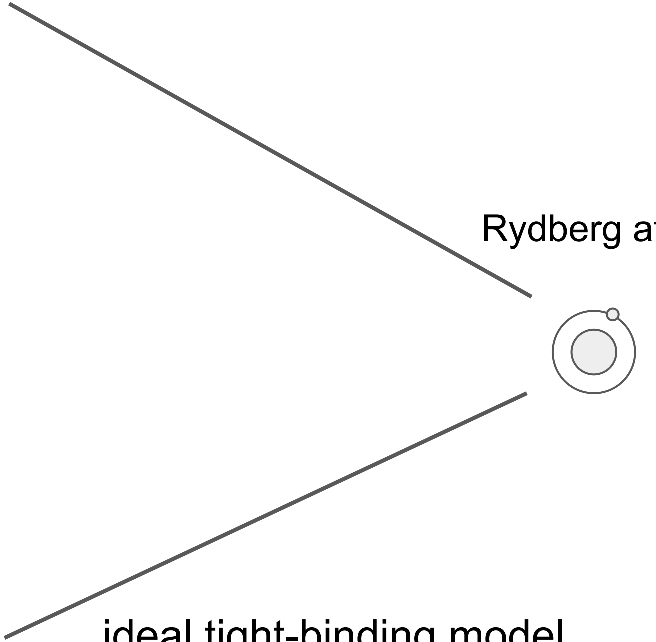


coupled by microwaves

$\mathbb{Z}_n$ potential from microwave detuning

θ from relative microwave phases

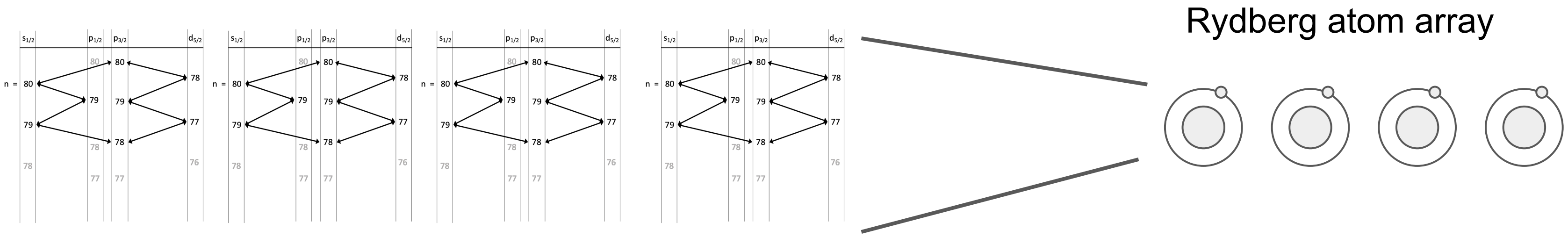
Rydberg atom



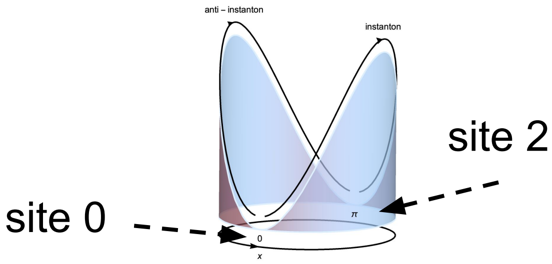
ideal tight-binding model obtained by rotating wave approximation (RWA)

Rydberg synthetic dimension

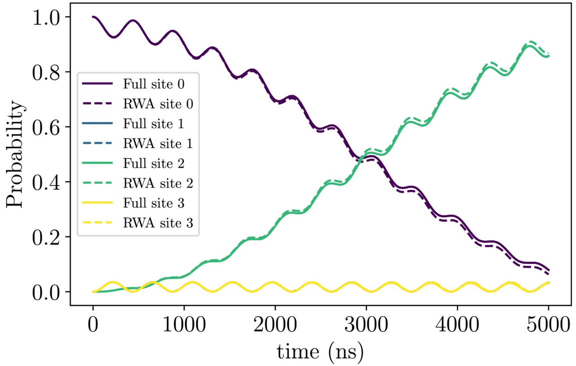
"2d" encoded in "1d"?



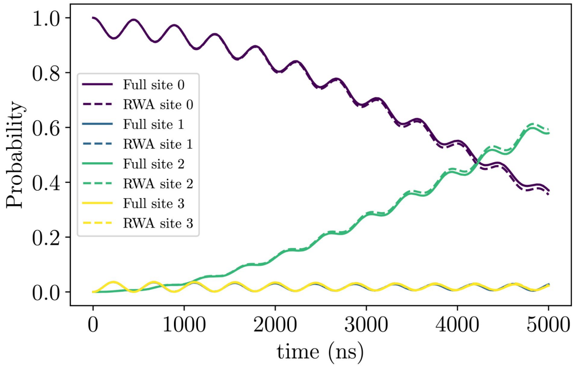
Numerical results from real-time dynamics



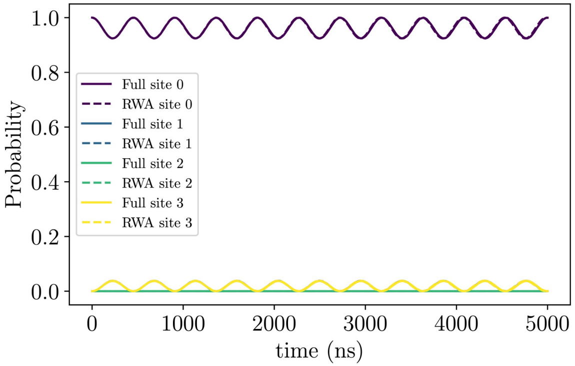
$n = 2,\ \omega = 2 \qquad n_s = 4 \text{ sites}$



$\theta = 0$



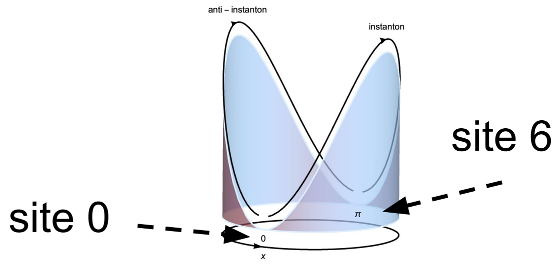
$\theta = \pi/2$



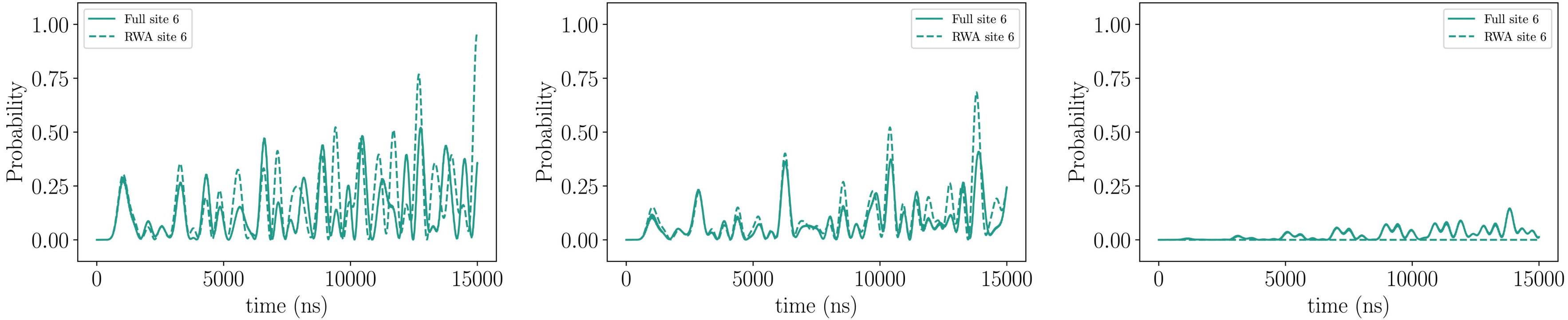
$\theta = \pi$

$$P_\theta\left(0 \to 0; t\right) \approx \frac{1}{2}\left(1 + \cos\left(\omega_{\text{tun}} t\right)\right) \qquad \omega_{\text{tun}} \propto \cos\left(\theta/2\right)$$

Numerical results from real-time dynamics



$n = 2,\ \omega = 2.5 \qquad n_s = 12$ sites



$\theta = 0 \qquad \theta = \pi/2 \qquad \theta = \pi$

$$P_\theta\left(0 \to 0; t\right) \approx \frac{1}{2}\left(1 + \cos\left(\omega_{\text{tun}} t\right)\right) \qquad \omega_{\text{tun}} \propto \cos\left(\theta/2\right)$$

(fast oscillations due to overlap with highly excited states)

Conclusions and outlook

- QM on the circle with θ can have ’t Hooft anomaly/global inconsistency and quantum tunneling in real-time.
- Toward quantum simulations of QFTs with θ.
- Weak-coupling charge-n Schwinger model dimensionally reduces to QM.
- A Rydberg synthetic dimension encodes 1d using 0d.
- Our results demonstrate possibility of an analog quantum simulation experiment with realistic experimental parameters.
- This work may be generalized to more interesting theories, including 4d Yang-Mills.