Tensor Network simulation of strong coupling U(N)

Pascal Milde & Jacques Bloch & Robert Lohmayer

University of Regensburg

Lattice symposium, 30 July 2021

Introduction

- Apply HOTRG methods to strong coupling U(N) in 3d and 4d
- Reproduce MC results of Rossi and Wolff (1984) (Metropolis) and of Adams and Chandrasekharan (2003) (Directed path algorithm) $\rightarrow$ theory of dimers and monomers
- Spontaneous chiral symmetry breaking at small masses

Partition Function

$$Z = \int D\Psi D\bar{\Psi} dU \exp(S(\bar{\Psi}, \bar{\Psi}, U))$$

with action $S(\bar{\Psi}, \bar{\Psi}, U)$.

$$S(\bar{\Psi}_a, \bar{\Psi}_a, U) = \frac{1}{2} \sum_{\mu,x} \Gamma_\mu(x)[\bar{\Psi}(x)U_\mu(x)\Psi(x+\hat{\mu}) - \bar{\Psi}(x+\hat{\mu})U_\mu(x)^\dagger \Psi(x)]$$
$$+m \sum_x \bar{\Psi}(x)\Psi(x)$$

- $\bar{\Psi}(x), \Psi(x)$ N-dimensional fermion vectors
- $\Gamma_\mu(x)$ staggered phase factor, including temperature in time parameter t
- $U_\mu(x) \in U(N)$
- m fermion mass

Tensorization of U(N) model

Dualization analogous to Rossi and Wolff(1984):

$$Z(m) = \sum_{[k]} \prod_x T^{(x)}_{k_{x,1},k_{x-\hat{1},1},\dots,k_{x,d},k_{x-\hat{d},d}}(m)$$

with local tensor

$$T^{(x)}_{k_{x,1},k_{x-\hat{1},1},\dots,k_{x,d},k_{x-\hat{d},d}}(m) = \left(\prod_{\mu}^{d} \sqrt{\alpha_{k_{x,\mu}}}\sqrt{\alpha_{k_{x-\hat{\mu},\mu}}}\right) \frac{1}{2^{\sigma_x}} \frac{N!}{(N-\sigma_x)!} m^{N-\sigma_x}$$
$$t^{k_{x,\hat{1}}+k_{x-\hat{1},\hat{1}}} \Theta(N-\sigma_x)$$

where

- $\sigma_x = \sum_{\mu}^{d} k_{x,\mu} + k_{x-\hat{\mu},\mu}$
- $\alpha_k = \frac{(N-k)!}{k!N!}$
- $\sum_{[k]} \equiv$sum over all tuples
- t temperature parameter

Tensor Network

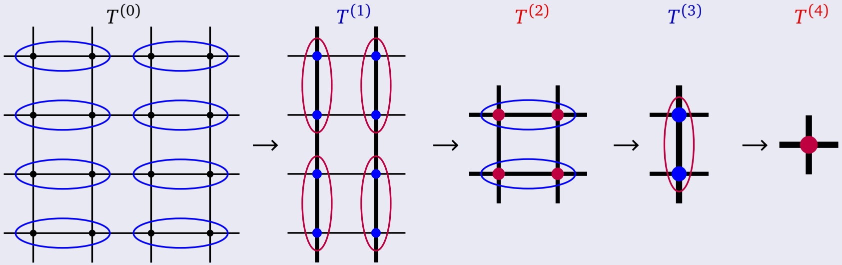
- Statistical physics: d-dimensional tensor network, which gets contracted over all indices to yield one number: the partition function or an impurity.
- Tensor network consists of V **identical** tensors T order $2d$.
- HOTRG (higher order tensor renormalization group, Xie et al. 2012): recursively reduce the number of local tensors by factor two by contracting pairs of adjacent tensors and truncating the emerging higher order tensor using modified HOSVD (higher order singular value decomposition).
- HT-HOTRG faster and cheaper approximation of HOTRG by transforming initial tensor

$$T_{tt'xx'yy'zz'} \approx \sum_{abcd} B^{(t)}_{tt'a} B^{(x)}_{xx'b} B^{(y)}_{yy'c} B^{(z)}_{zz'd} C_{abcd}$$

and factorize also at every step.

HOTRG in 2d

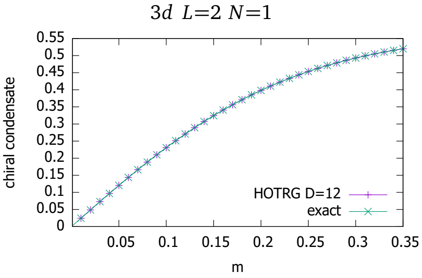
Alternating contractions

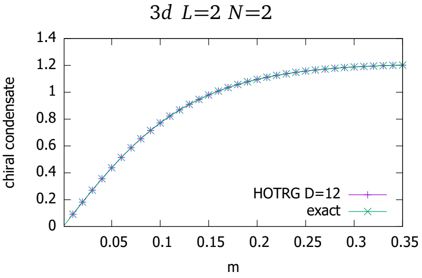


Possible contraction orderings

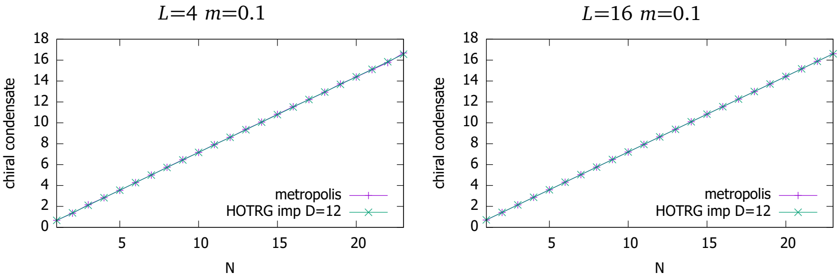
- alternating contractions (as above)
- spatial or temporal blocking: completely contract one direction → reduce order of tensor by 2
- improved contraction ordering (ICO): contract the direction yielding the smallest HOSVD truncation error

Comparison to exact results (Adams & Chandrasekharan, 2003)





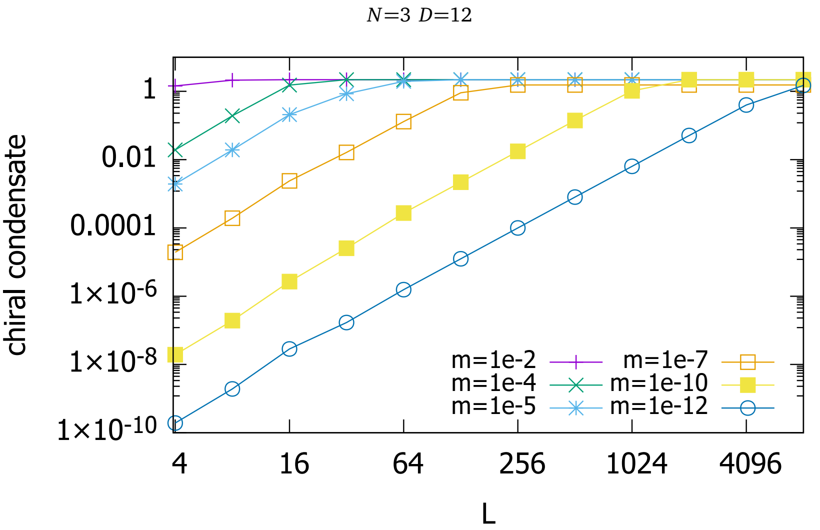
N-dependence 3d



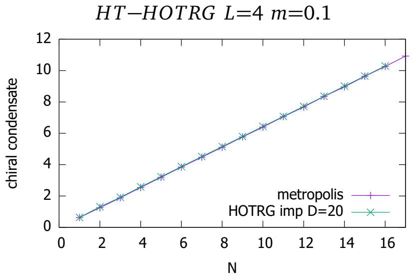
$\lim_{N\to\infty} \frac{<\bar{\Psi}\Psi>}{N}$

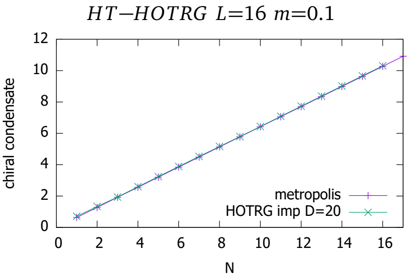
	L=4	L=16
Metropolis	0.723(2)	0.72311(4)
HOTRG (D=12)	0.7228(1)	0.7218(4)

Spontaneous symmetry breaking 3d



N-dependence 4d

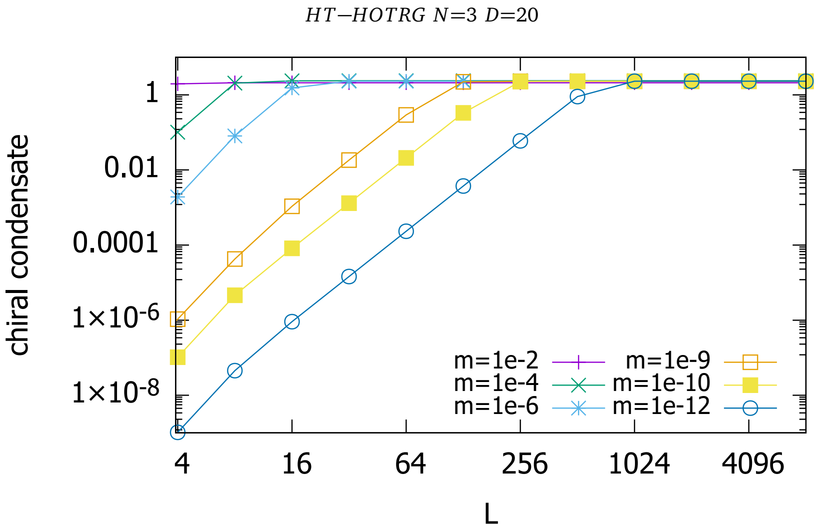




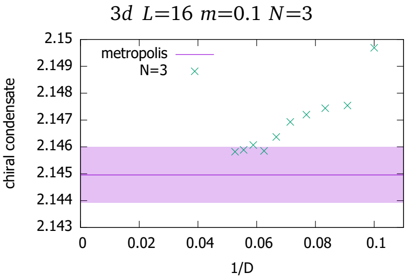
$\lim_{N\to\infty} \frac{<\bar{\Psi}\Psi>}{N}$

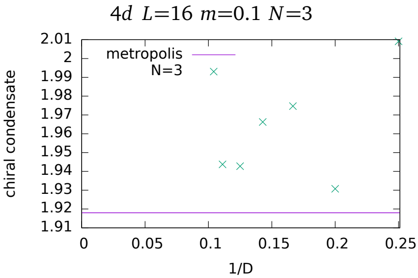
	L=4	L=16
Metropolis	0.6438(6)	0.64279(1)
HT-HOTRG (D=20)	0.644(2)	0.637(2)

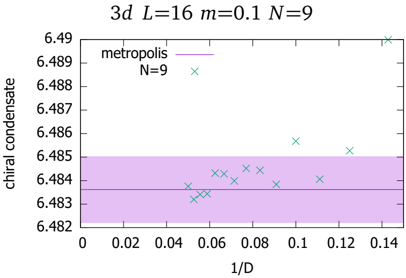
Spontaneous symmetry breaking 4d

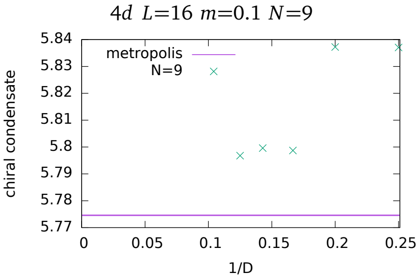


D-dependence 3d and 4d









Summary

Summary

- Applied HOTRG on strong coupling U(N)
- HOTRG verified for small lattice
- Spontaneous symmetry breaking verified
- In 4d additional HT approximation reduces complexity

Outlook

Improvements and extensions

- Apply tensor networks to strong coupled SU(3) (Christina Jäger)
- Improve efficiency 4d → go to larger D
- Further validation of the HT-HOTRG factorization to improve efficiency of 3d and 4d

Issues

- What affects the accuracy?
  - Mainly due to choice of D?
  - Is the additional HT-truncation affecting accuracy?
  - How large are the numerical rounding effects? These may grow with D.