Tensor Network Simulations of a Manifestly Gauge-invariant LGT in 1+1D

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Motivation

(Long-term) Goal: Q. Simulation of QCD = SU(3) Lattice Gauge Theory (LGT) in 3+1 D

(Near-term) Goal: Simulate SU(2) Lattice Gauge Theory (LGT) in 1+1, 2+1 D

Traditional approach: Path-integral Monte Carlo in "Euclidean" spacetime.

Issue: Sign problem, which limits access to

- Finite density systems
- Real-time dynamics

Alternative approach: Tensor networks

- TN simulation of Schwinger model
- TN simulation of SU(2) LGT in 1+1 D

Tensor Networks

1+1d QED/Zn

Banuls et al (2013) JHEP 2013(11), 158 & PRD 92 (2015) 034519 & PRD 93 (2016), 094512.
Zapp et al (2017) PRD, 95(11), 114508.
Magnifico PRD 99, no. 1 (2019): 014503 & PRB 100 (2019): 115152.
Butt et al (2020) PRD, 101(9), 094509.
....

2+1d QED/Zn TN

Robaina et al arXiv:2007.11630 (2020) Physical Review Letters, 126 (2021),
Emonts et al PRD 102, no. 7 (2020): 074501.
...

3+1d QED TN

Magnifico et al arXiv:2011.10658 (2020).
....

Weichselbaum Annals of Physics 327 (2012): 2972-3047.
Kühn et al JHEP (2015) 130, 553
Silvi et al Quantum 1 (2017): 9 & PRD 100, (2019): 074512.
Banuls PRX 7, no. 4 (2017): 041046.
....

SU(2) LGT

Hamiltonian formulation:

$$H = H_E + H_M + H_I + H_B$$

$$H_E = \frac{g^2 a}{2} \sum_{n,i} E^2(n,i)$$

$$H_M = m \sum_{n,i,\alpha} (-1)^n \psi^{\alpha\dagger}(n) \psi^\alpha(n)$$

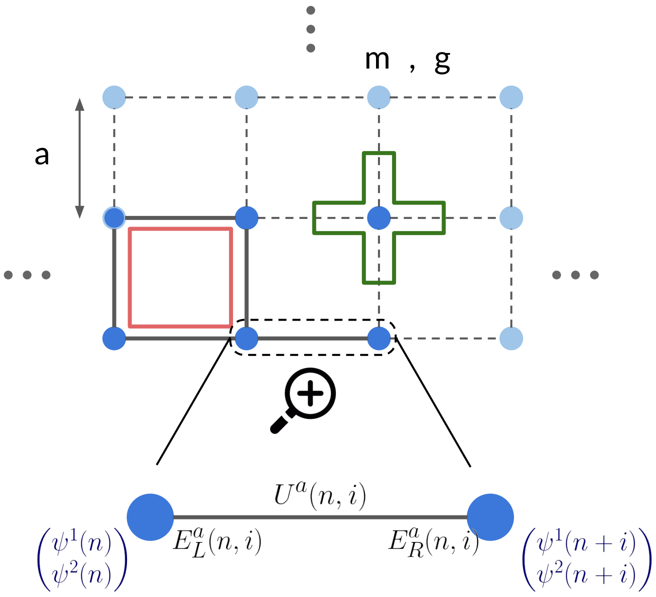
$$H_I = \frac{1}{2a} \sum_{n,i,\alpha\beta} (-1)^n \psi^{\alpha\dagger}(n) U_{\alpha\beta}(n,i) \psi^\beta(n+i)$$

$$H_B = \frac{2a}{g^2} \sum_{\square} (\mathrm{Tr}\, U_{\square} + h.c.)$$

Gauss' Laws: $G^a|phys\rangle = 0$

$$G^a(n) = \sum_i [E_L^a(n,I) - E_R^a(n-i,i)] - \psi(n)^\dagger \frac{\sigma^a}{2} \psi(n)$$

$$[G^a, G^b] = i f_c^{ab} G^c \neq 0 \quad (!)$$



Challenging!

- Noncommuting constraints
- Cannot eliminate gauge fields in d>1

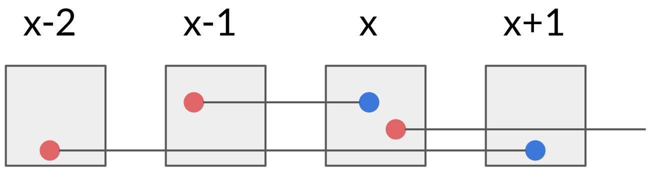
(as in, e.g., Banuls et al, PRX no. 4, (2017):041046)

Want:

- Gauge-invariant operator set
- Locality
- Simpler constraints

⇒ Loop-String-Hadron formulation of SU(2) LGT!

(I. Raychowdhury, J. Stryker, Phys. Rev. D **101**, 114502, 2020)



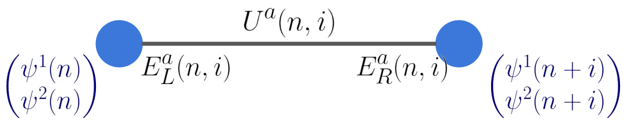
$|n_i, n_o, n_l\rangle$

Loop-String-Hadron (LSH) formulation of SU(2) LGT

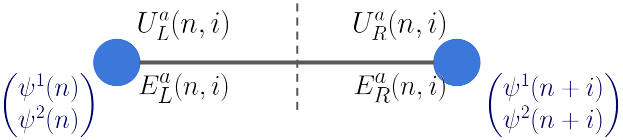
The LSH formulation provides a way out:

(see #95, I. Raychowdhury)

Kogut-Susskind



LSH



Basis:

Kogut-Susskind:

$$|n^1, n^2\rangle \otimes |j, m, m'\rangle \otimes |n^1, n^2\rangle$$

$$n^1, n^2 \in \{0,1\},\ m, m' \in \{-j, \ldots, j-1, j\}$$

LSH:

$$|n_i, n_o, n_l\rangle \otimes |n_i, n_o, n_l\rangle$$

$$n_i, n_o \in \{0,1\},\ n_l \in \{0,1,2,\ldots\}$$

Gauss' Law:

Kogut-Susskind:

$$G^a(n) = \sum_i [E_L^a(n,I) - E_R^a(n-i,i)] - \psi(n)^\dagger \frac{\sigma^a}{2}\psi(n)$$

$$[G^a, G^b] = i f_c^{ab} G^c \neq 0 \quad (!)$$

LSH:

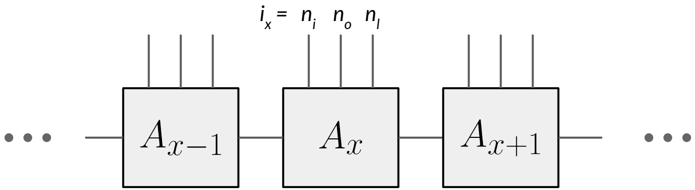
$$\boxed{N_L(n) = N_R(n+1)}$$

$$N_L = n_l + n_o(1-n_i)$$
$$N_R = n_l + n_i(1-n_o)$$

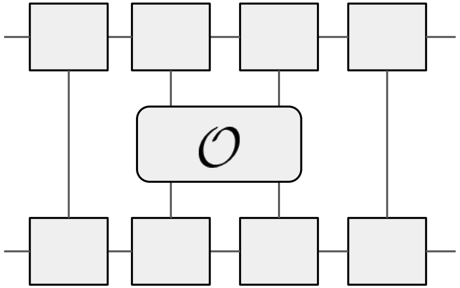
In this work

We build an MPS for 1+1D SU(2) LGT in the LSH formulation.

$$|\psi\rangle = \sum_{\{i_1,\ldots,i_N\}} \left[(A_1)^{v_1}_{i_1} \ldots (A_x)^{v_{x-1}v_x}_{i_x} \ldots (A_N)^{v_{N-1}}_{i_N} \right] |i_1 i_2 \ldots i_x \ldots i_N\rangle$$



$$\langle\psi|\mathcal{O}|\psi\rangle$$



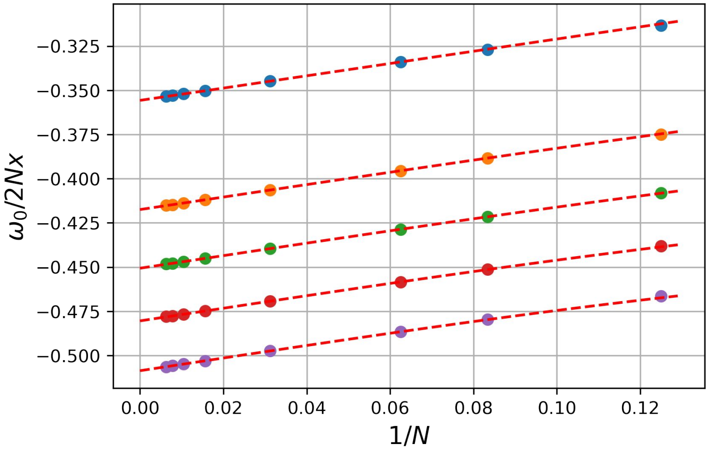
Benchmarks: (preliminary)

- Continuum ground state energy
- Continuum vector mass (1st excited state)
- Real-time dynamics

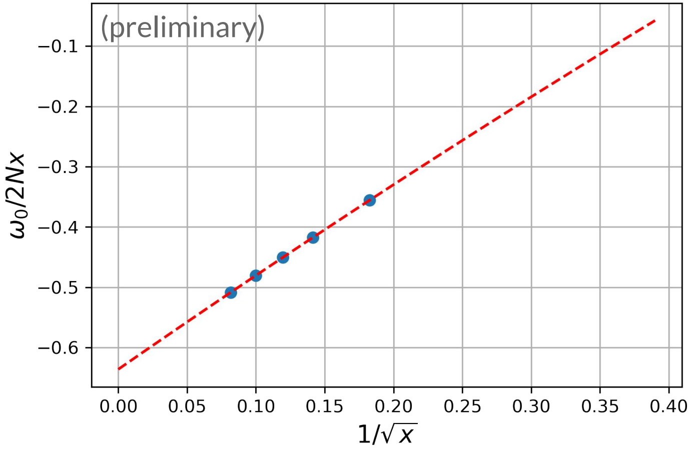
Continuum limits: Ground state energy

In 1+1D, it is predicted that the E0/(2Nx) → -2/π. For finite lattices, energy density goes as: E0/(2Nx) ~ a + b/N + c/N^3 + …

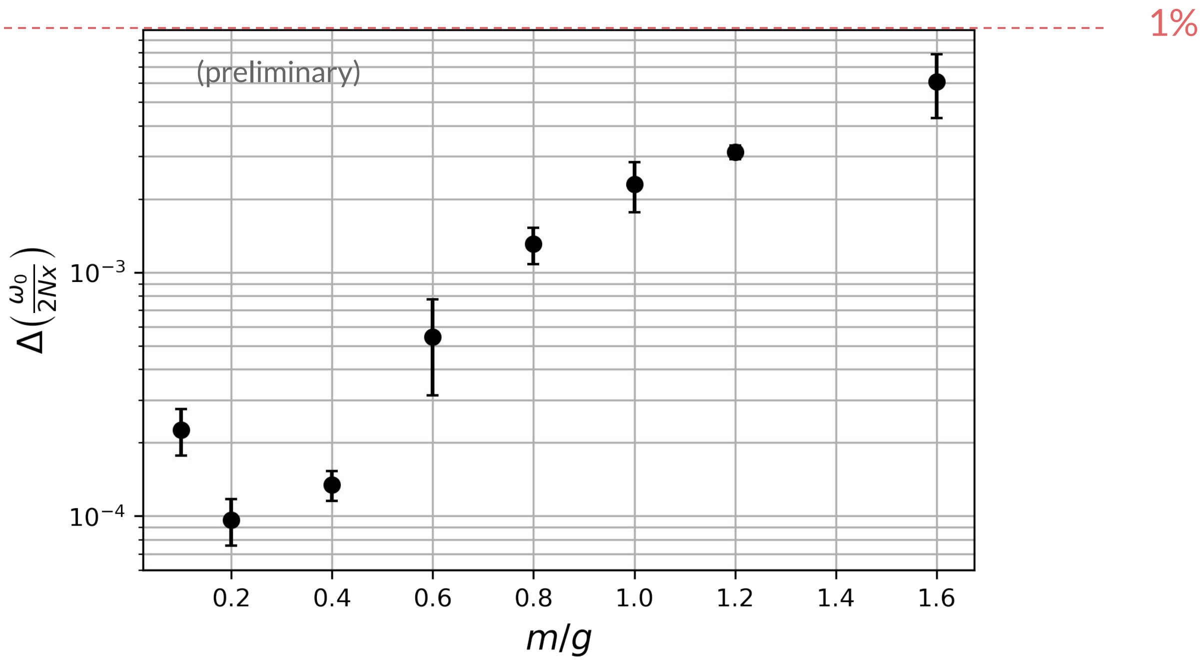
We fit the finite-lattice energies to a cubic function that mirrors the above series (up to N=160 sites). The x=1/(ga) limit fits well to a quadratic function.



(m/g = 1)



Continuum limits: Ground state energy

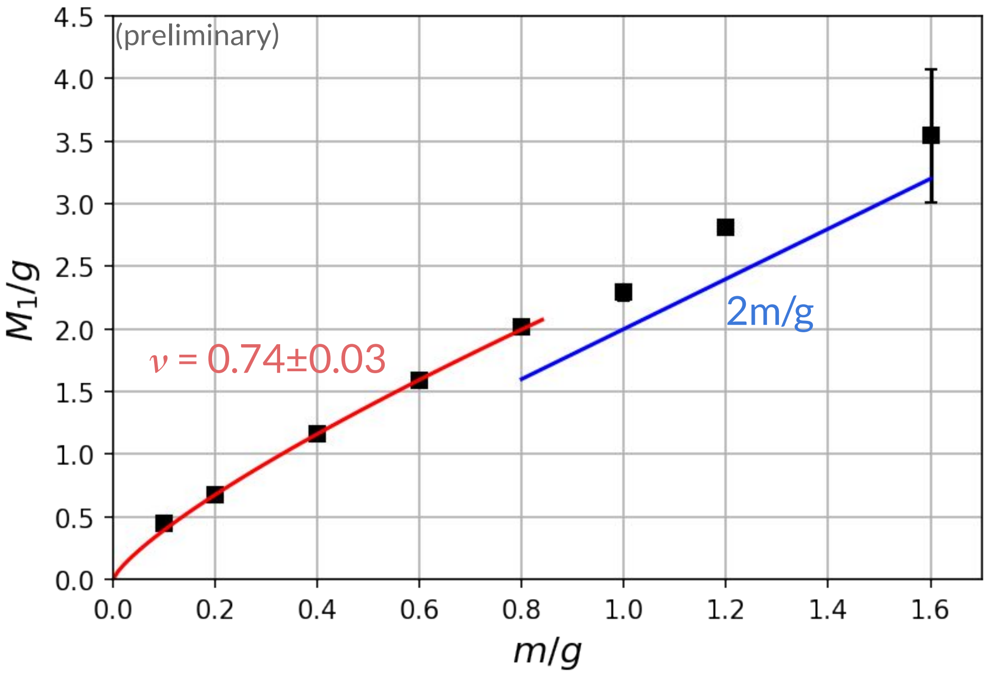


Continuum limits: Vector mass

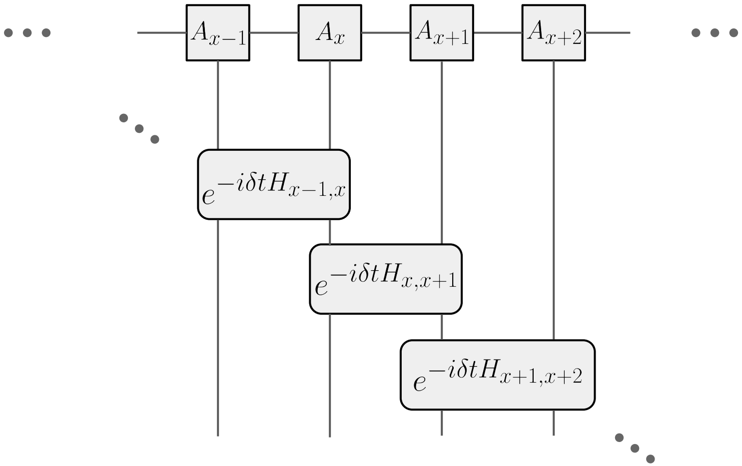
In the limit of strong coupling, it is predicted that $M_1/g \sim (m/g)^\nu$ where ν = ⅔.

In the weak coupling limit, M_1/g -> $2m/g$.

Our numerics are consistent with previous studies. (e.g. Banuls et al, PRX no. 4, (2017):041046).



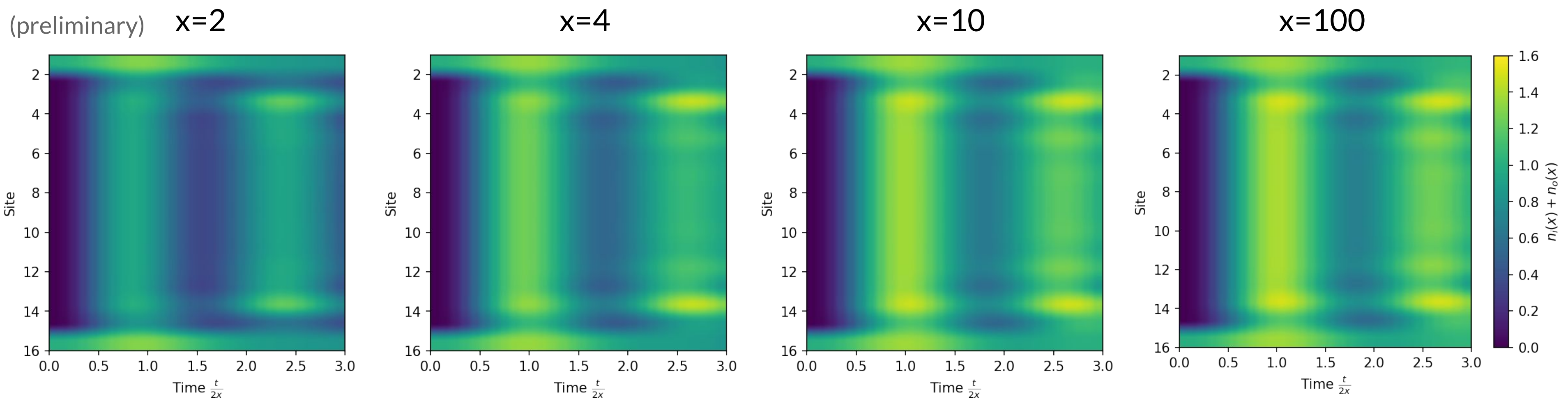
Dynamics



Time Evolving Block Decimation (TEBD):

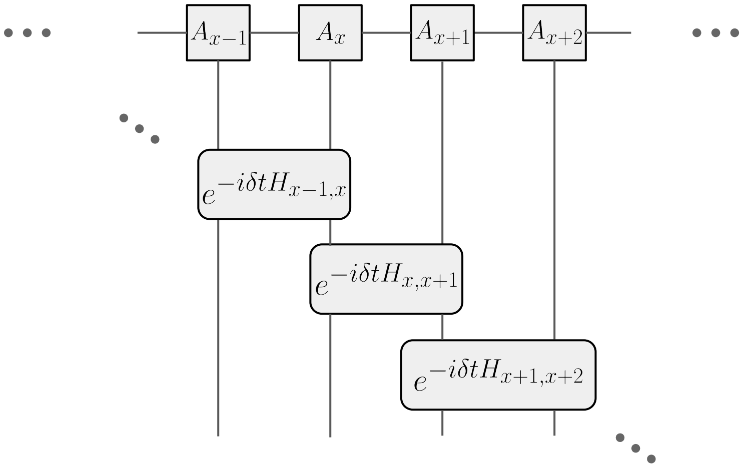
Time evolution of a quark/anti-quark pair at the ends of the chain, vs. lattice spacing x (strong → weak coupling limit):

(preliminary)



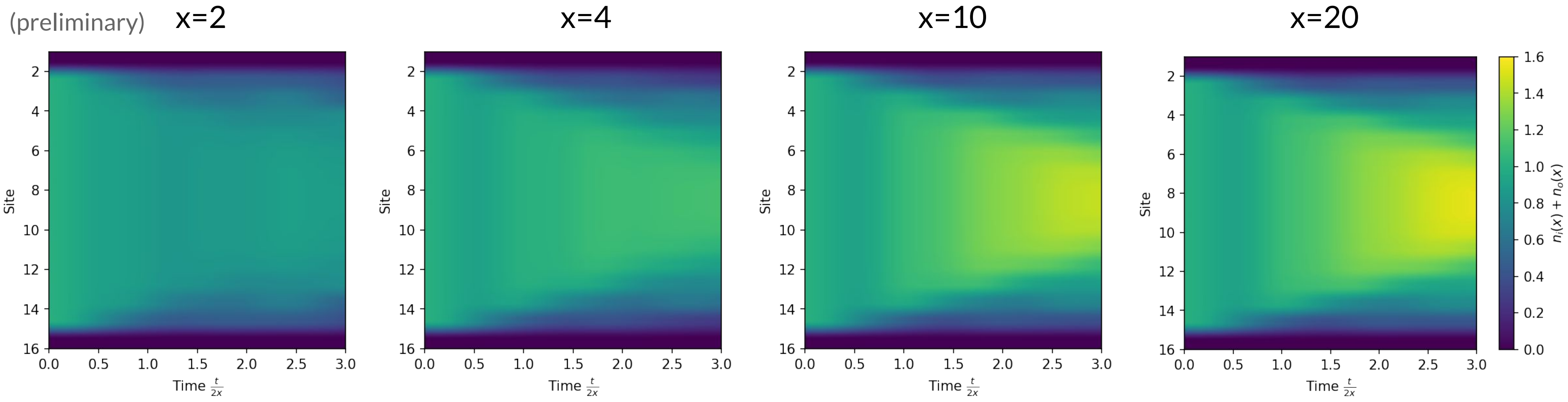
Dynamics

Time Evolving Block Decimation (TEBD):



Time evolution of a quark/anti-quark pair at the ends of the chain, vs. lattice spacing x:

(preliminary)



Dynamics

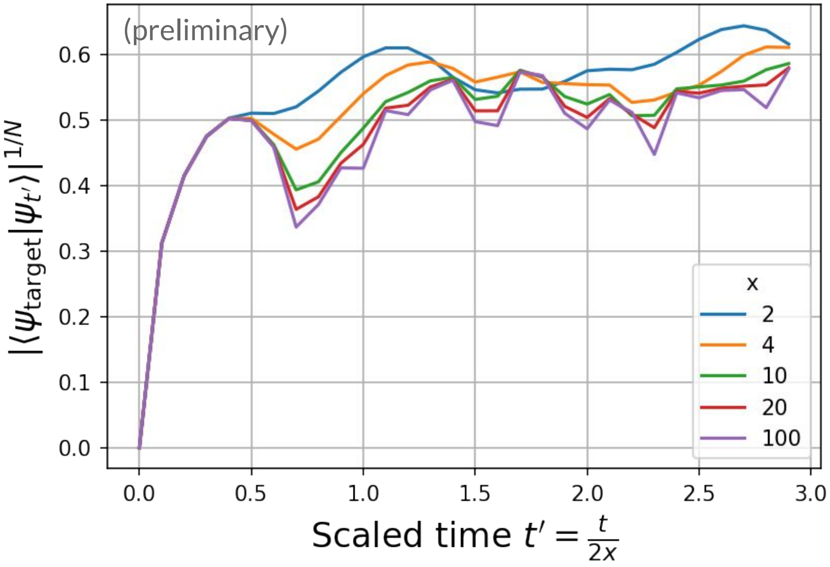
Probe of string breaking vs. time

State at time t:

$$|\psi_t\rangle = e^{-iHt} \Big| \quad \Big\rangle$$

Compute overlap with target state:

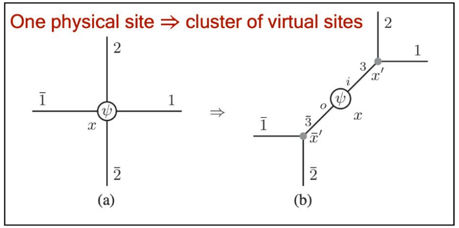
$$|\psi_{\text{target}}\rangle = \Big| \quad \Big\rangle$$



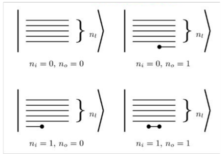
Outlook

- Continuum real-time dynamics
- There is a natural generalization to 2+1D using a PEPS construction. All operators are local and gauge-invariant.
- Tensor networks can provide a blueprint for quantum state preparation.

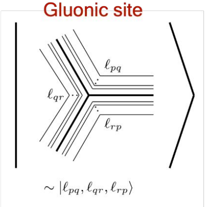
LSH Formalism: 2+1 d

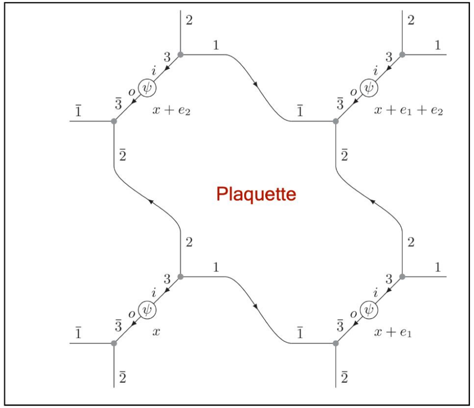


Matter site



Gluonic site





Matter-Gauge interactions are same as in 1d

Thank you!