

# Comparison of topology changing update algorithms

Lattice 2021

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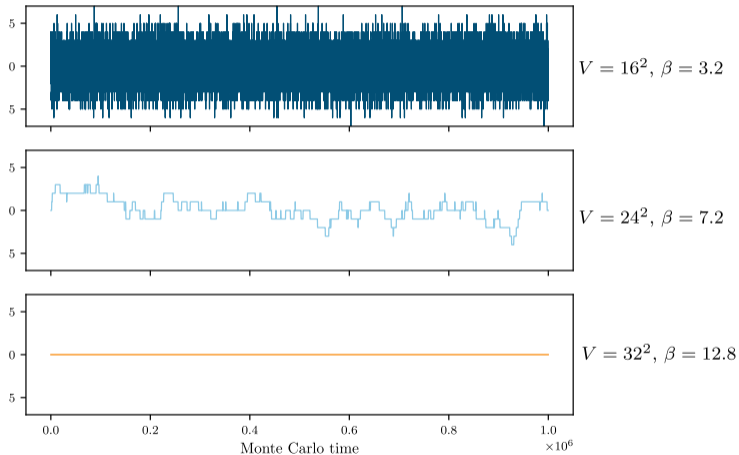


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# MOTIVATION

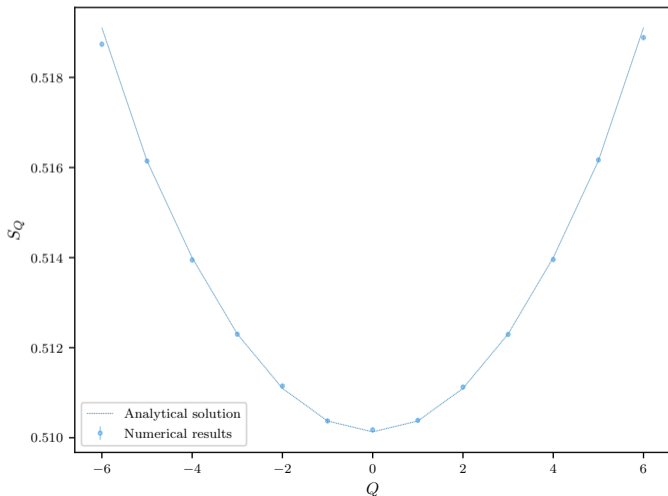
# Topological freezing in 2-dimensional $U(1)$



All 2-dimensional  $U(1)$  results on LCP  $V/\beta = 80$

# Topological freezing - Consequences

Even the most fundamental observable (gauge action) is affected!





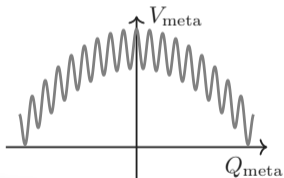
# UPDATE ALGORITHMS

# Three approaches

## Metadynamics

[Laio et al '16]

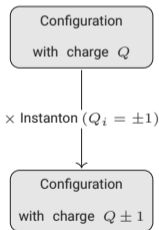
Add history-dependent bias potential to action



## Instanton updates

[Fucito et al '84]

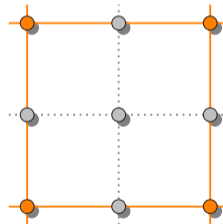
Multiply configurations with instantons



## Multiscale thermalization

[Endres et al '15]

Simulate on coarse lattice and fine-grain

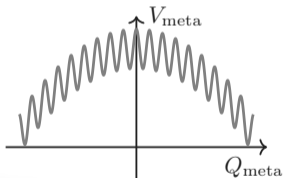


# Three approaches

## Metadynamics

[Laio et al '16]

Add history-dependent bias potential to action



- Use non-integer topological charge

$$Q_{\text{meta}} := \frac{1}{2\pi} \Im \left( \sum_n P_{01}(n) \right)$$

- After every accepted update, also update  $V_{\text{meta}}$ 
  - Keep track via linearly interpolating histogram
- Add bias potential to action

$$S_{\text{meta}} = S + V_{\text{meta}}$$

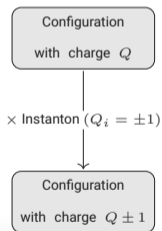
- Reweight observables back to desired distribution

# Three approaches

## Instanton updates

[Fucito et al '84]

Multiply configurations  
with instantons



- Link-wise multiplication of initial configuration with  $Q_i = 1$  or  $Q_i = -1$  instanton
- Add Metropolis accept/reject-step
- For examples:  
See talk by Dürr yesterday and [Dürr '12]
- Similar approach: Next talk by Albandea

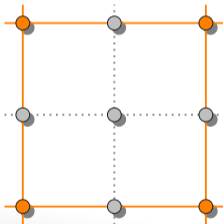


# Three approaches

## Multiscale thermalization

[Endres et al '15]

Simulate on coarse lattice  
and fine-grain

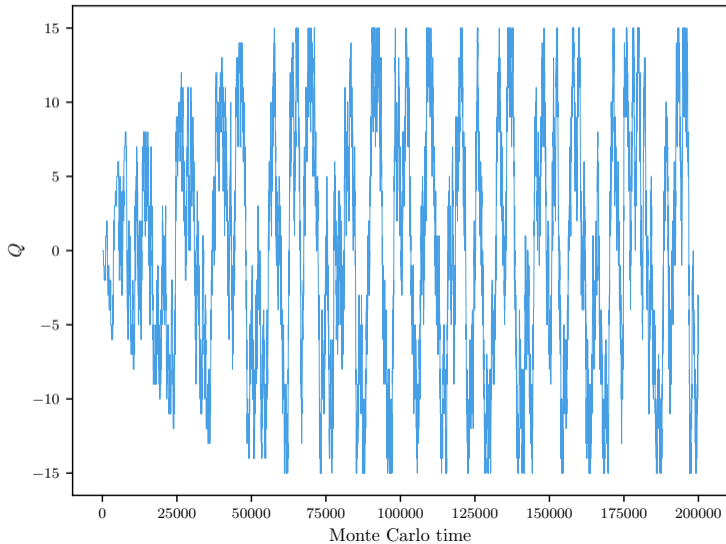


- Strategy: Start from coarse lattice with well-sampled topology and fine-grain
- **Fine-graining**
  - $U_{\mu}^f(\mathbf{n}) = U_{\mu}^c(\mathbf{n}/2)$
  - $U_{\mu}^f(\mathbf{n} + \hat{\mu}) = \mathbb{1}$
- Interpolation
  - Set gray links to  $\mathbb{1}$
  - Apply smearing to gray links
- Apply rethermalization updates

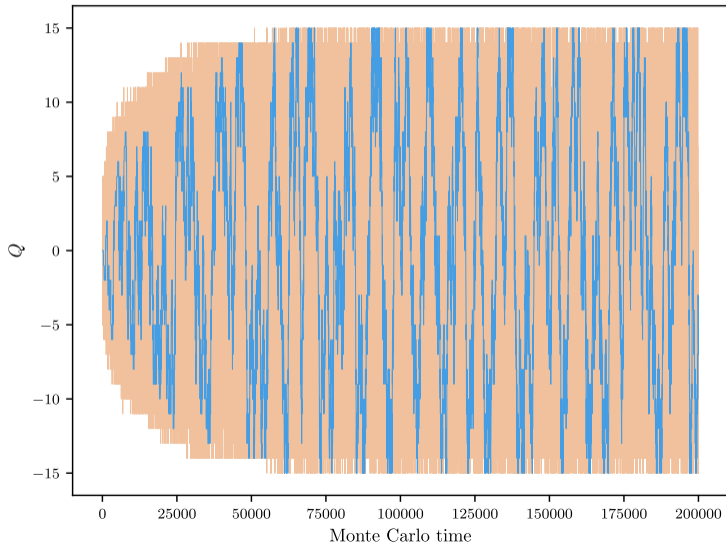


# RESULTS

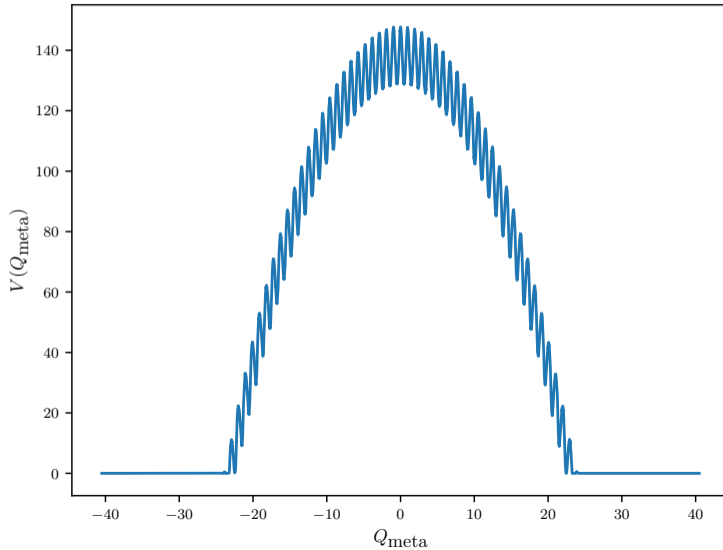
# Metadynamics



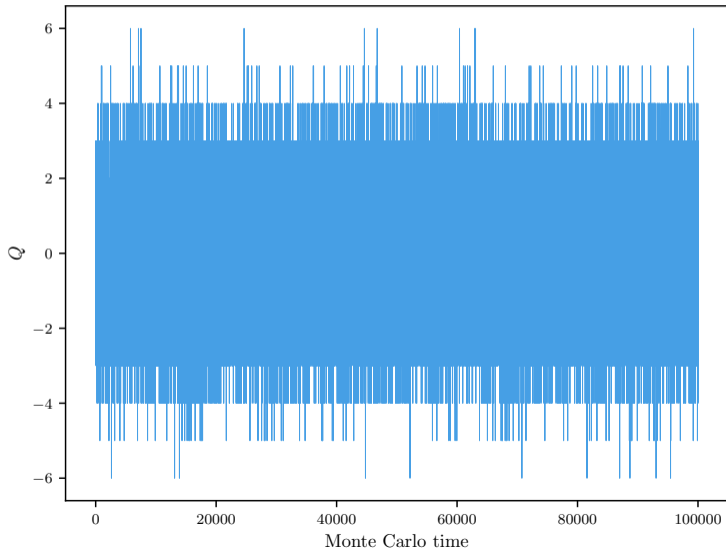
# Metadynamics + Instanton update



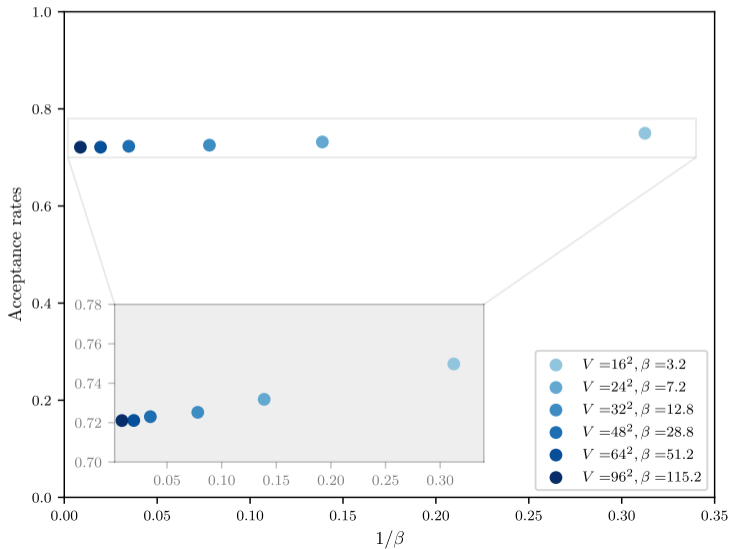
# Evolution of Metapotential



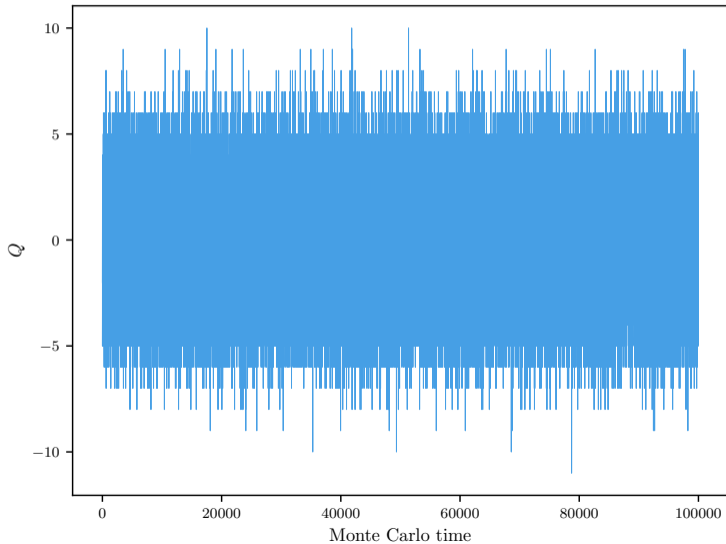
# Instanton update



# Instanton update

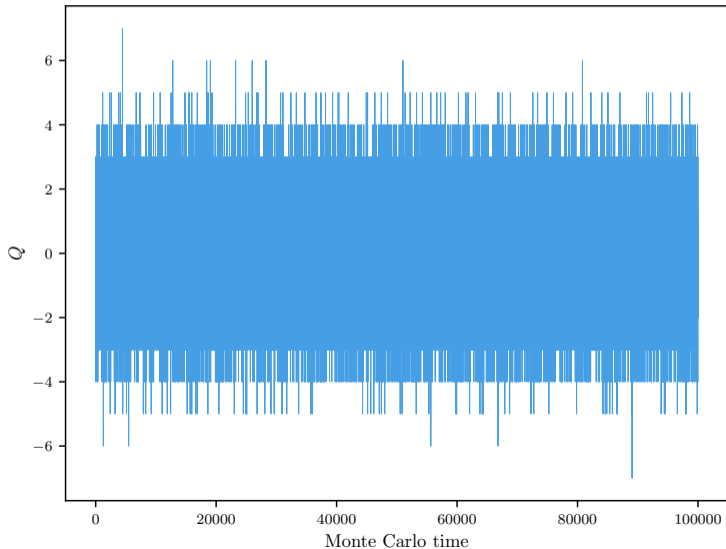


# Multiscale

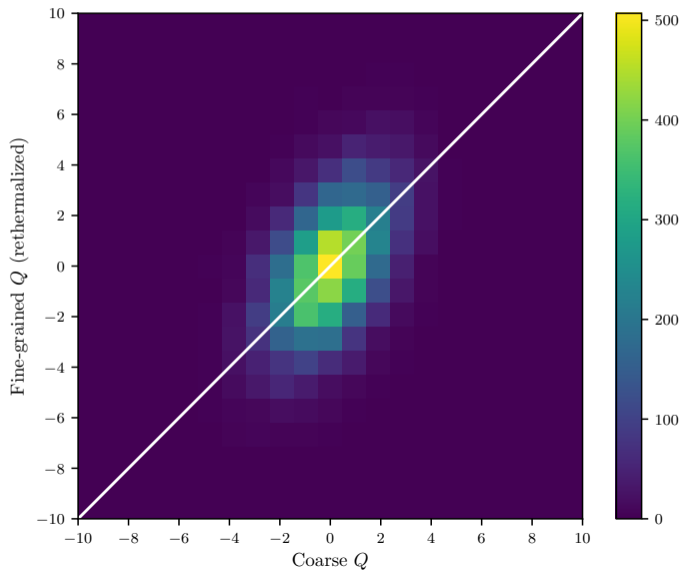




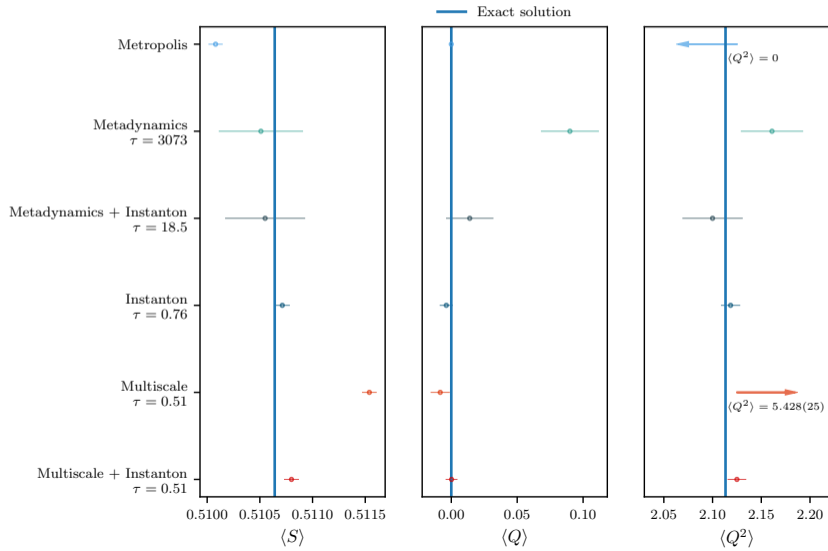
# Multiscale + Instanton update



# Multiscale charge correlation



# Comparison of results



# Comparison of results

Analytical solution: [Elser '02] and 2× [Bonati et al '19]

Approach	$\langle S \rangle$	$\langle Q \rangle$	$\langle Q^2 \rangle$	$\tau_{\text{int}}$
Analytical solution	0.510 641	0	2.113 304	—
Metropolis	0.510 080(68)	0	0	—
Metadynamics	0.510 51(40)	0.090(22)	2.161(32)	3073
Metadynamics + Instanton	0.510 55(38)	0.014(18)	2.100(31)	18.5
Instanton	0.510 713(72)	-0.0038(50)	2.1185(99)	0.76
Multiscale	0.511 539(70)	-0.0084(73)	5.428(25)	0.51
Multiscale + Instanton	0.510 800(71)	0.0002(45)	2.1250(96)	0.51

# Extension to $SU(3)$

- **Metadynamics**

- Requires much higher metapotential (see next point)

- **Instanton update**

- Suffers from high action penalties when jumping from sector to sector
- $V = 8^4, \beta = 6.0: \Delta S \sim 3000$
- $V = 16^4, \beta = 6.0: \Delta S \sim 12000$

- **Multiscale thermalization:**

- Successfully used in  $SU(3)$  as a thermalization procedure [Endres et al '15]
- Correct susceptibility?

- **Combine all three approaches?**

- Build up metapotential with instanton update on coarse lattices
- Progressively go to finer lattices and correct metapotential

# Thank you for your attention!

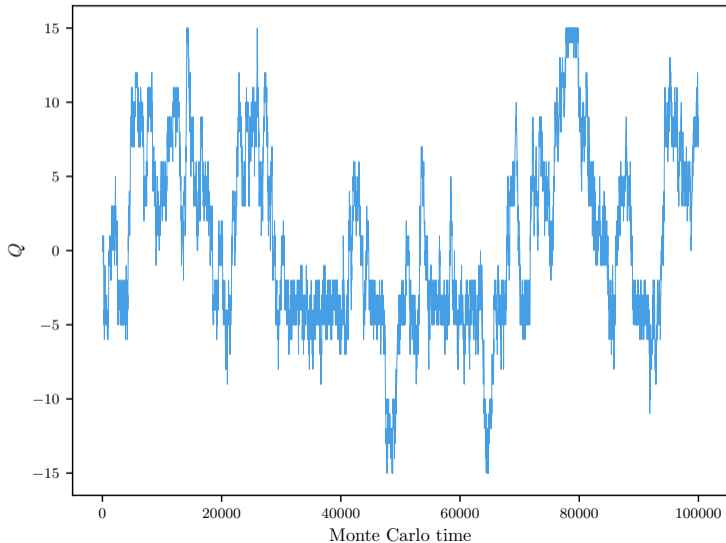
[timo.eichhorn@protonmail.com](mailto:timo.eichhorn@protonmail.com)





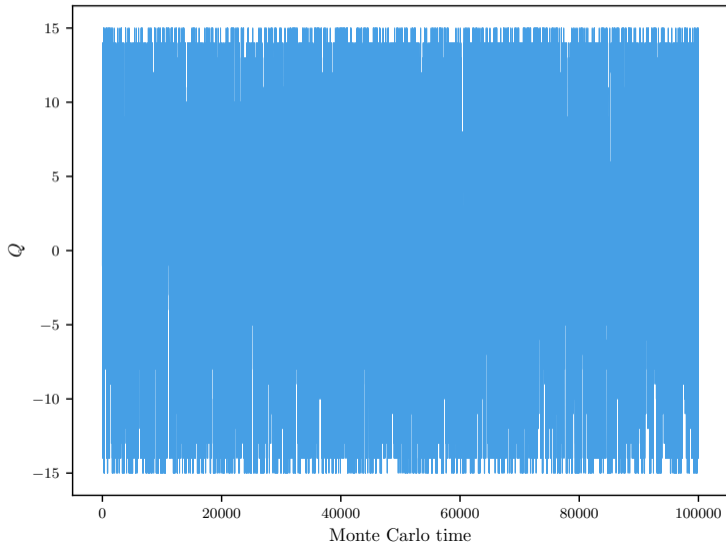
BACKUP SLIDES

# Metadynamics - Static potential





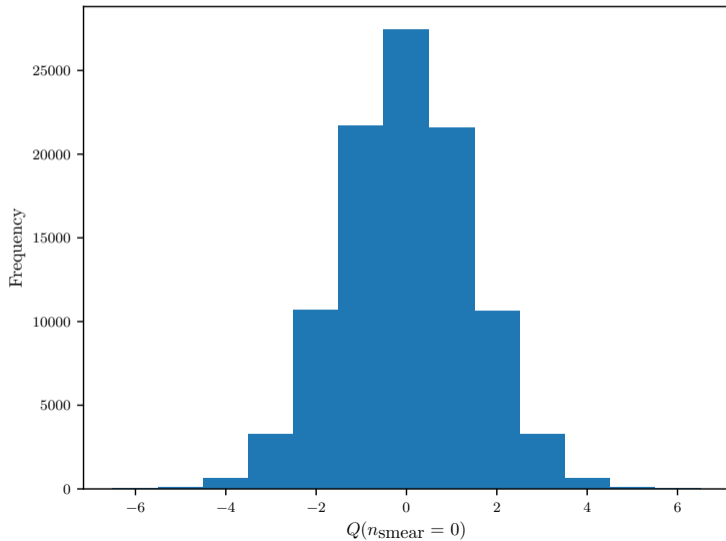
# Metadynamics - Static potential



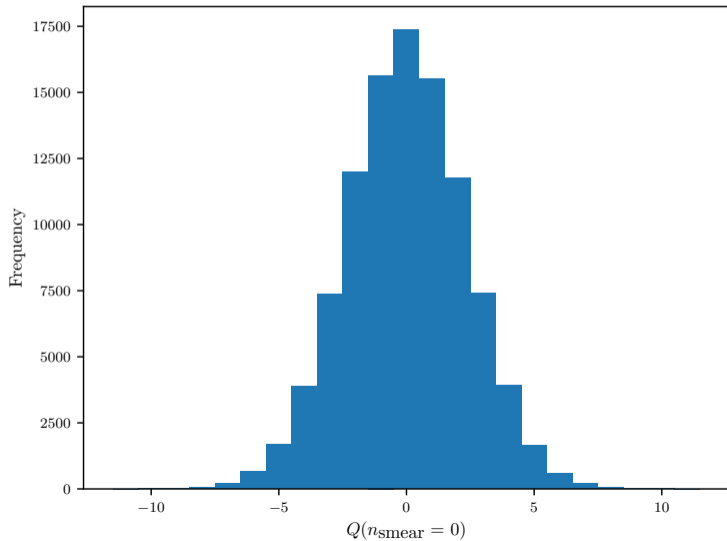
# $SU(3)$ instantons

- Main difference to 2-dimensional  $U(1)$ : Instantons no longer delocalized
- In  $D$  dimensions
  - $D < 4$ : Instantons spread out
  - $D = 4$ : Instantons size-independent
  - $D > 4$ : Instantons contract

# Instanton $Q$ histogram



# Multiscale $Q$ histogram



# Multiscale + Instanton update $Q$ histogram

