Qubit Regularization of Asymptotic Freedom

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with Tanmoy Bhattacharya (LANL), Alex J. Buser (Caltech), Shailesh Chandrasekharan (Duke U.), Rajan Gupta (LANL)

“Digitization” of QFTs for digital quantum computers

- Traditional lattice regularization for bosons = ∞-dim local Hilbert space. Implied by the bosonic commutation relations

$$[\phi_x, \pi_y] = i\delta_{x,y} \tag{1}$$

- But digital quantum computers need a **finite dimensional** local Hilbert space
- Need to truncate the Hilbert space somehow...
- Several approaches towards finding a “digitization”
  - Field-space digitization **[Jordan, Lee, Preskill, 2011, ...]**
  - Loop-string hadrons **[Raychoudhary et al, 2020, ...]**
  - Single-particle digization **[Barata et al, 2020, ...]**
  - Tensor networks **[Meurice, 2020, ...]**
  - Discrete subgroups for gauge theories **[Lamm et al, ...]**
  - D-theory, quantum-link models **[Brower et al, 2004, ...]**
  - ...

“Digitization”

- Most approaches to digitization: truncate the Hilbert space (to n qubits), then reproduce the traditional lattice Hamiltonian by taking $n \to \infty$, and then take the continuum limit like in traditional lattice models

$$\text{Digitized model} \xrightarrow{n\to\infty} \text{Traditional lattice model} \xrightarrow{a\to 0} \text{continuum QFT} \quad (2)$$

- Is it necessary to do this 2-step procedure?

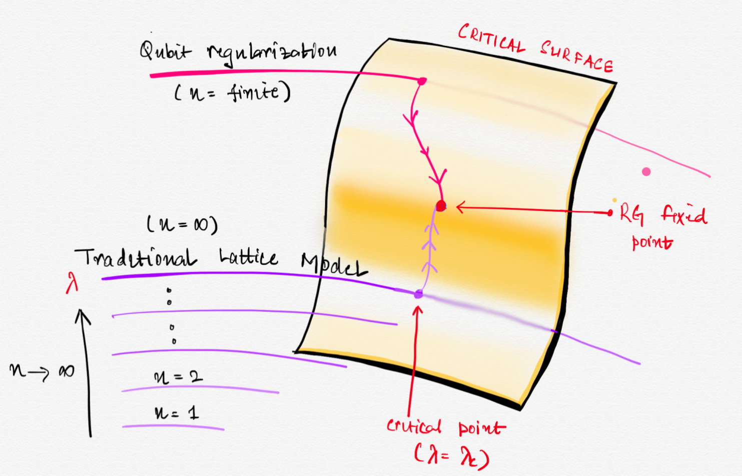
“Digitization”

- However, that is not necessary
- **Wilson’s insight: QFT = Second-order phase transitions**
- Even with finite n (#qubits per lattice site) one can obtain continuum limits of field theories

Qubit regularization of field theories

- Continuum limit: tune to a second-order critical point of a quantum lattice Hamiltonian
- This defines a procedure to obtain a continuum QFT
- **Qubit regularization:**
  a quantum lattice Hamiltonian acting on a finite-dimensional local Hilbert space (kept fixed) which reproduces a desired QFT in the vicinity of a quantum critical point.

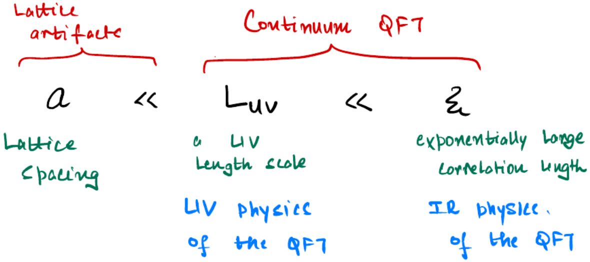
Regularization from a Wilsonian RG perspective



Qubit regularization of asymptotic freedom

- For some QFTs, it is quite easy to construct such “qubit” models.
- For example, the Ising model reproduces the ϕ^4 theory in $d = 3, 4$ dimensions
- Question:
  can asymptotically free theories can be regularized with a finite-dimensional local Hilbert space?

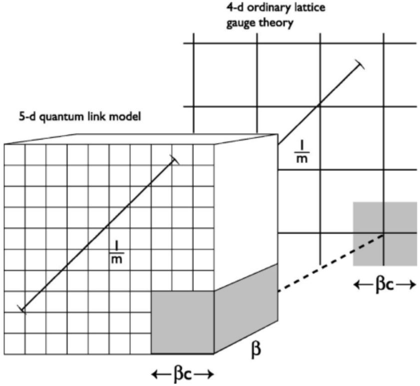
The challenge of asymptotic freedom



- To get the continuum limit, we need to recover both the IR physics and the UV physics

D-theory: a regularization of asymptotic freedom

- D-theory is a qubit regularization of asymptotic freedom **[Wiese, 1999; Brower et al, 2004]**
- The marginally relevant operator for a D dimensional system is obtained from the size of an extra dimension L_y ($D+1$ dimensions)
- However, the number of qubits grows as L_y increases



[Brower, Chandrasekharan, Wiese, 1999]

A toy model of asymptotic freedom

- **$O(3)$ nonlinear sigma model in 1+1 dimensions**
- Continuum action

$$S[\vec{n}(x)] = \frac{1}{2g^2} \int d^2x \, \partial_\mu \vec{n} \cdot \partial^\mu \vec{n} \tag{3}$$

  with $\vec{n} \in \mathbb{R}^3$ and $|\vec{n}| = 1$.
- toy model for QCD: asymptotic freedom, dynamical mass generation, dimensional transmutation

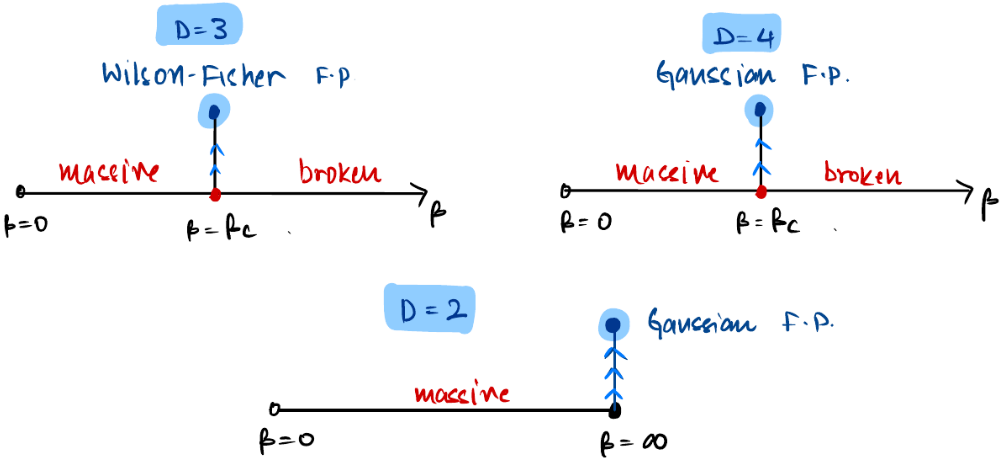
Traditional lattice regularization

- **$O(3)$ nonlinear sigma model in 1+1 dimensions**
- Lattice regulated action:

$$S = -\beta \sum_{\langle xy \rangle} \vec{\phi}_x . \vec{\phi}_y \quad (4)$$

- 2d $O(3)$ NLSM is the continuum QFT which emerges in the $\beta \to \infty$ limit of the lattice model

$O(N)$ NLSM in D dimensions: traditional lattice model



Aside: qubit $O(N)$ NLSM in $D \geq 3$ dimensions

- It is easy to obtain the 3d and 4d continuum field theories associated to the $O(N)$ NLSM.
- Just need to get the symmetries right on both sides of the second-order phase transition
- We constructed a qubit regularization of the $O(N)$ nonlinear sigma models in $D = 3, 4$ spacetime dimensions
  - $O(3)$ = 4-dim local Hilbert space **(2 qubits)** **[HS, Chandrasekharan, 2019]**
  - $O(N)$ = (N+1)-dim local Hilbert space **[HS, 2019]**
- Local hilbert space at site x

$$\mathcal{H}_x = \mathbf{1} \oplus \mathbf{\mathit{N}} \tag{5}$$

1 singlet ("vacuum") state and N fundamental ("particle") states

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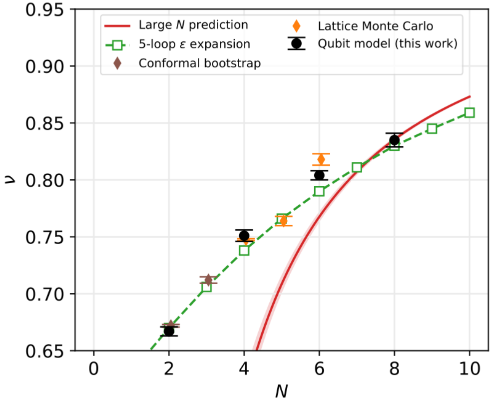


Critical exponents for the qubit-regularized $O(N)$ models in $D = 3$, $N = 2, \ldots, 8$

HS, Chandrasekharan (2019), HS (2019)

Probing the continuum limit for asymptotically free theories

- To probe the universal behaviour of the continuum limit, we can use the **step scaling function** as a convenient tool **[Luscher, Weisz, Wolff, 1991]**
- Put the asymptotically free theory in a box of size L (natural length scale)
- Define a dimensionless renormalized coupling $\bar{g}^2(L)$
  - For example, we can choose $\bar{g}^2(L) = M(L)L$, where $M(L)$ is the finite-volume mass gap
- All dimensionless observables depend only on the renormalized coupling $\bar{g}^2(L)$.

Step scaling function

- We will look at the universal function $F(z)$ defined by

$$\frac{\xi(\beta,2L)}{\xi(\beta,L)} \equiv F\left(\xi(\beta,L)/L\right) \tag{7}$$

where β is a bare coupling and $z = \xi(\beta,L)/L$ is the renormalized coupling

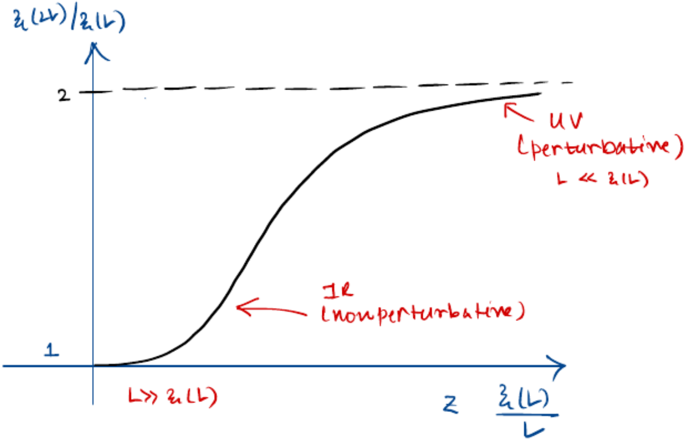
- $\xi(\beta,L)$ is a definition of finite-volume correlation length: the "second-moment" correlation length

$$\xi(L) = \frac{\sqrt{\tilde{G}(0)/\tilde{G}(2\pi/L) - 1}}{2\sin(\pi/L)} \tag{8}$$

- Easy to measure

Step scaling function: qualitative behaviour

- IR physics: $z \to 0$
- UV physics: $z \to \infty$ (accessible from perturbation theory)



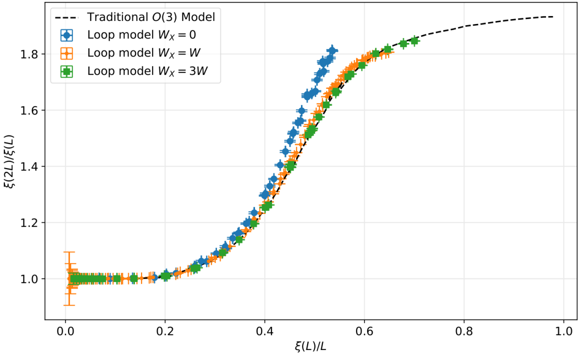
Finding the right two-qubit model

- Goal: Find a regularization of the $O(3)$ NLSM with only **two qubits** per lattice site.
- Strategy: Look in the space of $O(3)$-invariant two-qubit models and compare against the known step scaling function – reproduce both UV and IR physics.

Results: some (not-so-successful) attempts

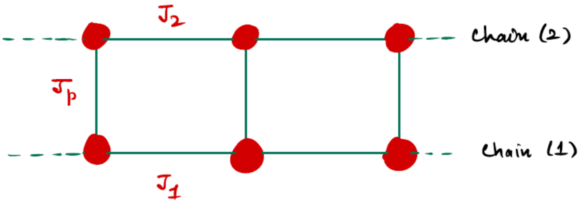
Naive attempt: use the qubit models that worked in $3d$ and $4d$

- Add diagonal bonds as well
- $\xi(L)/L$ saturates $\implies$ **Low-energy EFT with a finite cutoff, but not a continuum QFT**
- **Neidermeyer, Wolff (2016)** studied a similar model



Exploring spin ladders

- Nearest-neighbor two-qubit models = Spin ladders
- $O(3)$ invariant.
- Consider:

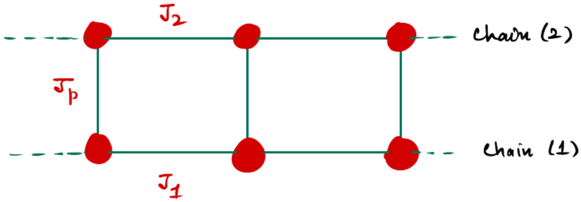


$$H = J_p \sum_x H_{(x,1),(x,2)} + J_1 \sum_x H_{(x,1),(x+1,1)} + J_2 \sum_x H_{(x,2),(x+1,2)} \qquad (9)$$

with $H_{x,y} = \vec{S}_x \cdot \vec{S}_y$.

- Rich phase diagram.
- **Could this have the right critical point?**

Exploring spin ladders

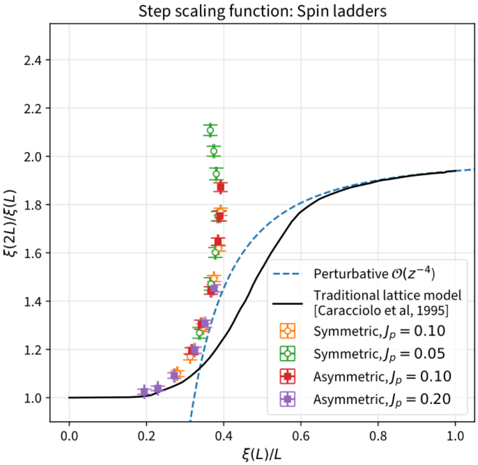


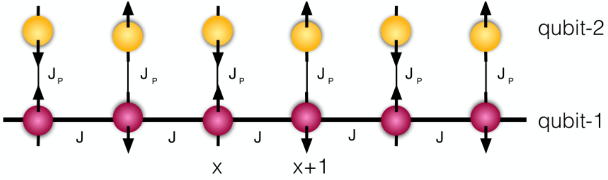
$$H = J_p \sum_x H_{(x,1),(x,2)} + J_1 \sum_x H_{(x,1),(x+1,1)} + J_2 \sum_x H_{(x,2),(x+1,2)} \tag{10}$$

- We consider three cases:
  1. Symmetric ladder: $J_1 = J_2 \gg J_p$
  2. Asymmetric ladder: $J_1 = J_2 \gg J_p$
  3. Heisenberg comb: $J_1 \gg J_p \gg J_2$

Results: Spin ladder, symmetric and asymmetric

- Two weakly coupled chains
  - Symmetric ladder: $J_1 = J_2 \gg J_p$
  - Asymmetric ladder: $J_1 \gg J_2 \gg J_p$
- Again, the spin ladders describe the low-energy physics correctly **[Shelton, Narseyan, Tsvelik, 1996]**
- But not the UV physics



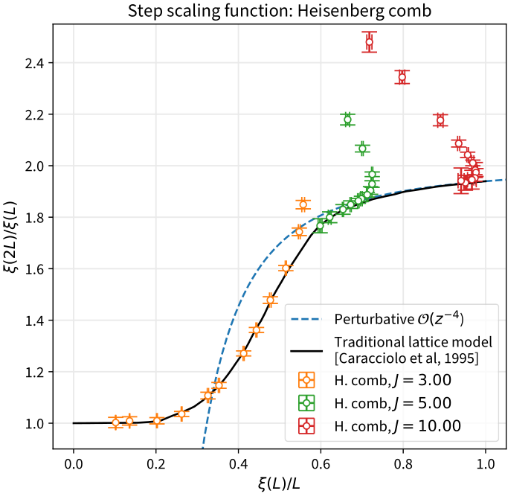


- Finally, we consider the limit $J_1 \gg J_p \gg J_2$
- Hamiltonian

$$H = \sum_i \quad J_p \, H_{(i,1),(i,2)} + J \, H_{(i,1),(i+1,1)} \tag{11}$$

- Set $J_2 = 0, \; J_p = 1$. Continuum limit: $J \to \infty$.

Results: Heisenberg comb

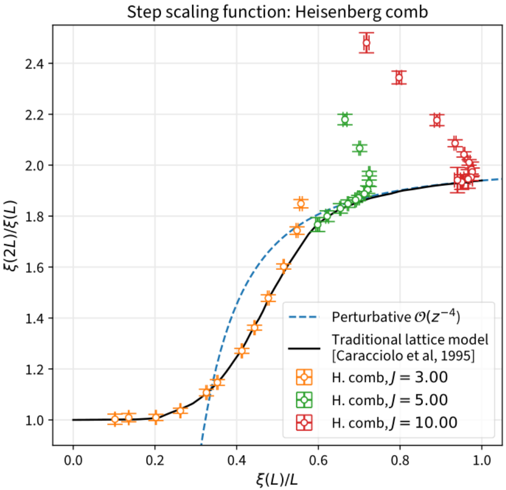


Indeed, the step scaling function is reproduced for $L > L_{\text{min}}(J)$!

Results: Heisenberg comb

- Infinite volume correlation lengths

J	ξ	L_{min}
3.0	25	30
5.0	600	100
10.0	2×10^5	400



We can reproduce the UV physics up to correlation lengths of 2×10^5!

Conclusions

- "Qubit" regularization: continuum limit of a desired QFT with a finite-dim local Hilbert space.
- Such regularizations seem very natural for digital quantum computers, unlike traditional lattice regularizations
- Questions: Can all QFTs be regularized in such a way? In particular, AF theories like QCD? What is the minimum number of qubits required?
- We now have strong evidence that a $2d$ asymptotically free QFT, the $O(3)$ sigma model can be reproduced with just 2 qubits per lattice site
- The space of such non-traditional formulations of lattice QFTs is quite rich and important for near-term quantum computers

Thank you!