

First study of twist-3 PDFs & GPDs for the proton from Lattice QCD

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In collaboration with:

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Twist-classification of PDFs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \dots$$

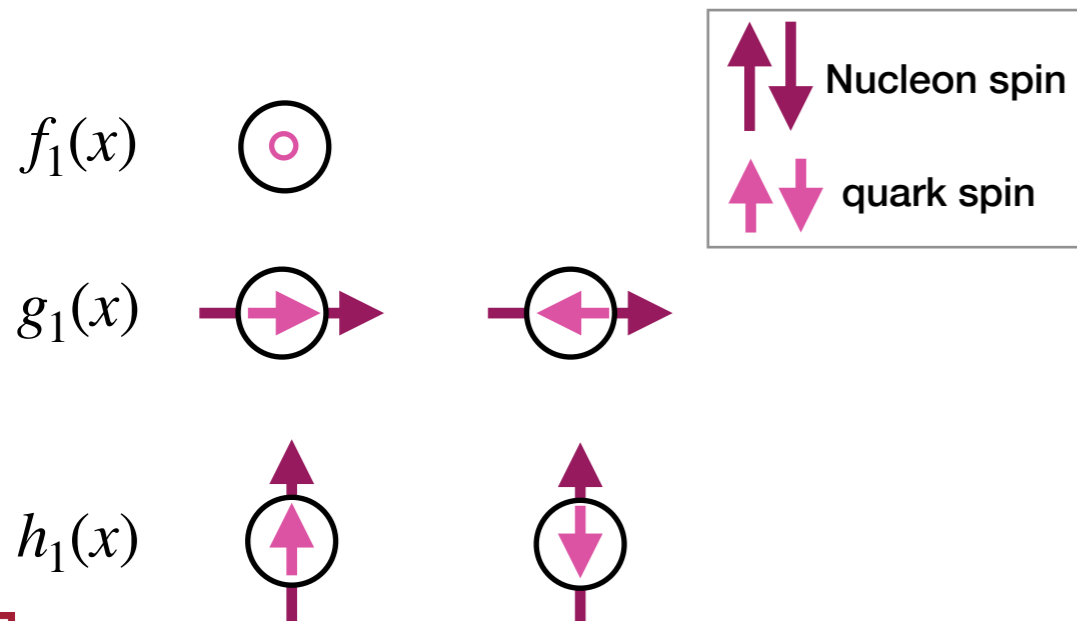
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Twist-2 ($f_i^{(0)}$)

Quark \ Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$f_1(x)$ number density		
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T			$h_1(x)$ transversity

Probabilistic interpretation



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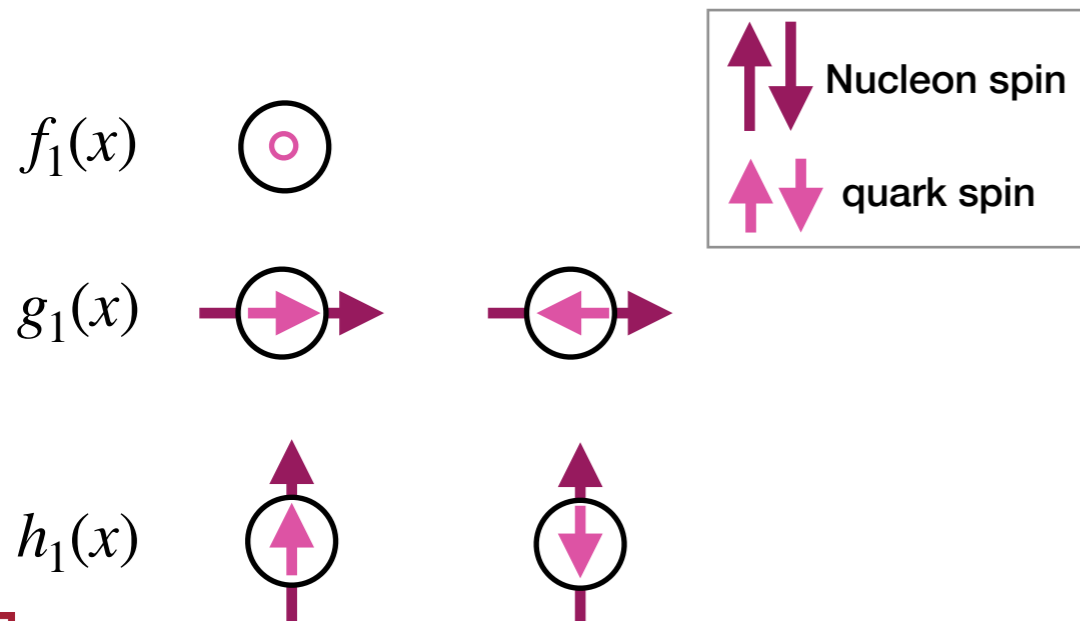
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Twist-3 ($f_i^{(1)}$)

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U	$e(x)$		
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Probabilistic interpretation



- ★ Lack density interpretation, but can be **sizeable**
- ★ Kinematically suppressed
Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities
(see A. Metz talk)
- ★ Contain info on quark-gluon-quark correlators

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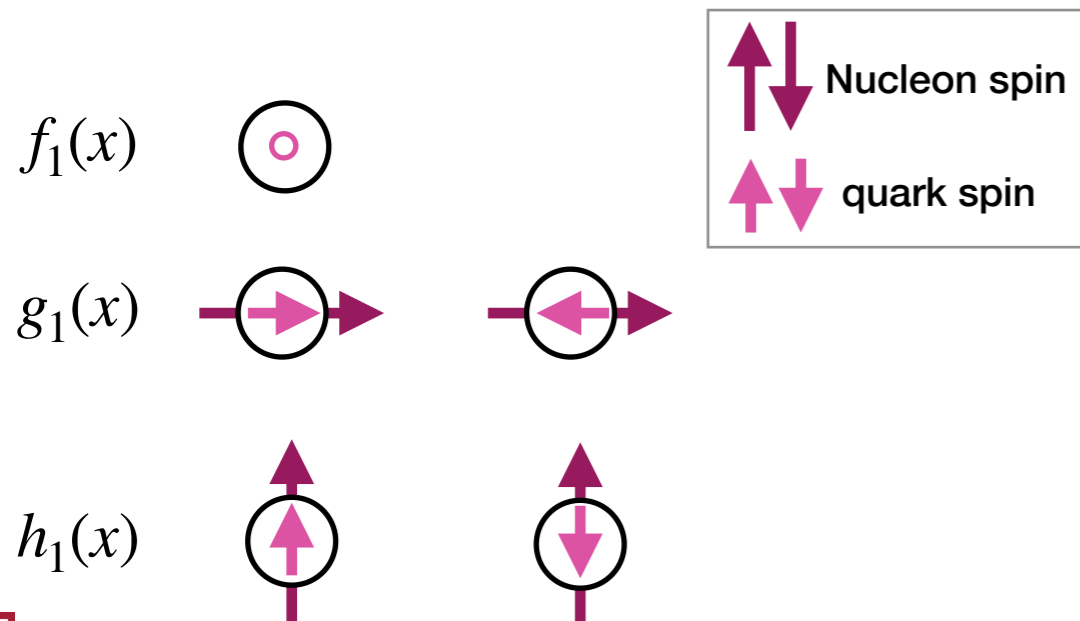
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This talk
+ twist-3
 \widetilde{G}_i GPDs

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Parameters of calculations

★ $N_f=2+1+1$ twisted mass (TM) fermions & clover term

★ Ensemble parameters:

Name	β	N_f	$L^3 \times L_T$	a [fm]	M_π	$m_\pi L$
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★ Twist-3 u-d PDFs: $g_T(x)$, $h_L(x)$

P_3 [$\frac{2\pi}{L}$]	P_3 [GeV]	N_{conf}	N_{meas}
2	0.83	194	1552
3	1.25	731	11696
4	1.67	1644	105216

- Double statistics for $g_T(x) : \gamma^1 \gamma^5$ & $\gamma^2 \gamma^5$

- $T_{sink} = 1.12$ fm

★ Twist-3 u-d GPDs: $\widetilde{G}_1(x, \xi, t)$, $\widetilde{G}_2(x, \xi, t)$, $\widetilde{G}_3(x, \xi, t)$, $\widetilde{G}_4(x, \xi, t)$

P_3 [GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$ [GeV ²]	ξ	N_{meas}
0.83	(2,0,0)	0.69	0	4288
1.25	(2,0,0)	0.69	0	4288
1.25	(2,2,0)	1.39	0	4288
1.67	(2,0,0)	0.69	0	4288

- We utilize $\gamma^1 \gamma^5$ & $\gamma^2 \gamma^5$
 $\pm \vec{Q}, \pm P_3$

- $T_{sink} = 1$ fm

Twist-3 PDFs

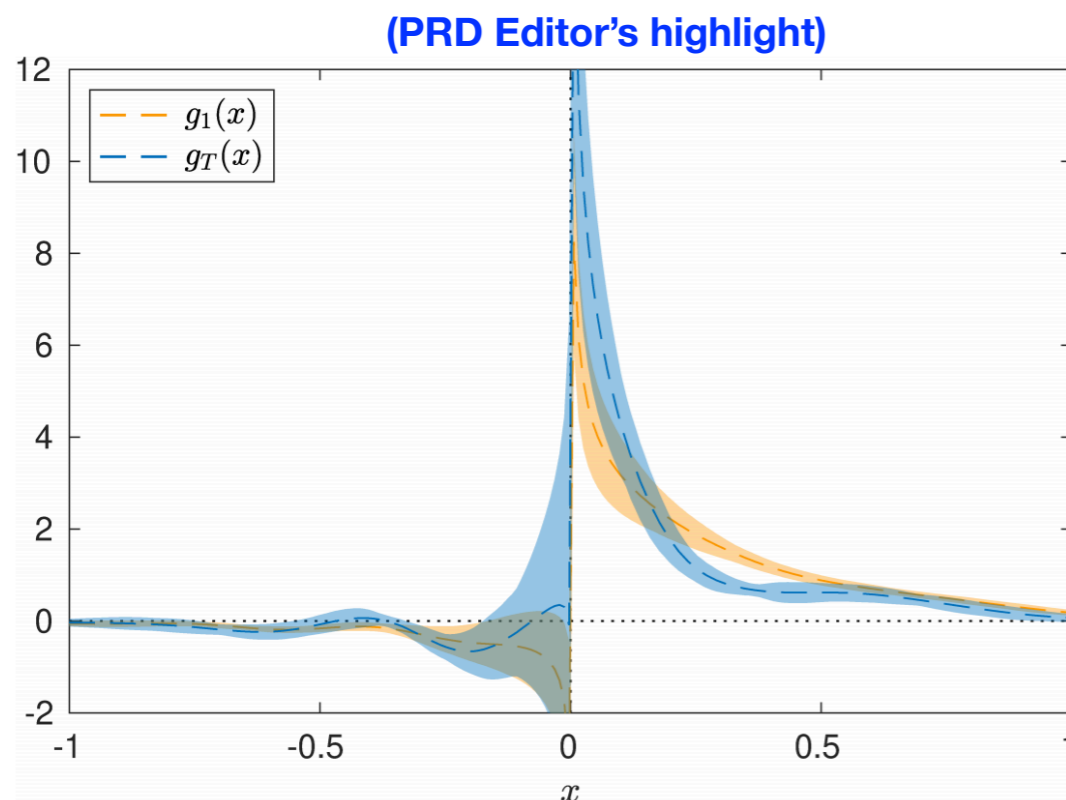
- *New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$*
S.Bhattacharya, K.Cichy, M.C, A.Metz, A.Scapellato, F.Steffens, **PRD 102 (2020), 111501 (Editors' Suggestion)**,
[\[arXiv:2004.04130\]](#)
- *One-loop matching for the twist-3 parton distribution $g_T(x)$*
S.Bhattacharya, K.Cichy, M.C, A.Metz, A.Scapellato, F.Steffens, **PRD 102 (2020) 3, 034005**, [\[arXiv:2005.10939\]](#)
- *The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$*
S. Bhattacharya, K. Cichy, M.C, A. Metz, A. Scapellato, F. Steffens, **PRD 102 (2020) 114025**, [\[arXiv:2006.12347\]](#)
- *Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case*
S. Bhattacharya, K. Cichy, M.C, A. Metz, A. Scapellato, F. Steffens, [\[arXiv:2107.02574\]](#)

Twist-3 $g_T(x)$ PDF

- $g_2(x)$ ($g_2(x) = g_T(x) - g_1(x)$) related to the transverse force acting on the active quark in DIS off a transversely polarized nucleon immediately after it has absorbed the virtual photon [M. Burkardt, *Phys.Rev.D* 88 (2013) 114502, arXiv:0810.3589]
- can be separated from twist-2 helicity PDF in scattering longitudinally polarized leptons from a transversely polarized target (at a target polarization angle of 90° the twist-2 PDF cancels)

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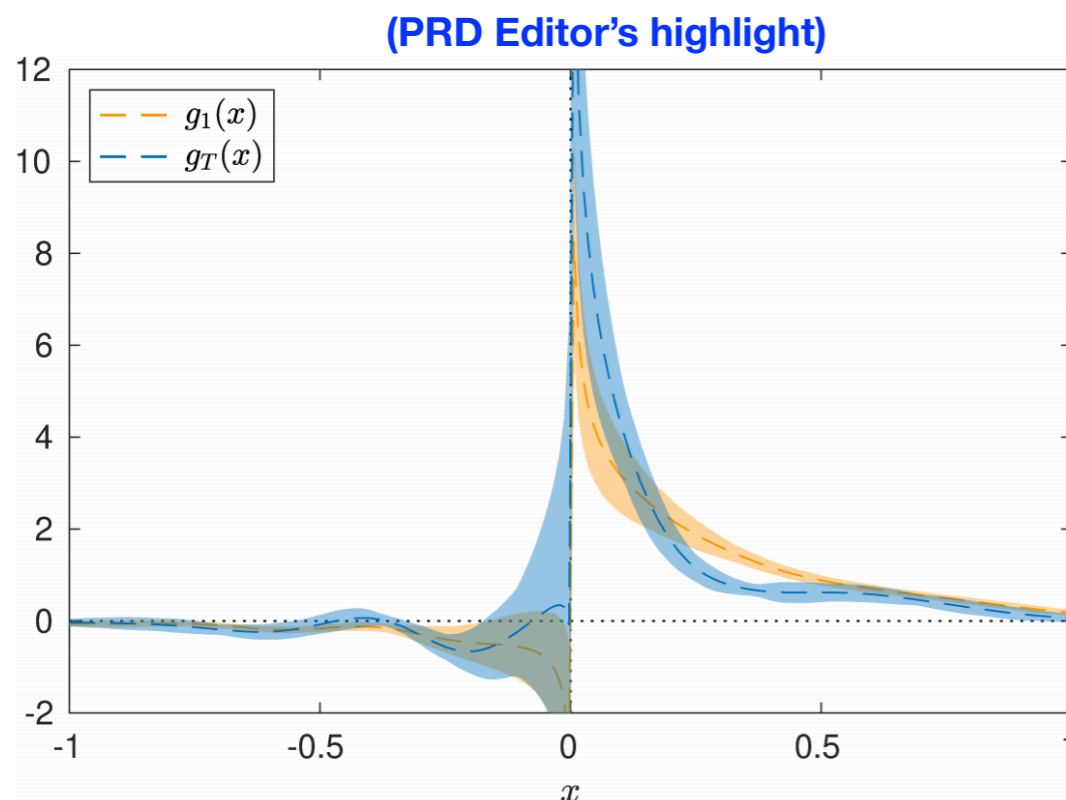


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Burkhardt-Cottingham sum rule important check

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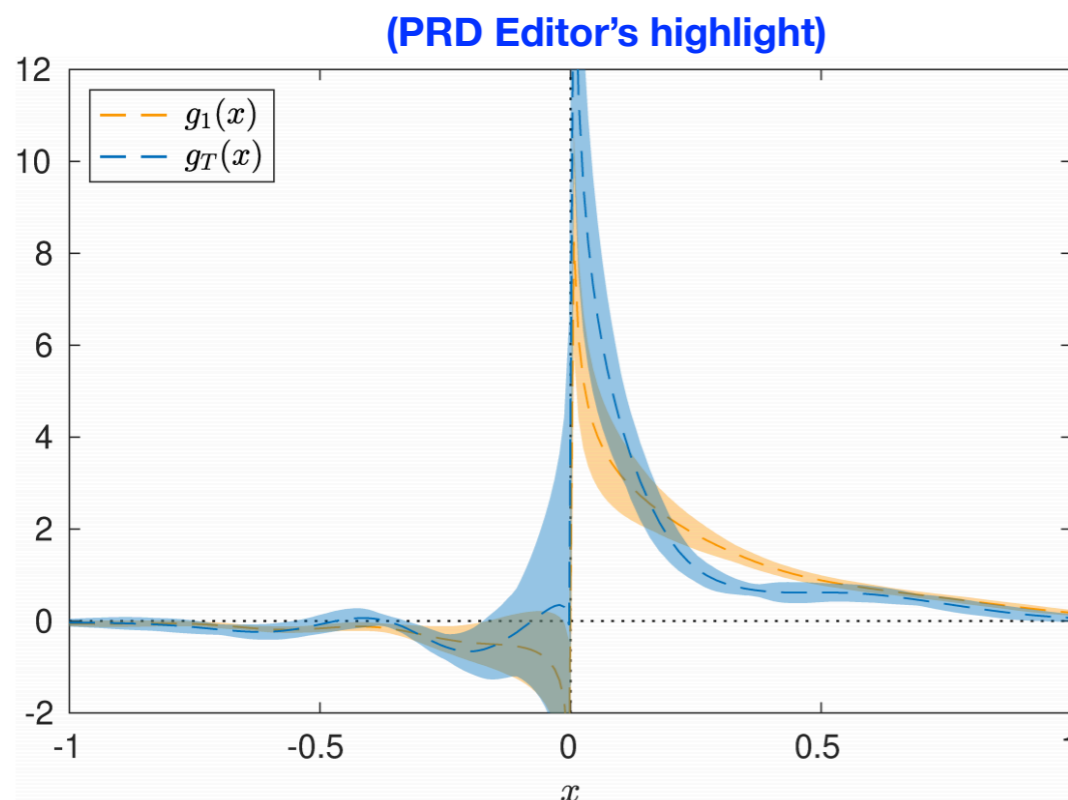
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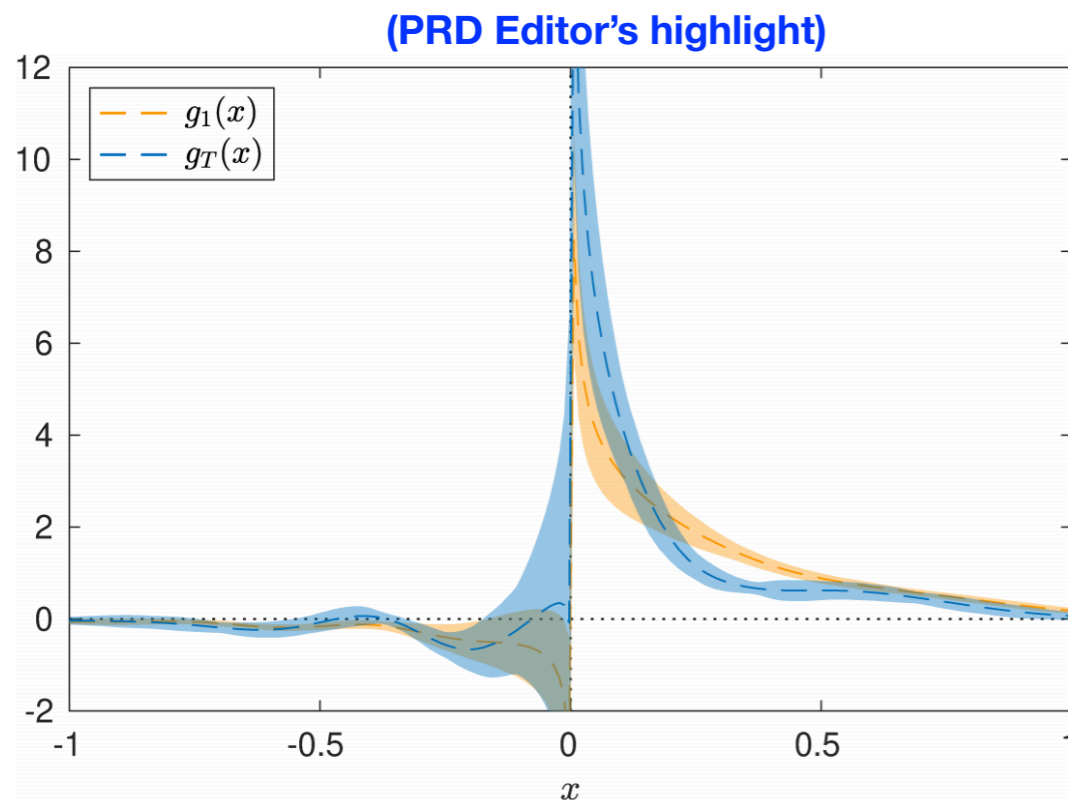
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$$\int_{-1}^1 dx g_1(x) - \int_{-1}^1 dx g_T(x) = 0$$

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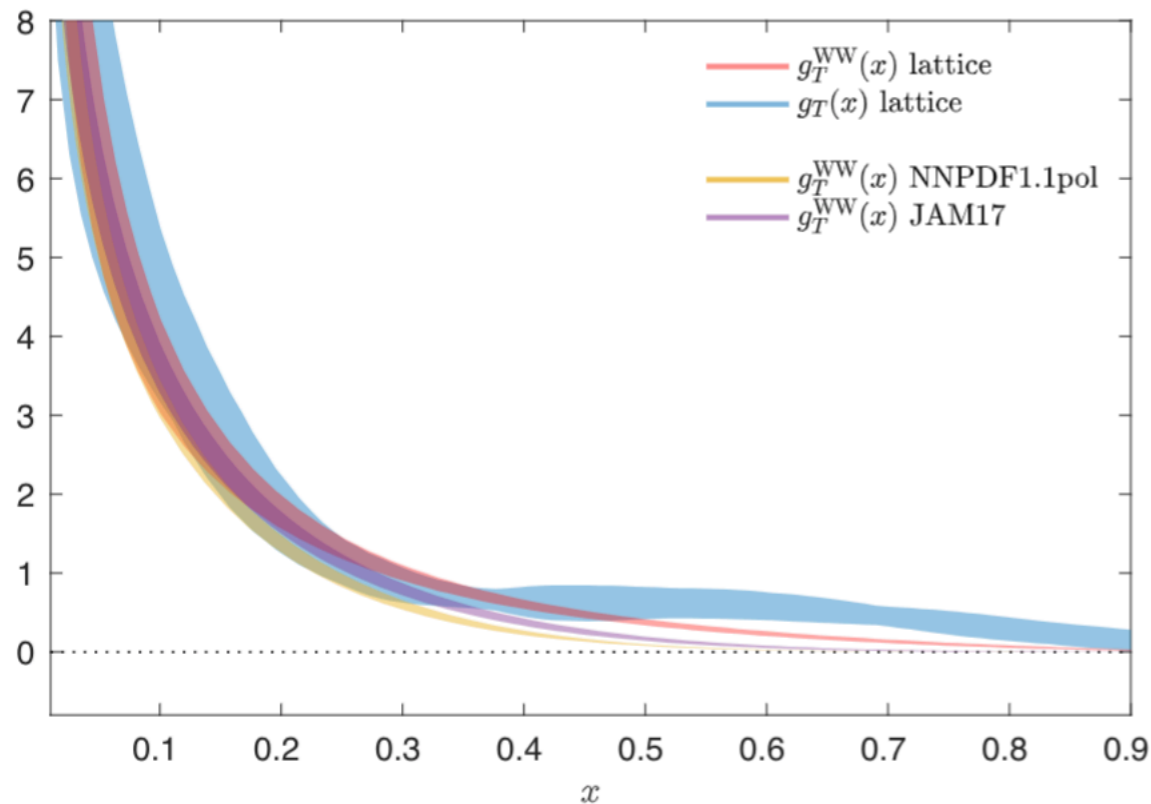
WW approximation:

- **twist-3 $g_T(x)$ determined by the twist-2 $g_1(x)$:** $g_T^{\text{WW}}(x) = \int_x^1 \frac{dy}{y} g_1(y)$

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- $g_T(x)$ agrees with $g_T^{WW}(x)$ for $x < 0.5$ (violations up to 30-40% possible)

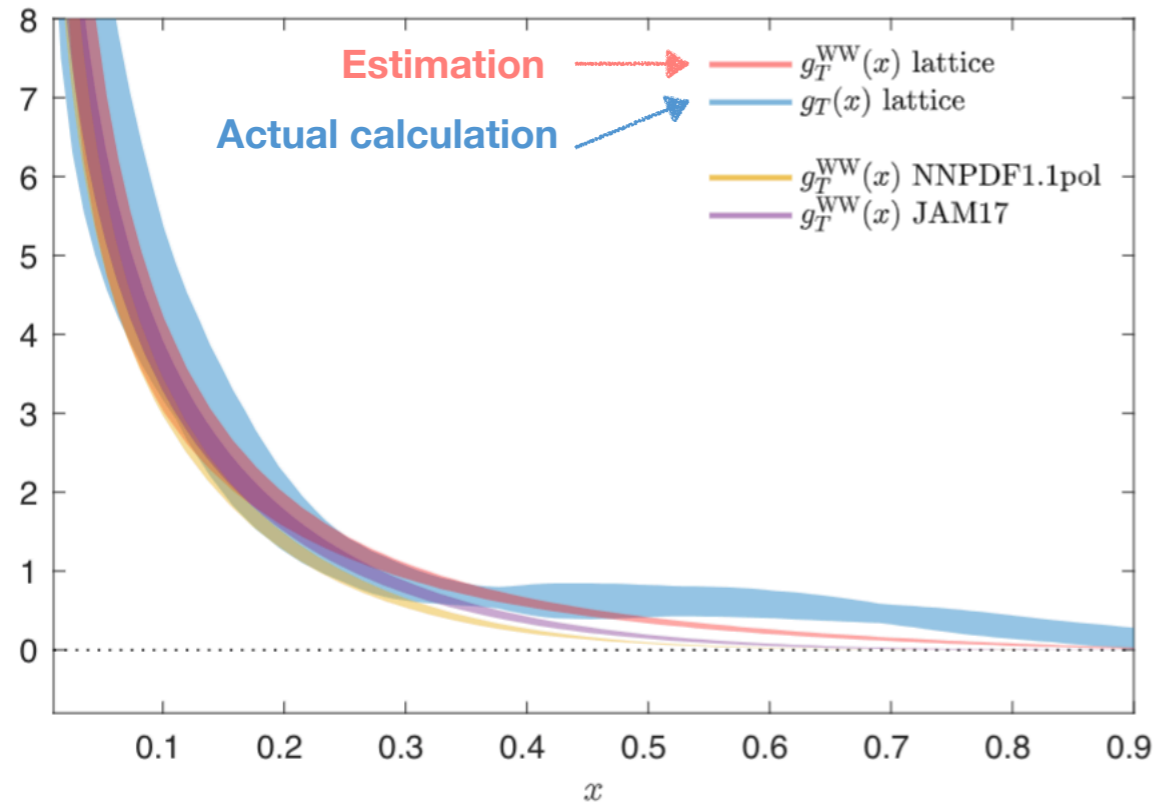
- **Violations of 15-40% expected from experimental data**

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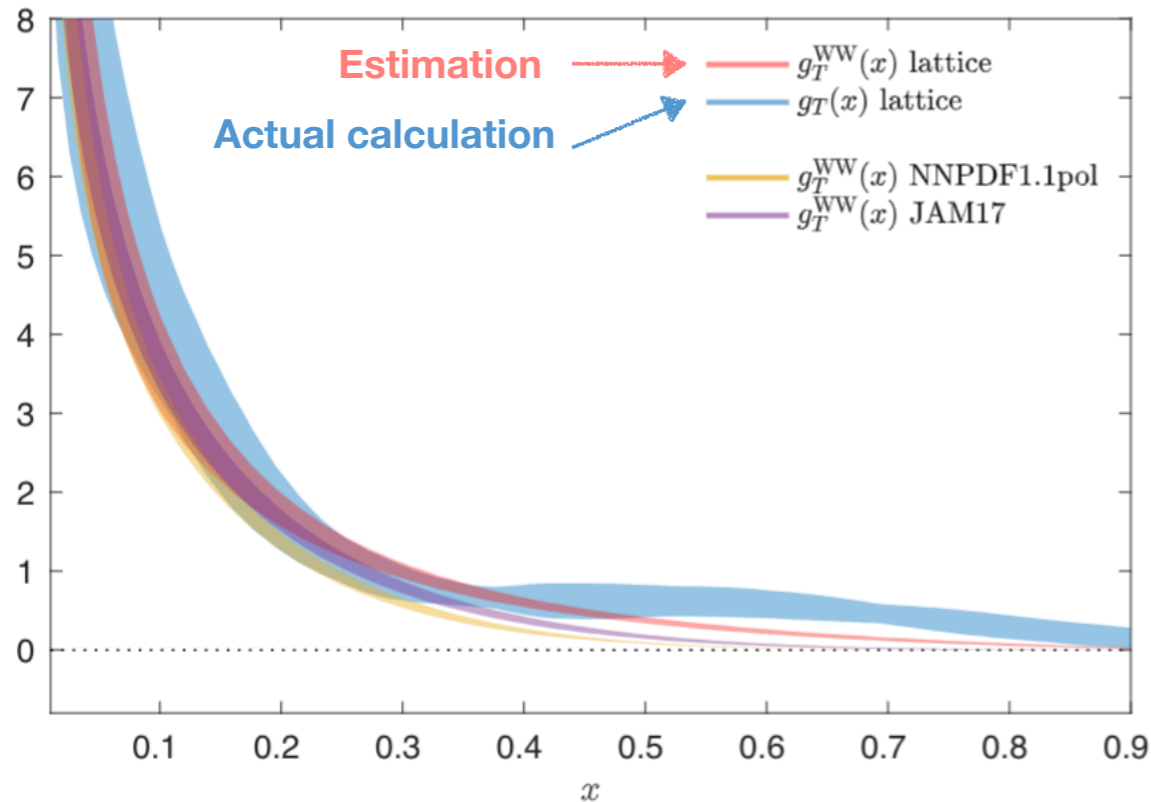
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d_2 -moment: $d_2 = \int dx 3 x^2 [g_T(x) - g_T^{WW}(x)]$

- **Investigated experimentally (JLab Hall A) and found $\mathcal{O}(10^{-3})$**

[D. Flay et al., PRD 94, 5 (2016) 052003, arXiv:1603.03612]

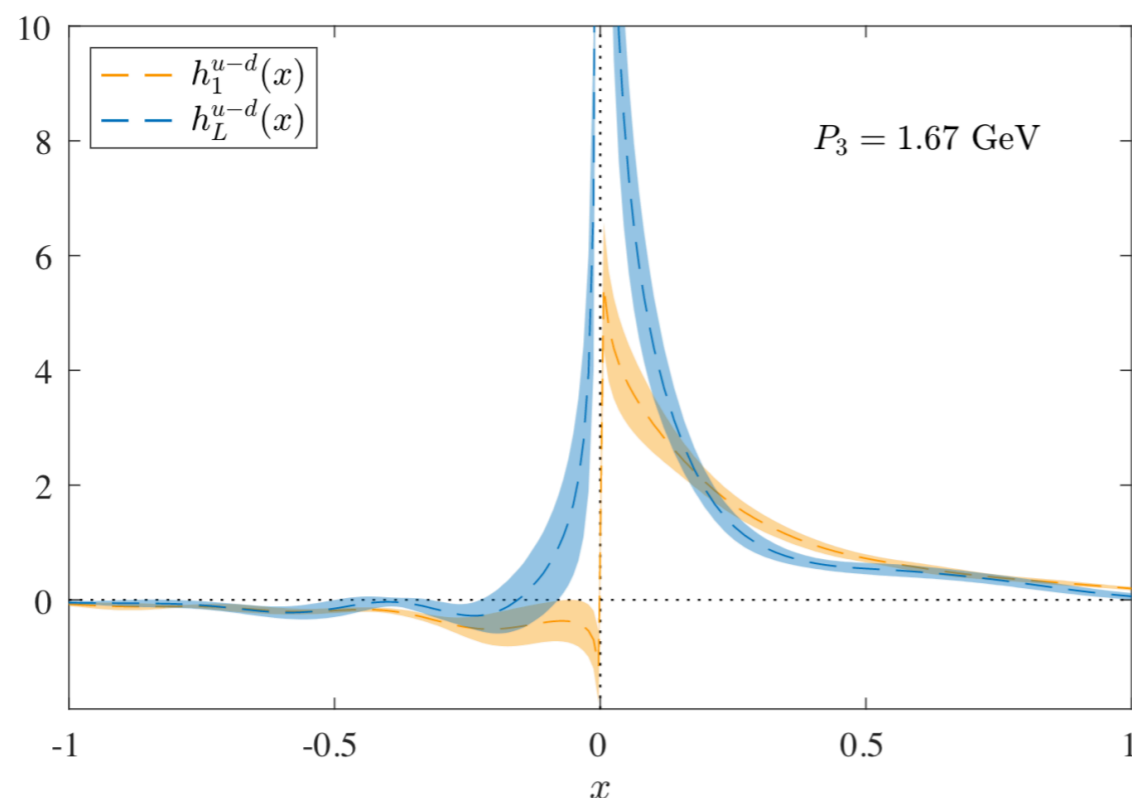
- **High-accuracy results are required to calculate d_2**

Twist-3 $h_L(x)$ PDF

- **Chiral odd: decouple from usual DIS, thus not trivial to extract**
- **$h_L(x)$ accessed via:**
 - **di-hadron single spin asymmetries (CLAS)**
[Gliske et al., PRD 90 (2014) 11, 114027, arXiv:1408.5721; A. Vossen, CIPANP 2018, arXiv: 1810.02435]
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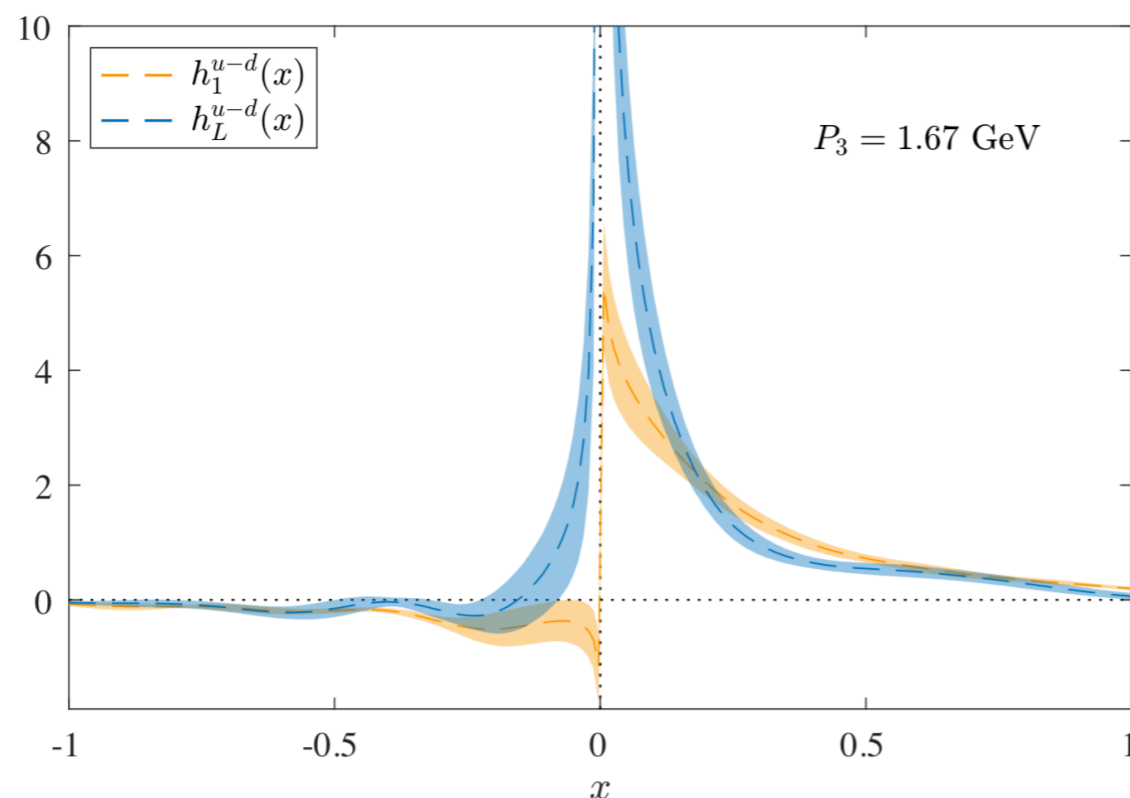
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Burkhardt-Cottingham valid for quasi- $h_L(x)$

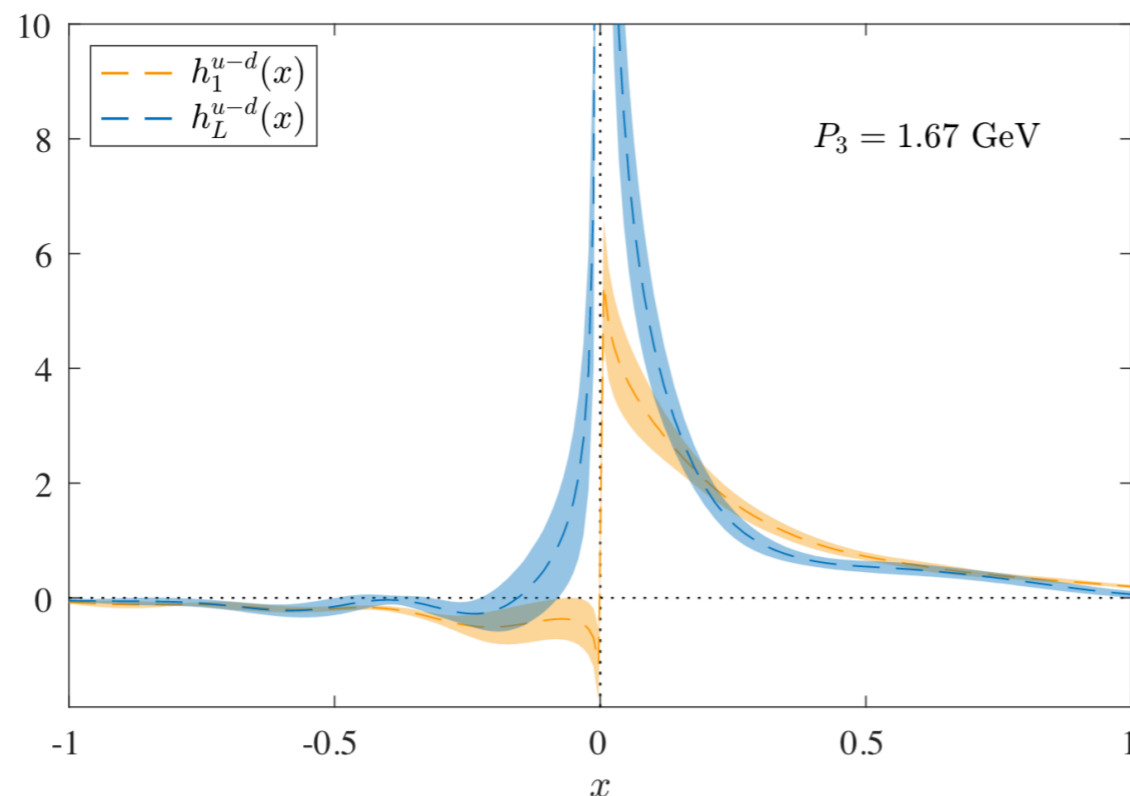
[S. Bhattacharya, A. Metz, arXiv:2105.07282]

$$\int_{-1}^1 dx \tilde{h}_L(x, P_3) = \int_{-1}^1 dx \tilde{h}_1(x, P_3) = g_T$$

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$$\int dx \tilde{h}_L(x, 0.83 \text{ GeV}) = 1.13(08), \quad \int dx \tilde{h}_1(x, 0.83 \text{ GeV}) = 1.02(07)$$

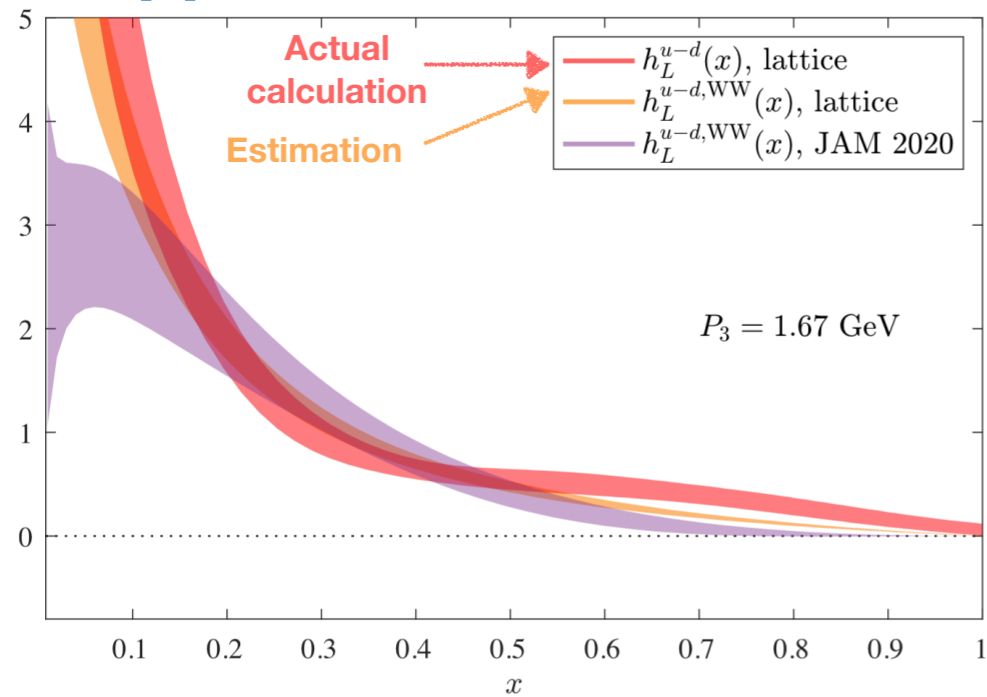
$$\int dx \tilde{h}_L(x, 1.25 \text{ GeV}) = 1.09(10), \quad \int dx \tilde{h}_1(x, 1.25 \text{ GeV}) = 1.07(08)$$

$$\int dx \tilde{h}_L(x, 1.67 \text{ GeV}) = 1.03(16), \quad \int dx \tilde{h}_1(x, 1.67 \text{ GeV}) = 0.94(10)$$

[S. Bhattacharya, K. Cichy, M.C, A. Metz, A. Scapellato, F. Steffens, arXiv:2107.02574]

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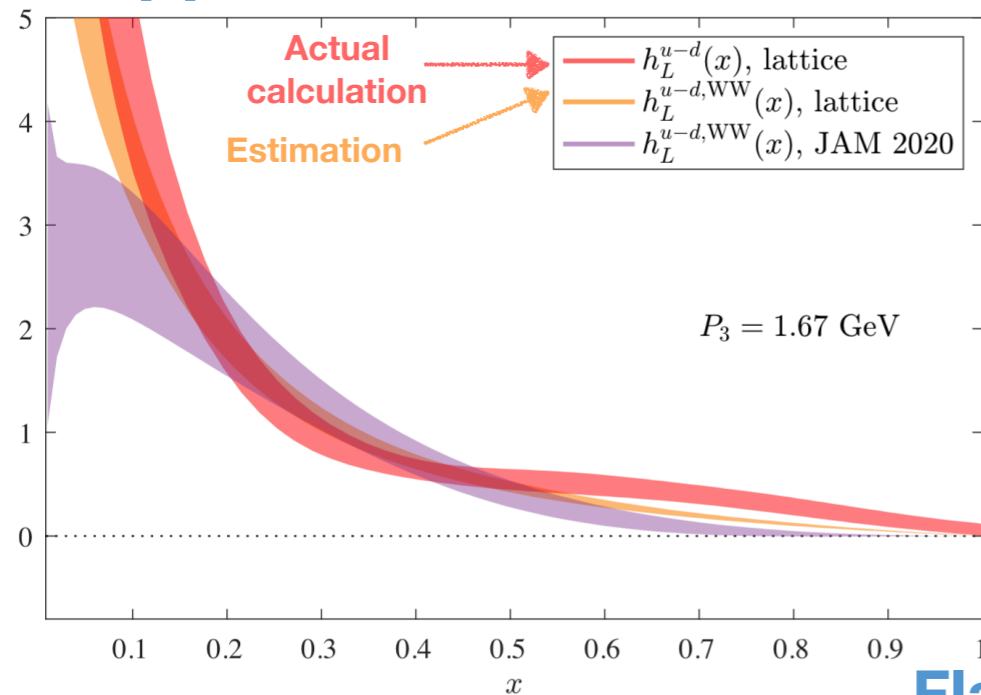


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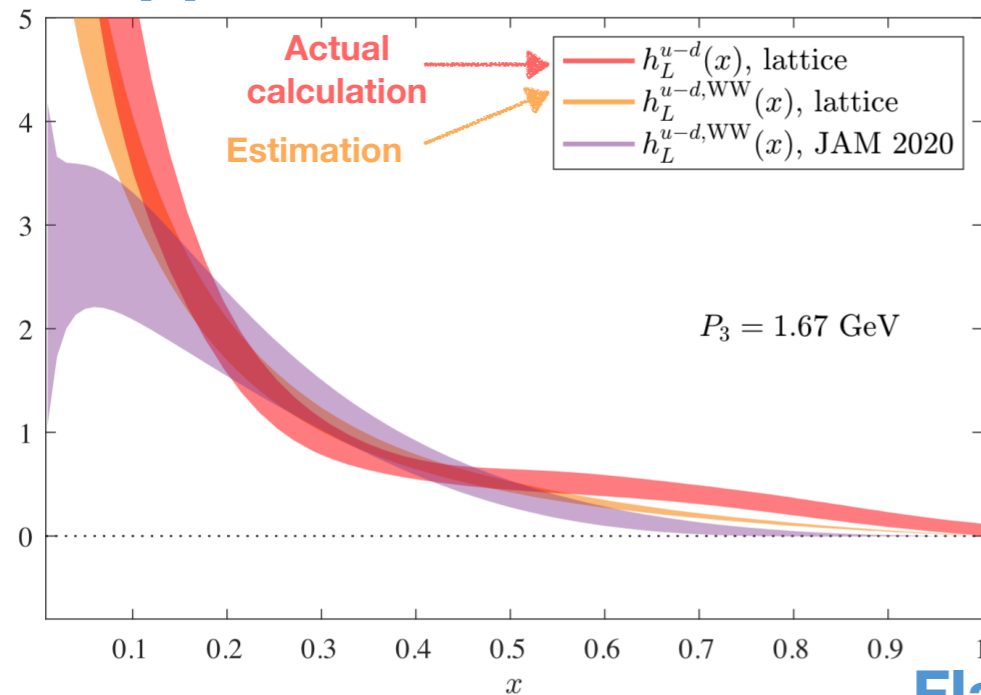
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- Disconnected contributions negligible for transversity PDF **F. Manigrasso talk** [C. Alexandrou et al., arXiv:2106.16065]

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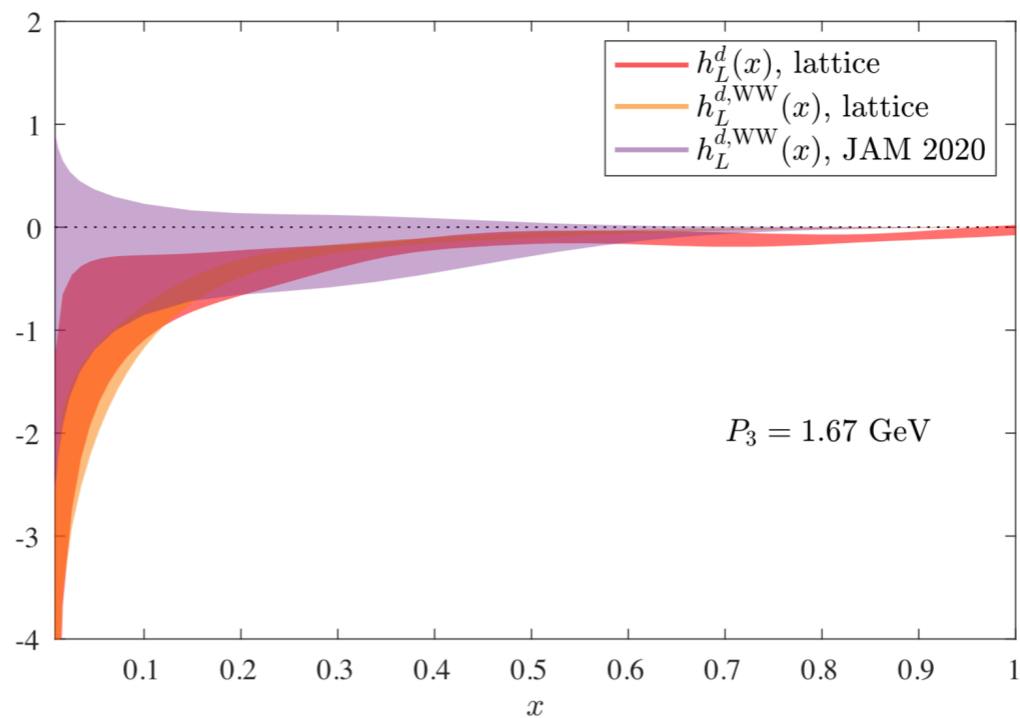
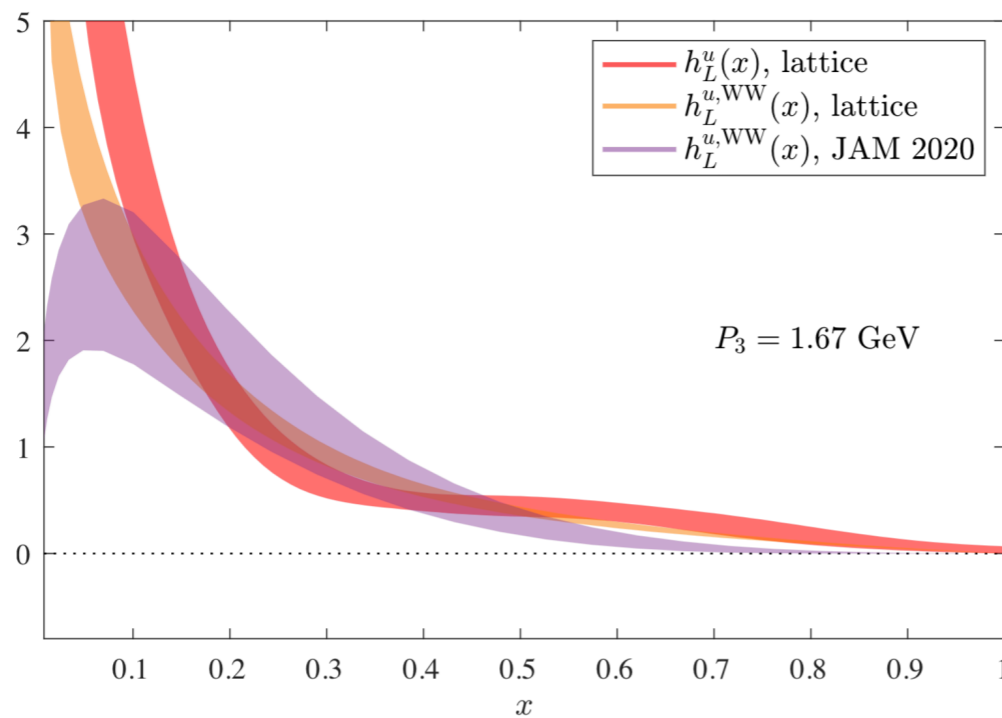


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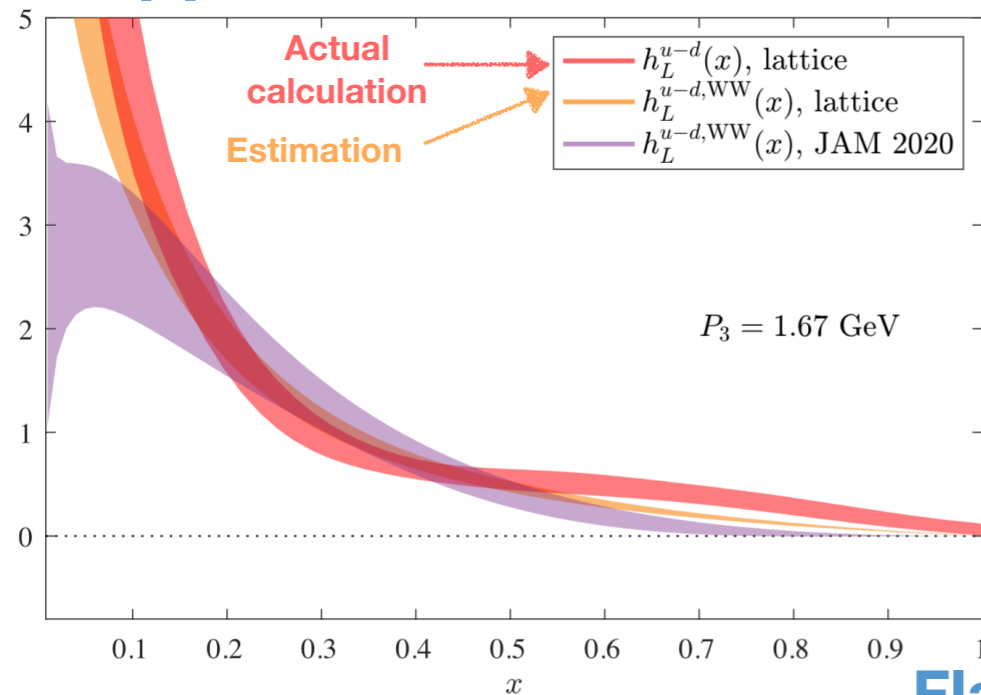
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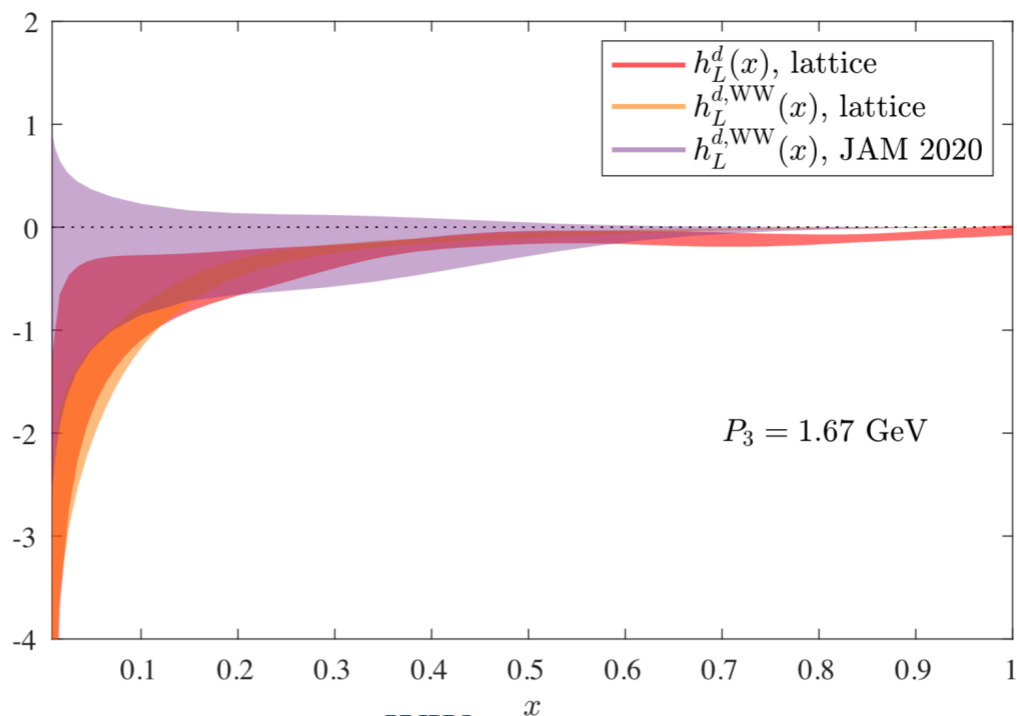
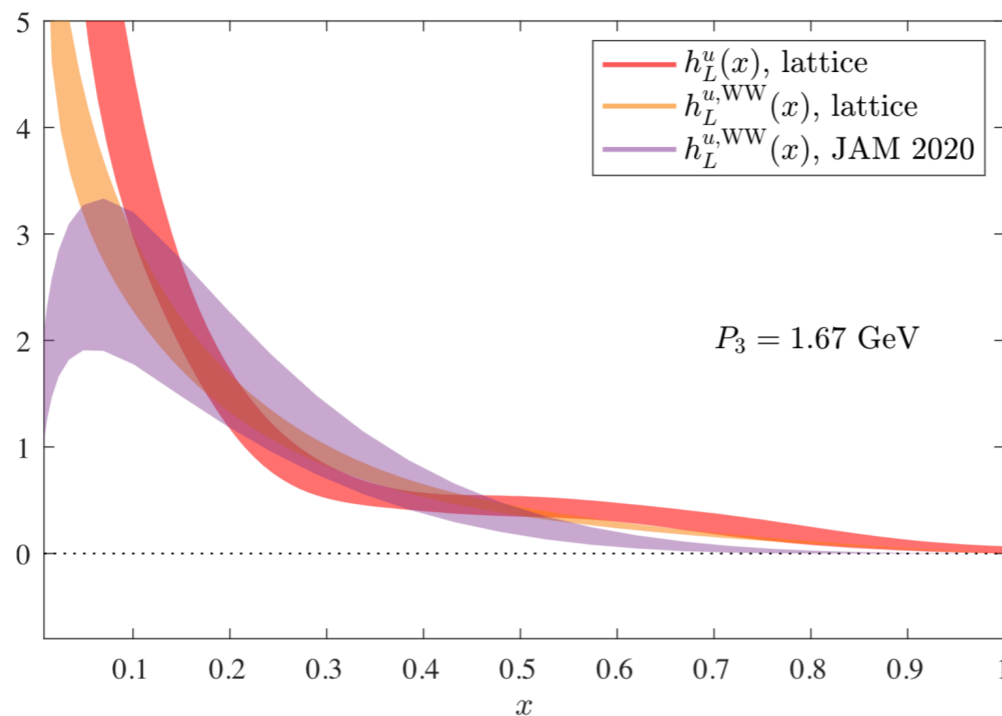


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- h_L^u dominant, and tension is observed between h_L and h_L^{WW}
- h_L^d negative and approaches zero faster than h_L^u

Twist-3 GPDs

- *Twist-3 partonic distributions from lattice QCD*
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Twist-3 GPDs

see also J. Dodson's poster

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Twist-3 GPDs ($\gamma^\mu \gamma^5$)

★ Transverse matrix element of axial operator

$$\tilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+} \tilde{H} + P^\mu \frac{\tilde{e}^+}{P^+} \tilde{E}$$

$$\begin{aligned} h^\mu &= \bar{u}(p') \gamma^\mu u(p), & e^\mu &= \bar{u}(p') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} u(p), & b &= \bar{u}(p') u(p), \\ \tilde{h}^\mu &= \bar{u}(p') \gamma^\mu \gamma^5 u(p), & \tilde{e}^\mu &= \frac{\Delta^\mu}{2m} \tilde{b}, & \tilde{b} &= \bar{u}(p') \gamma^5 u(p) \end{aligned}$$

$$+ \Delta_\perp^\mu \frac{\tilde{b}}{2m} (\tilde{E} + \tilde{G}_1) + \tilde{h}_\perp^\mu (\tilde{H} + \tilde{G}_2) + \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+} \tilde{G}_3 + \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+} \tilde{G}_4 \quad \mu = 1, 2$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et al., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

★ Sum Rules (generalization of Burkhardt-Cottingham)

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[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

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[A. Efremov, O. Teryaev, E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^1 dx x \tilde{G}_1(x, 0, t) = \frac{1}{2} \left[F_2(t) - \int_{-1}^1 dx x \tilde{E}(x, 0, t) \right],$$

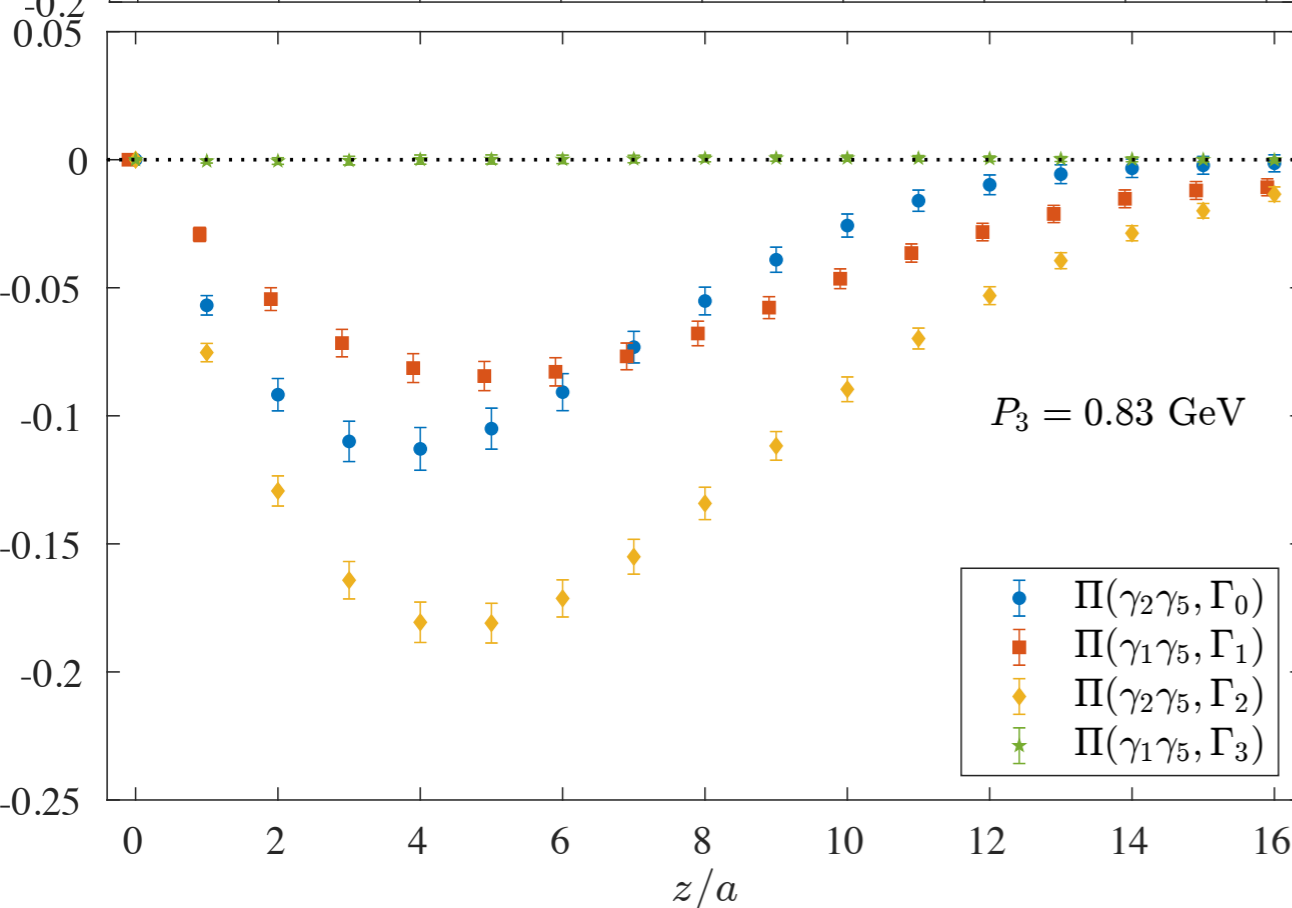
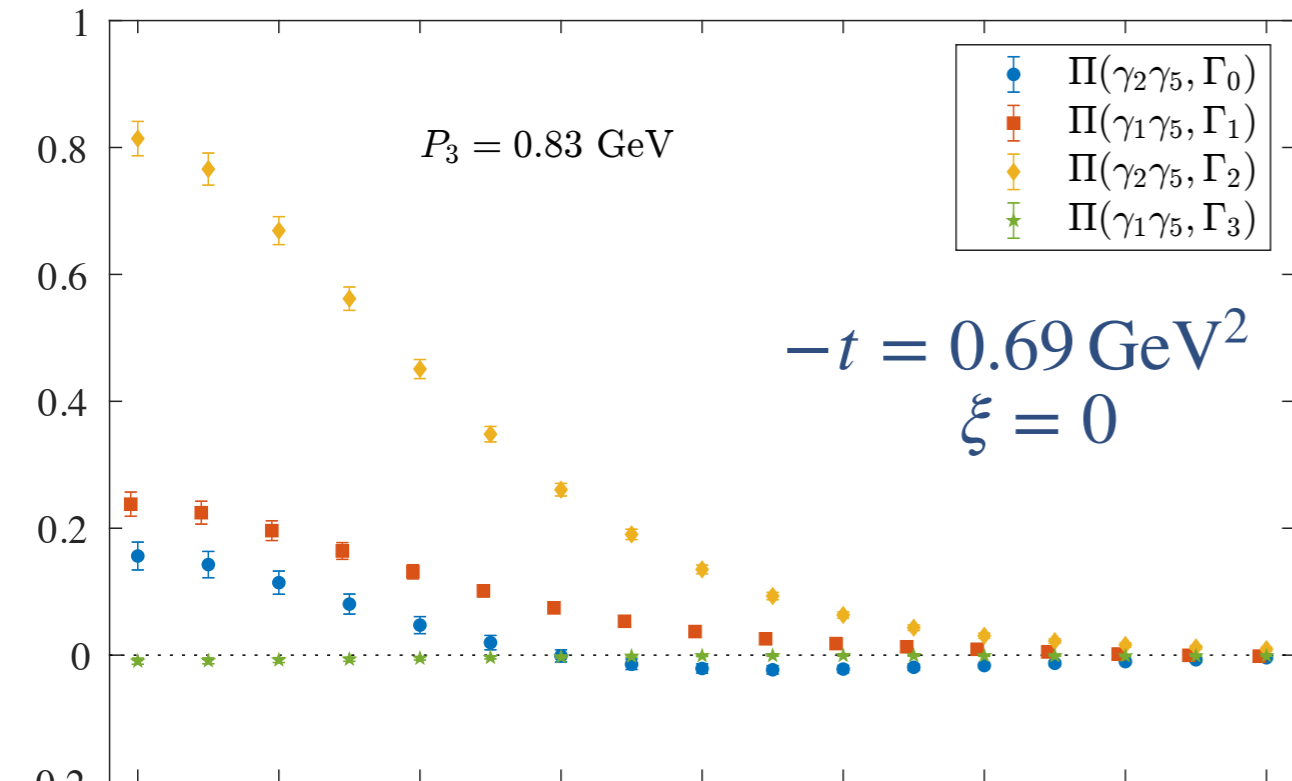
$$\int_{-1}^1 dx x \tilde{G}_2(x, 0, t) = \frac{1}{2} \left[-\frac{t}{4m^2} F_2(t) - \tilde{A}_{20}(t) \right],$$

$$\int_{-1}^1 dx x \tilde{G}_3(x, 0, t) = 0$$

$$\int_{-1}^1 dx x \tilde{G}_4(x, 0, t) = \frac{1}{4} G_E(t)$$

F_2 : Pauli FF
 G_E : electric FF
 $\tilde{A}_{20}, \tilde{B}_{20}$: axial GFFs

Bare matrix elements (ME)



★ $\Pi(\gamma^2\gamma^5, \Gamma_0)$ & $\Pi(\gamma^2\gamma^5, \Gamma_2)$:
disentangle $\widetilde{H} + \widetilde{G}_2, \widetilde{G}_4$

★ $\Pi(\gamma^1\gamma^5, \Gamma_1)$ and $\widetilde{H} + \widetilde{G}_2$:
disentangle $\widetilde{E} + \widetilde{G}_1$

★ $\Pi(\gamma^1\gamma^5, \Gamma_3)$ gives \widetilde{G}_3

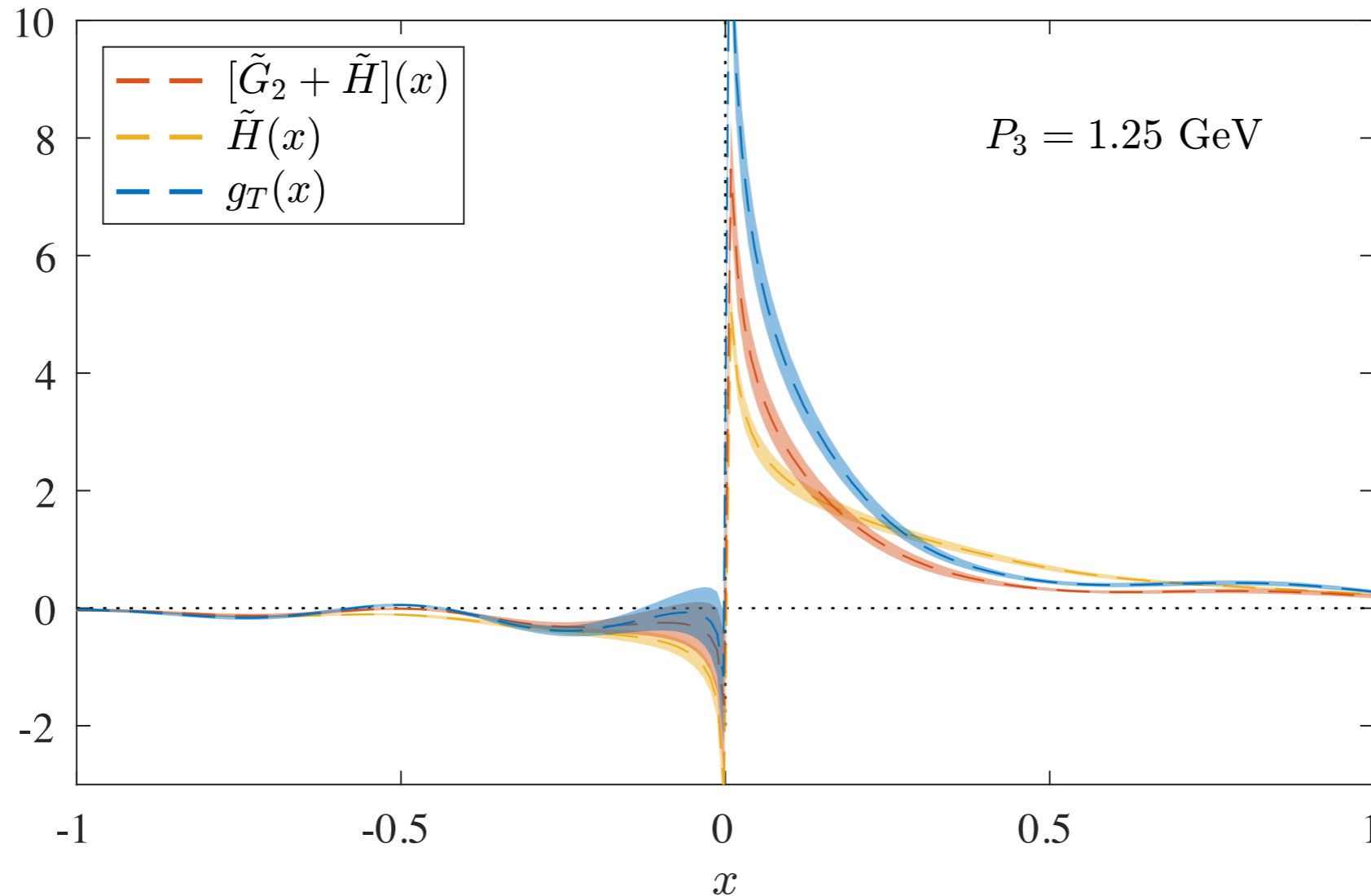
★ Similar picture for $P_3 = 1.25 \text{ GeV}$

★ Real part of ME: dominant

★ \widetilde{G}_3 is kinematically suppressed

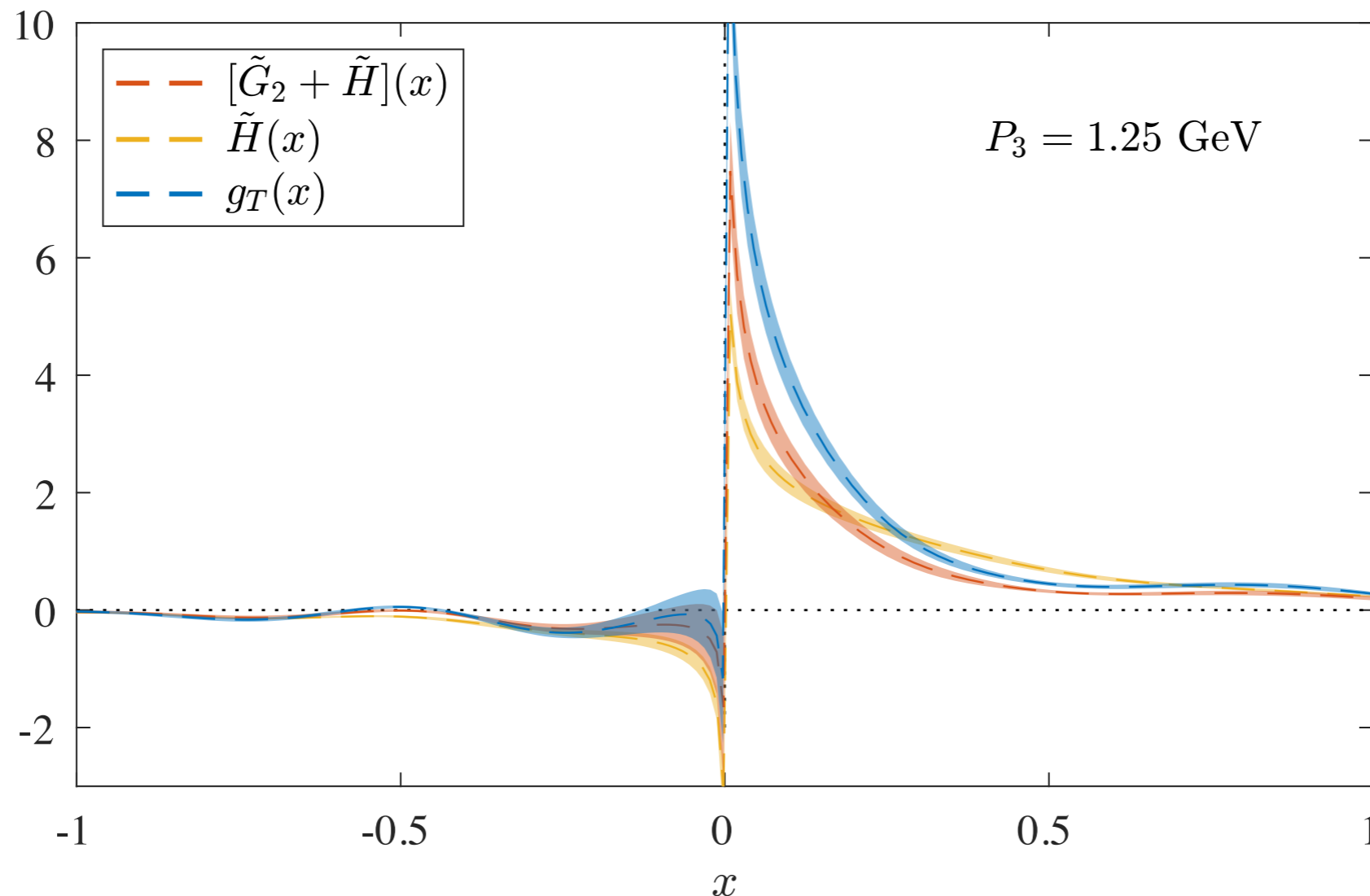
x-dependence of twist-3 GPDs

$$-t = 0.69 \text{ GeV}^2, \quad \xi = 0$$



x-dependence of twist-3 GPDs

$$-t = 0.69 \text{ GeV}^2, \quad \xi = 0$$



★ $g_T(x)$: dominant distribution

★ $\tilde{H} + \tilde{G}_2$ similar in magnitude to \tilde{H} For twist-2 GPDs see A. Scapellato's talk

★ \tilde{G}_2 is expected to be small

Concluding Remarks

- ★ quasi-PDFs formalism can be generalized beyond leading twist:
 $g_T(x)$, $h_L(x)$, $e(x)$
- ★ We address computationally expensive calculations
GPDs with signal comparable to PDFs
- ★ Several improvements - mixing with quark-gluon-quark correlator

Concluding Remarks

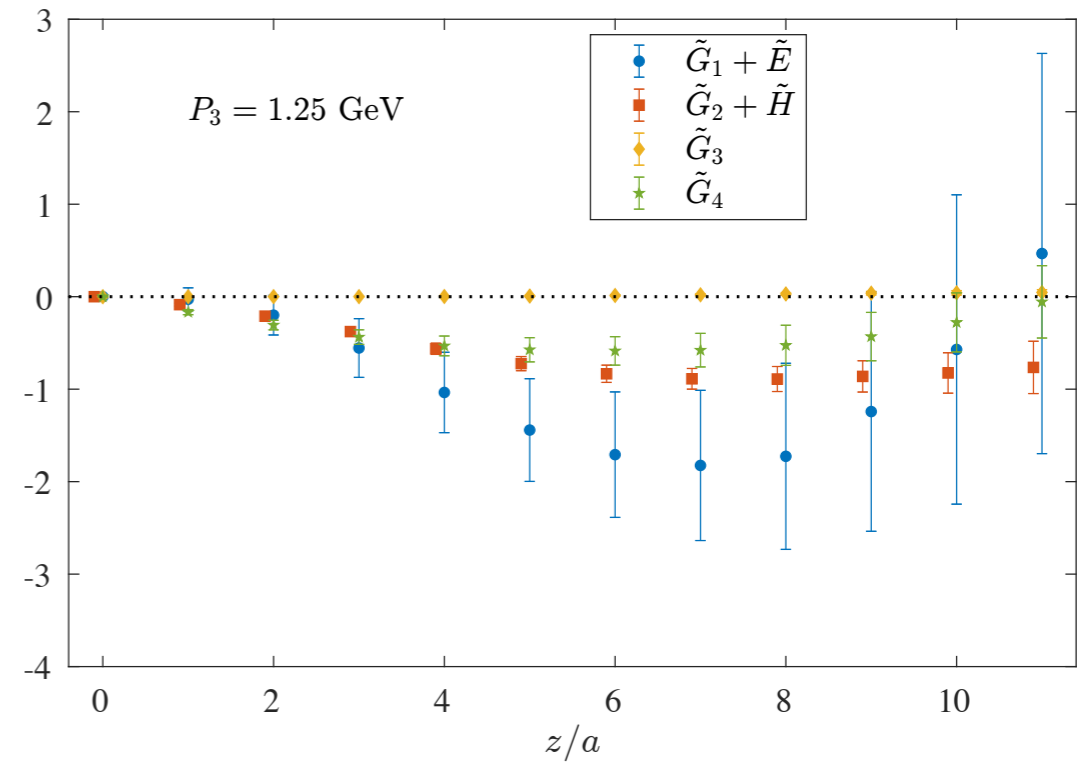
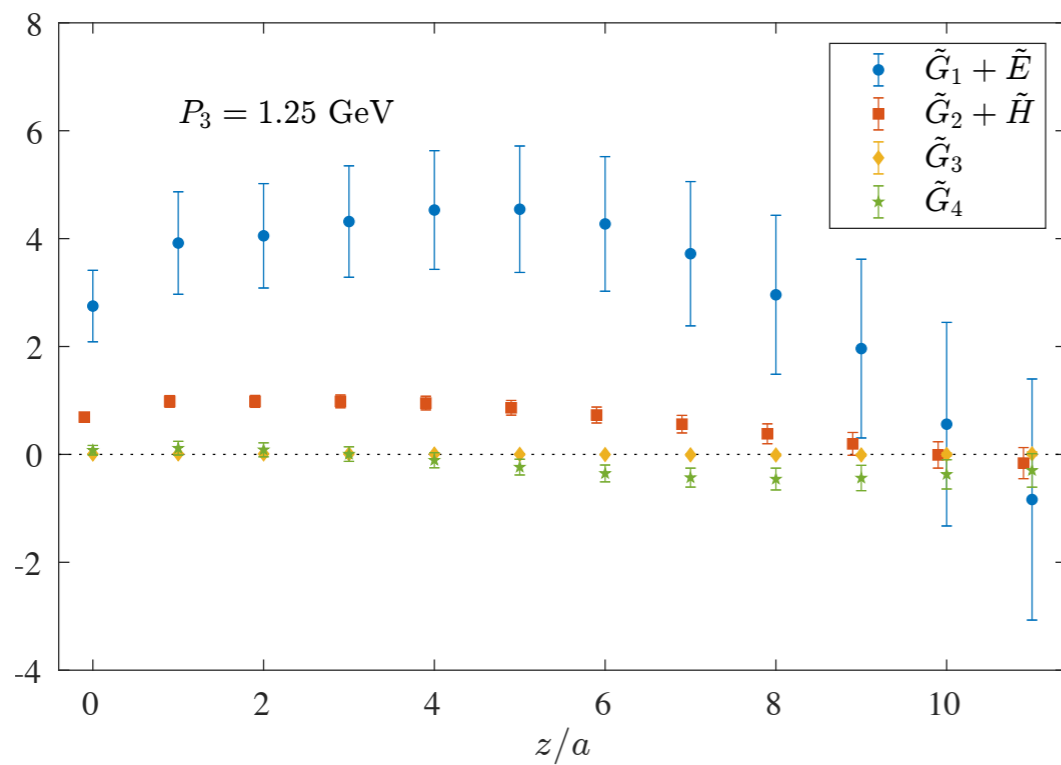
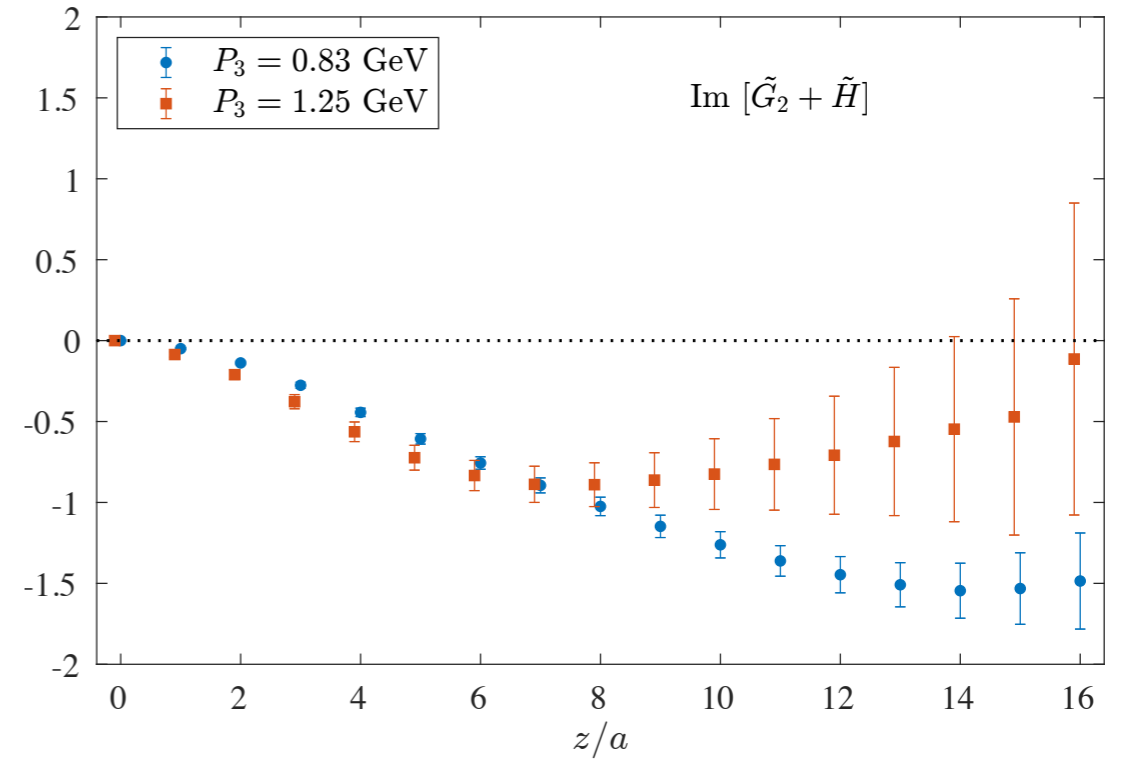
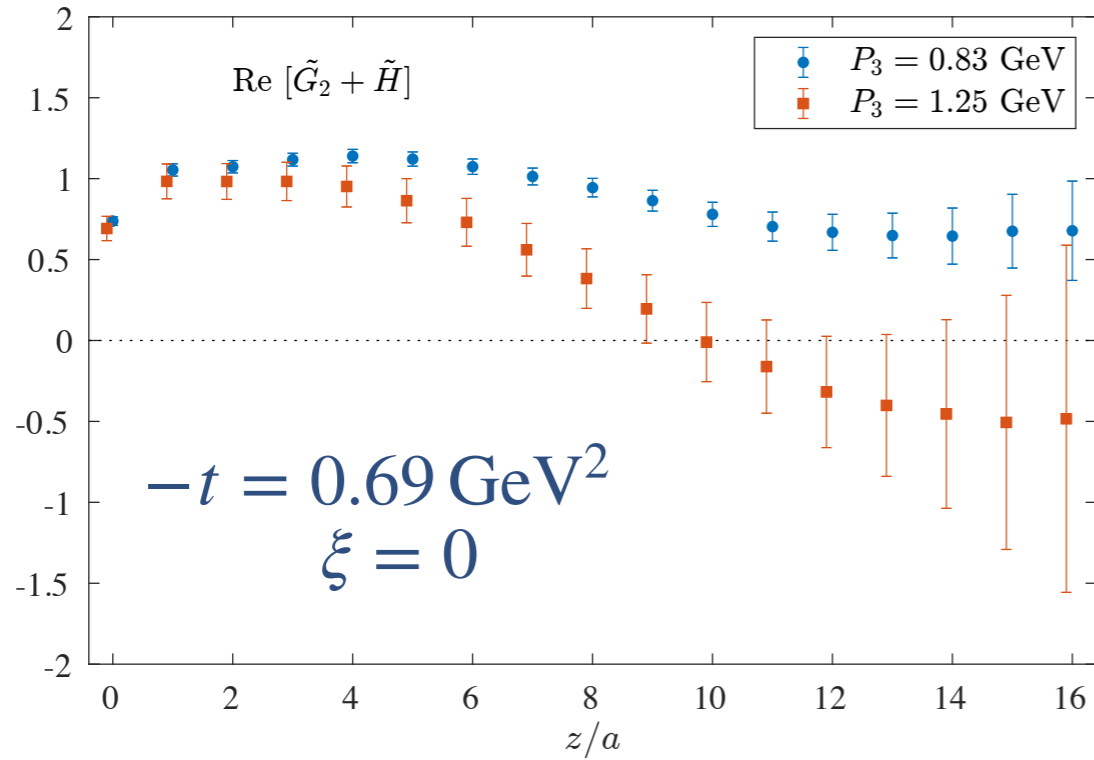
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Thank you

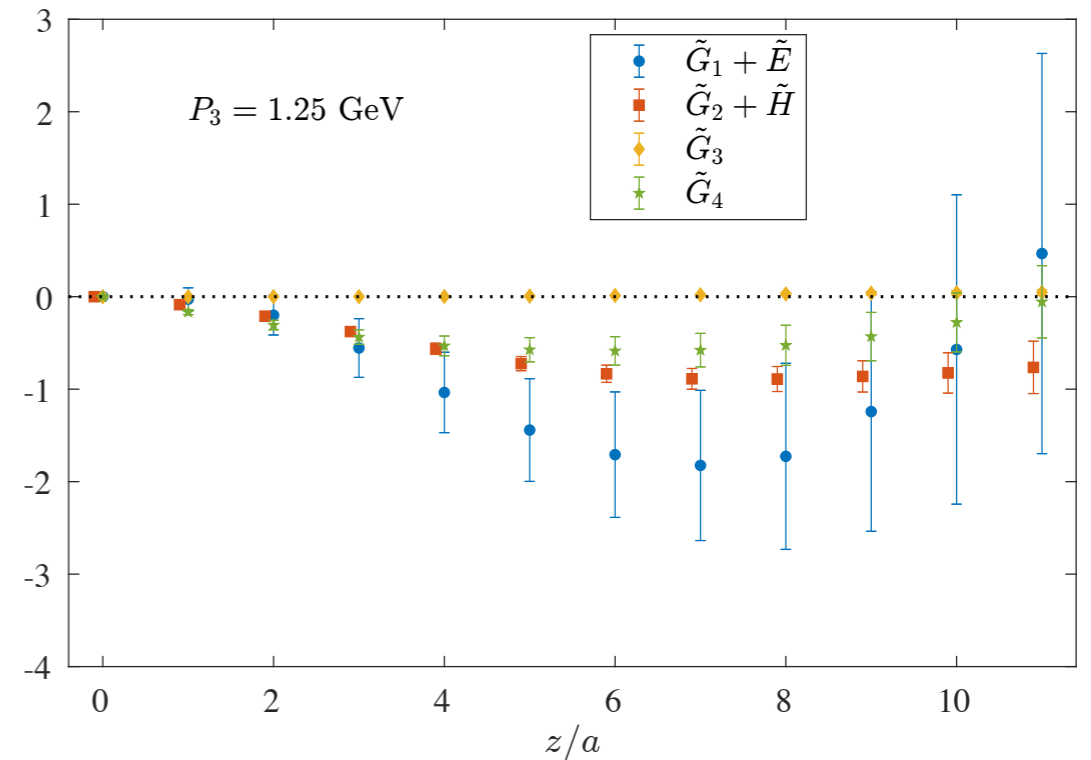
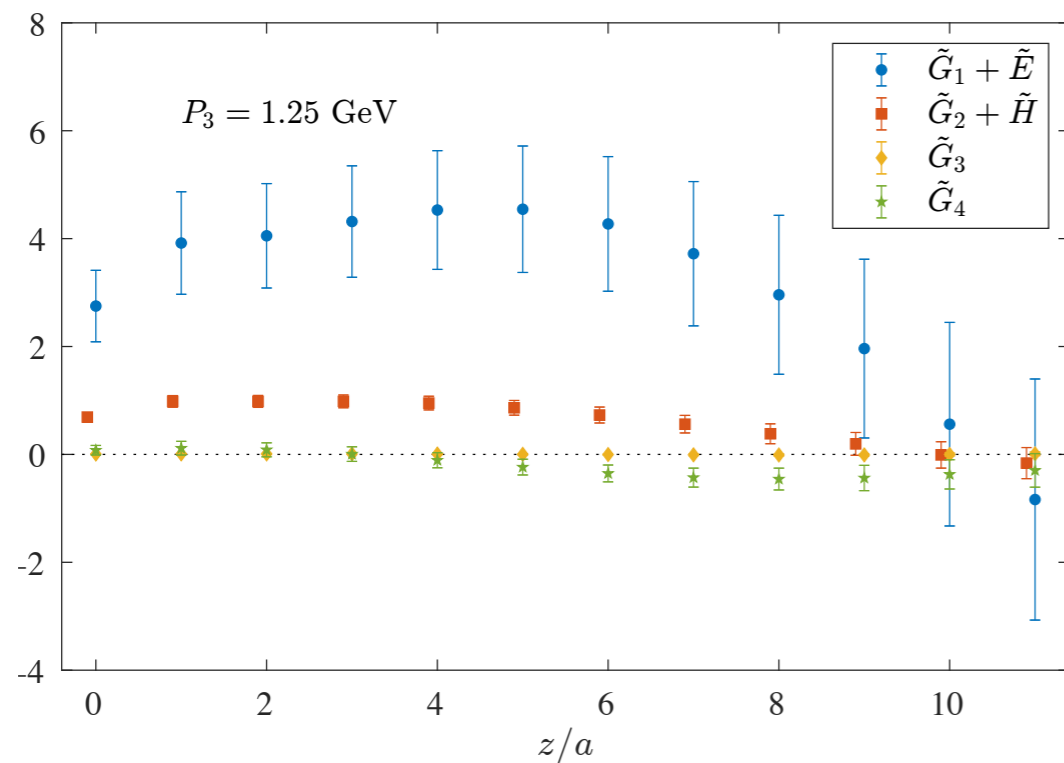
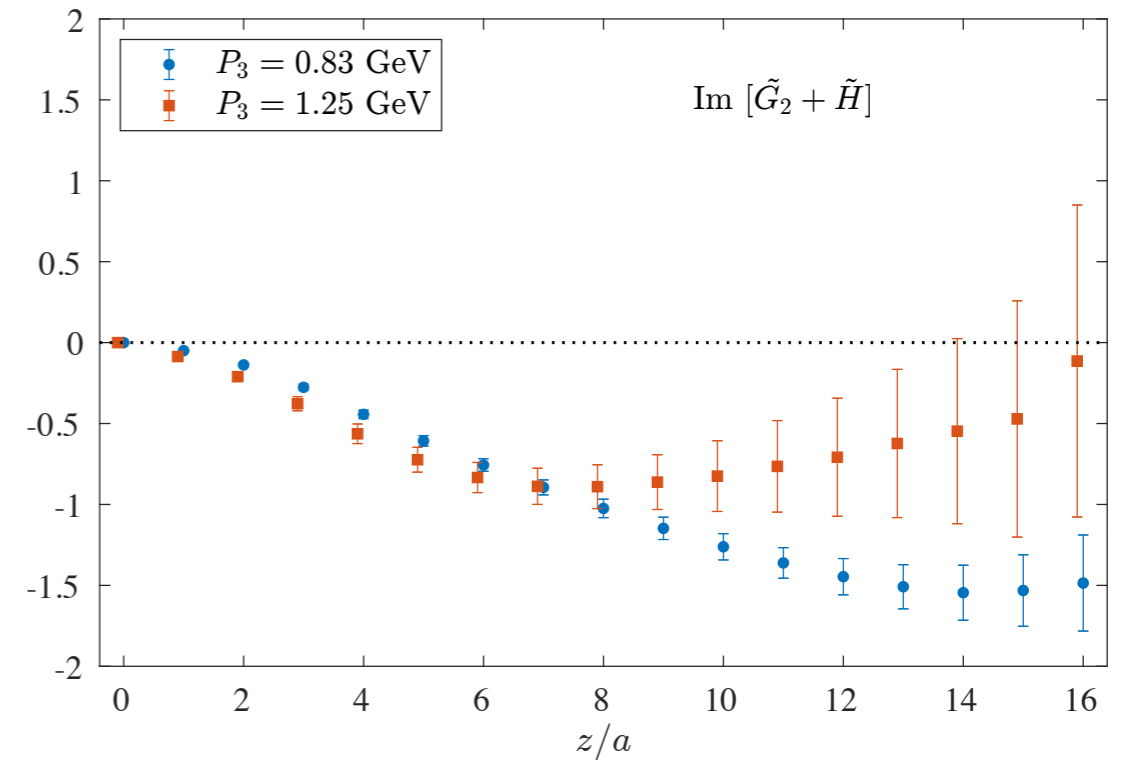
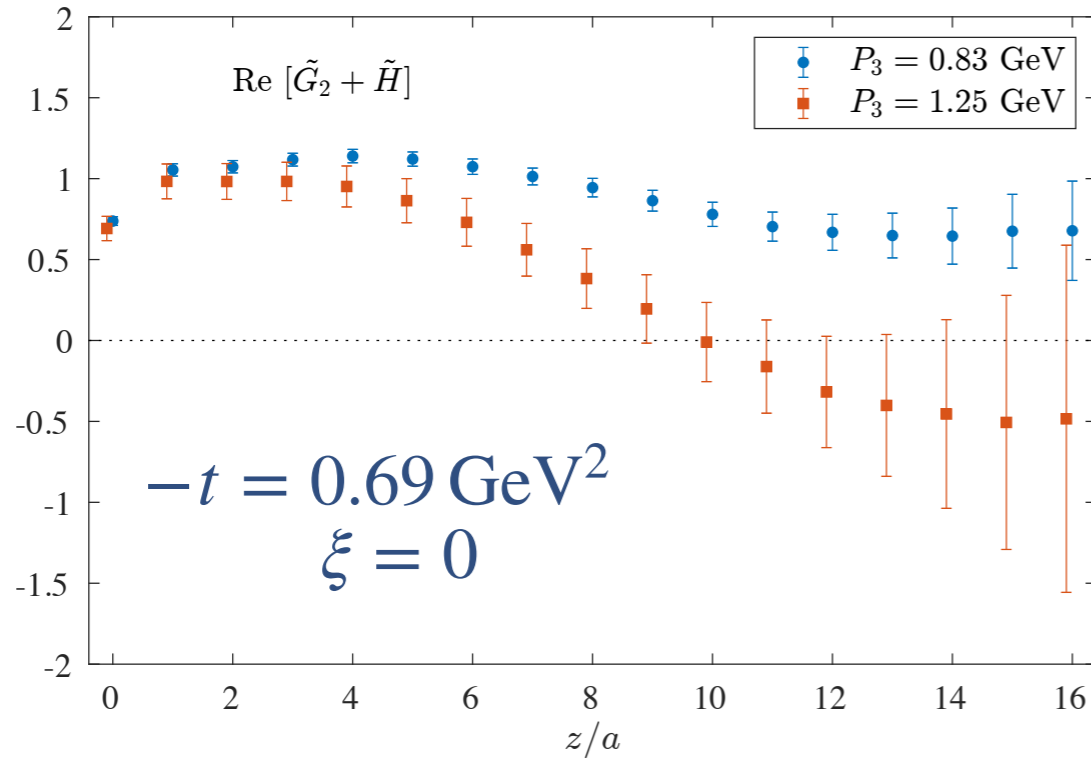


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Matrix elements decomposition



Matrix elements decomposition



★ $\tilde{E} + \tilde{G}_1$: highest contribution

★ \tilde{G}_3, \tilde{G}_4 : suppressed