

Pion and Kaon form factors using twisted-mass fermions

Joseph Delmar¹ Constantia Alexandrou^{2,3} Simone Bacchio^{2,3} Ian Cloët⁴ Martha Constantinou¹ Kyriakos Hadjiyiannakou^{2,3} Giannis Koutsou² Colin Lauer^{1,4} Alejandro Vaquero⁵

¹Temple University ²University of Cyprus ³The Cyprus Institute ⁴Argonne National Laboratory ⁵University of Utah

Abstract

We present a calculation of the connected contributions to the pion and kaon scalar, vector and tensor form factors. We use an ensemble of $N_f = 2 + 1 + 1$ maximally twisted mass fermions with clover improvement, with a pion mass of 260 MeV, and a kaon mass of 530 MeV. The lattice spacing of the ensemble is 0.093 fm and the lattice has a spatial extent of 3 fm. We analyze several values of the source-sink time separation to eliminate excited-states effects. The scalar and tensor form factors are converted to the $\overline{\text{MS}}$ scheme at a scale of 2 GeV. We also address the effect of SU(3) flavor symmetry breaking.

Motivation

- Form factors (FFs) give access to properties describing hadron structure
 - Scalar FFs: exploration of interplay between EHM mechanism and Higgs boson interaction
 - Vector FFs: electromagnetic properties
 - Tensor FFs: useful for beyond the Standard model studies
- An important QCD property is spontaneous chiral symmetry breaking (SCSB)
 - QCD Lagrangian would have full chiral symmetry
 - Would give rise to massless Goldstone bosons (i.e. pions and kaons)
 - Mass difference of pion and kaon: interplay between QCD dynamics and quark-mass effect
- Limited experimental data
 - Pion less studied than the nucleon. Kaon structure is very limitedly studied

Lattice Setup

- Ensemble of $N_f = 2 + 1 + 1$ twisted mass fermions

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

- Each matrix element decomposes to one form factor (spin-0 mesons) with a normalization $C = 1/\sqrt{4E(p)E(p')}$ and $E(p) = \sqrt{m^2 + p^2}$:

$$\langle p' | \mathcal{O}_S | p \rangle = C F_S^{M,q}, \quad \langle p' | \mathcal{O}_V^\mu | p \rangle = -2i C P^\mu F_V^{M,q}, \quad \langle p' | \mathcal{O}_T^{\mu\nu} | p \rangle = i C \frac{(P^\mu \Delta^\nu - P^\nu \Delta^\mu)}{m_M} F_T^{M,q}$$

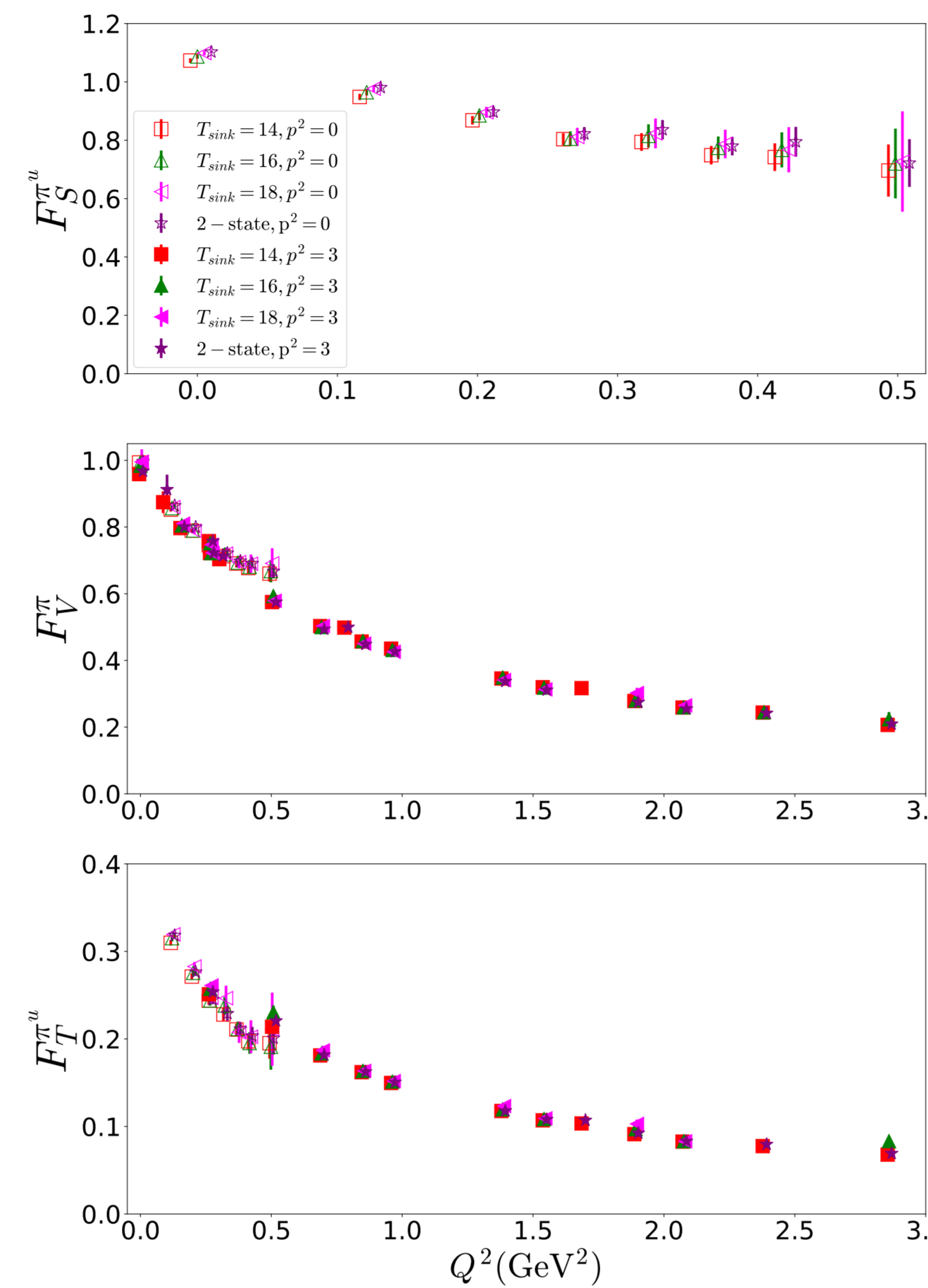
- We explore both rest and boosted frame ($p = 2\pi\mathbf{n}'/L$) for a wide range of Q^2

Statistics						
\mathbf{n}'	t_s/a	N_{confs}	N_{src}	$N_{p'}$	Total statistics	
(0,0,0)	12, 14, 16, 18, 20, 24	122	16	1	1,952	
($\pm 1, \pm 1, \pm 1$)	12	122	16	8	15,616	
($\pm 1, \pm 1, \pm 1$)	14, 16, 18	122	72	8	70,272	

- Study of excited-states effects: **plateau fit** (1.12-2.23 fm) and **2-state fit**

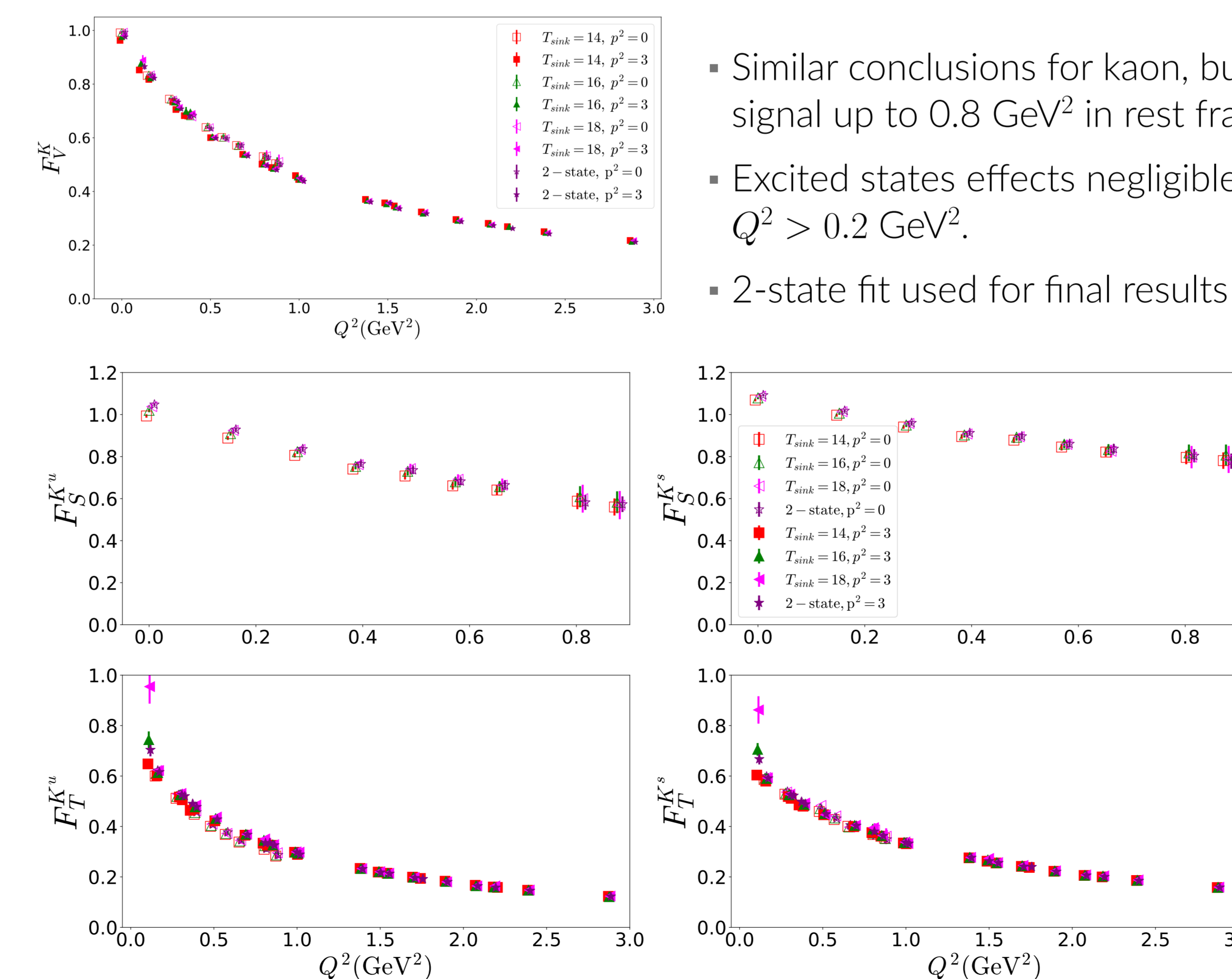
Results

Pion



- Rest frame: signal up to 0.5 GeV² ($|Q_{\text{max}}|/m_\pi \sim 2.5$)
- Boosted frame: signal up to 3 GeV² ($|Q_{\text{max}}|/E_0 \sim 2$)
- FFs in rest and boosted frame compatible
- Scalar FF only in rest frame (increased gauge noise)
- Tensor FF suppressed by quark mass (chirality flip on a quark line)
- Excited states contamination:
 - suppressed for $t_s \geq 1.3$ fm in vector FF
 - scalar and tensor FFs suffer from severe excited states ($t_s \geq 1.7$ fm)
- 2-state fit employed for final results

Kaon



- Similar conclusions for kaon, but signal up to 0.8 GeV² in rest frame
- Excited states effects negligible for $Q^2 > 0.2$ GeV².
- 2-state fit used for final results

Radii

- Parameterizing the form factors allows extraction of the radius
- 1-parameter or 2-parameter fit for scalar and vector
- $F_T(0)$ not determined from lattice data (vanishing kinematic factor)
- Fits on rest and boosted frame combined

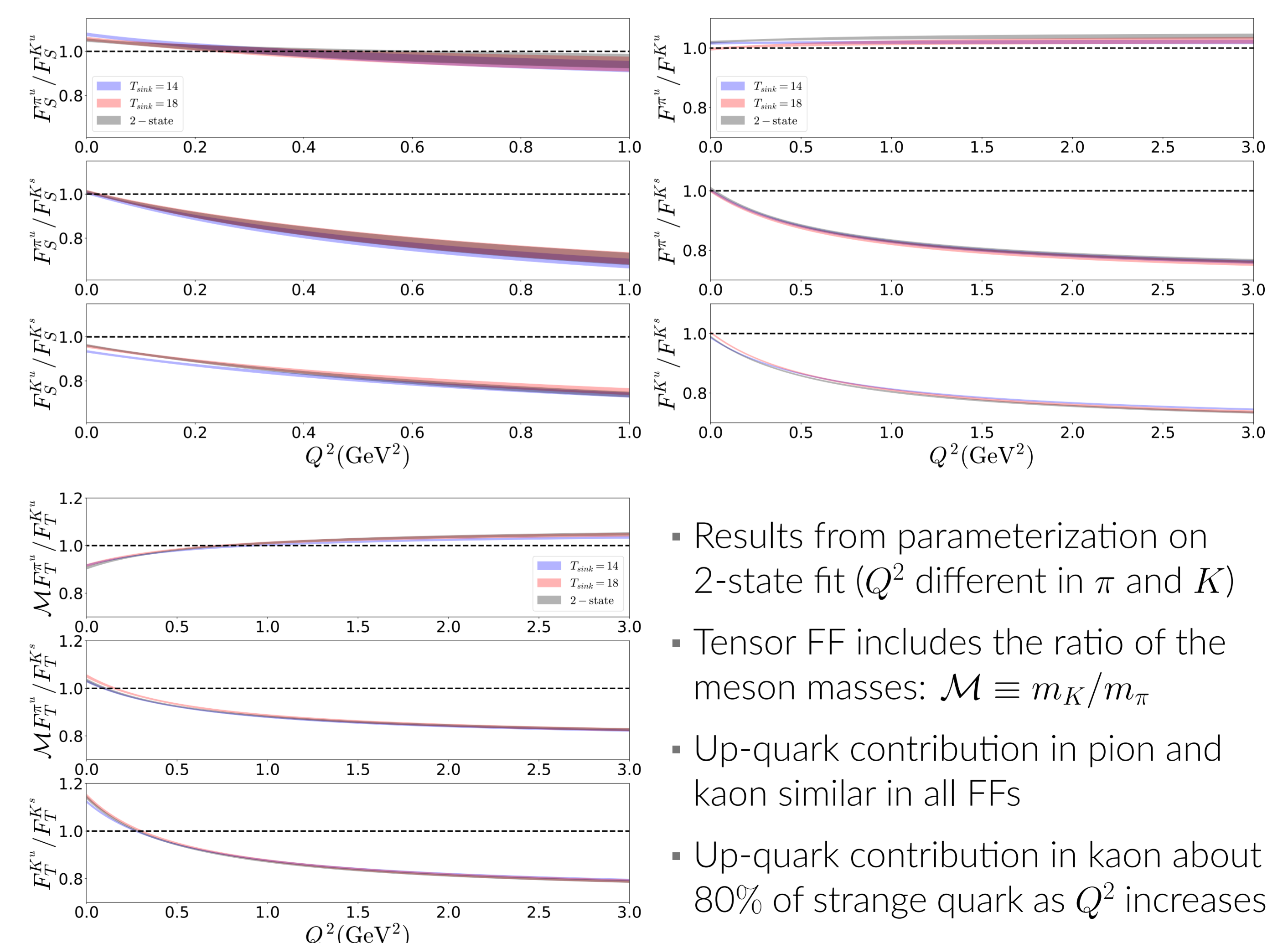
$$F^\Gamma(Q^2) = \frac{F^\Gamma(0)}{1 + \frac{Q^2}{M_\Gamma^2}}, \quad \langle r^2 \rangle^\Gamma = -\frac{6}{F^\Gamma(0)} \left. \frac{\partial F^\Gamma(Q^2)}{\partial Q^2} \right|_{Q^2=0} = \frac{6}{M_\Gamma^2}.$$

Preliminary

$\langle r^2 \rangle_{S,1}^u$ (fm ²)	$\langle r^2 \rangle_{S,2}^u$ (fm ²)	$\langle r^2 \rangle_{V,1}^u$ (fm ²)	$\langle r^2 \rangle_{V,2}^u$ (fm ²)	$\langle r^2 \rangle_{T,2}^u$ (fm ²)
0.258(28)	0.258(28)	0.3105(47)	0.3276(54)	0.3533(97)
$\langle r^2 \rangle_{S,2}^K$ (fm ²)	$\langle r^2 \rangle_{S,2}^s$ (fm ²)	$\langle r^2 \rangle_{V,2}^K$ (fm ²)	$\langle r^2 \rangle_{T,2}^K$ (fm ²)	$\langle r^2 \rangle_{T,2}^{K^*}$ (fm ²)
0.196(26)	0.0518(85)	0.2932(47)	0.4217(48)	0.2670(32)

- Results for scalar and vector radius lower than PDG value ($\langle r^2 \rangle^\pi = 0.434(4)$ fm², $\langle r^2 \rangle^K = 0.314(25)$ fm²) due to significant pion mass dependence
- Results agree with other lattice data at similar m_π (JLQCD, arXiv:0810.2590)

SU(3) Flavor Symmetry Breaking



- Results from parameterization on 2-state fit (Q^2 different in π and K)
- Tensor FF includes the ratio of the meson masses: $\mathcal{M} \equiv m_K/m_\pi$
- Up-quark contribution in pion and kaon similar in all FFs
- Up-quark contribution in kaon about 80% of strange quark as Q^2 increases