Nucleon Form Factors in the Continuum Limit from Clover-on-HISQ Formulation

Yong-Chull Jang

Physics Department, Columbia University

(PNDME Collaboration)

T. Bhattacharya, R. Gupta, B. Yoon (LANL), H.-W Lin (MSU)

The 38th International Symposium on Lattice Field Theory

LATTICE21
JULY 26-30 2021, ZOOM/GATHER@MIT

Outline

$$\langle N(\vec{p}_f)|A_\mu(\vec{Q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[G_A(Q^2)\gamma_\mu + q_\mu \frac{\widetilde{G}_P(Q^2)}{2M}\right]\gamma_5 u(\vec{p}_i)$$

$$\langle N(\vec{p}_f)|P(\vec{q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[G_P(Q^2)\gamma_5\right] u(\vec{p}_i)$$

$$\langle N(\vec{p}_f)|V_\mu(\vec{q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[F_1(Q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu \frac{F_2(Q^2)}{2M}\right] u(\vec{p}_i)$$

$$q = p_f - p_i,\ Q^2 = -q^2 = \vec{p}_f^{\,2} - (E-M)^2,\ \vec{p}_i = 0$$

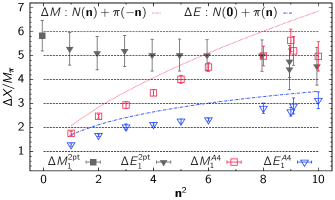
$$C^{\rm 3pt}_{\vec{q}}(\tau,t)[J] = \langle N|J|N\rangle c_{00} e^{-E_0 t} e^{-M_0(\tau-t)} \left\{1 + \sum_{(i,j)\neq(0,0)} d_{kl} e^{-(E_i-E_0)t} e^{-(M_j-M_0)(\tau-t)}\right\}$$

- G_A, $\widetilde{G}$, and G_P are axial, induced pseudoscalar, pseudoscalar from facators
- $F_1 - Q^2/(4M^2)F_2 = G_E$ and $F_1 + F_2 = G_M$ are electric and magnetic form factors

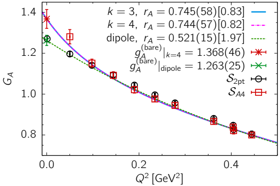
- Excited state analysis to isolate $\langle N|J|N\rangle$
- Axial form factors at the physical limit

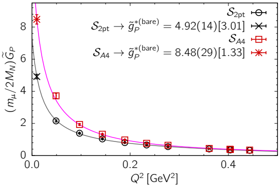
G_A, $\widetilde{G}_P$, and G_P with excited states from $C^{\mathrm{3pt}}_{\boldsymbol{p}\neq 0}[A_4]$, M_1^{A4} and E_1^{A4}

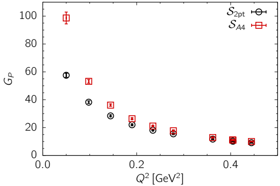
[Jang. et. al., PRL 124, 072002 (2020)]



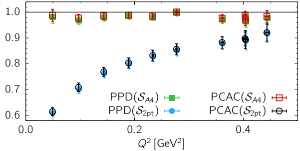
- Excited states with M_1^{A4} (E_1^{A4}) couple to the sink (source) operators and are
- different from the excited states from the two-point function fits (E_1^{2pt}, M_1^{2pt})
- The $\widetilde{G}_P$ and G_P are roughly doubled at small Q^2







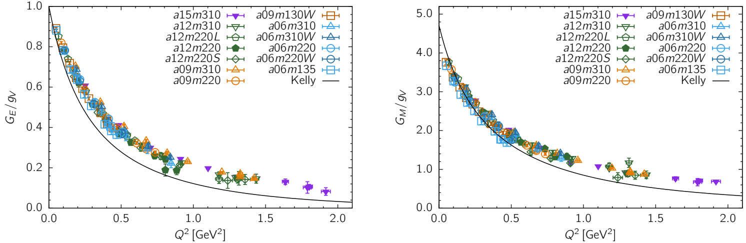
- $g_A = G_A(0)$ and $\langle r_A^2 \rangle = -(6/g_A) G_A'|_0$ become compatible with experiments
- PCAC is satisfied, and PPD (pion-pole dominance) follows PCAC



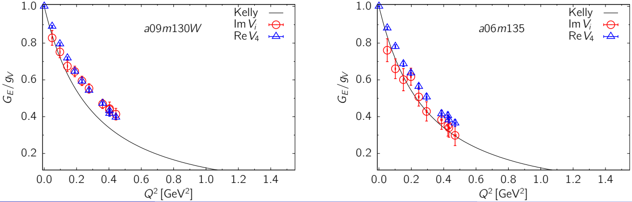
G_E and G_M also deviate from experiments

[Jang. et. al., PRD 101, 014507 (2020)]

- Kelly curve represents a parameterization of the experimental data
- resulting smaller $\langle r_E^2 \rangle$, $\langle r_M^2 \rangle$, and magnetic moment $\mu = G_M(0)$



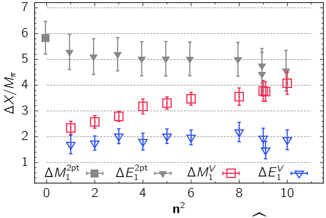
- Two G_E determinations by V_i and V_4 show a systematic deviation
- What is the excited states in $C^{3pt}[V_\mu]$?
- V_i ($i = 1, 2, 3$) have the same parity of the A_4

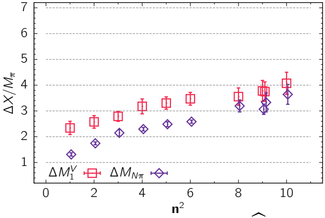


Excited States for Vector Current

$$\widehat{V}(\boldsymbol{q}) \equiv \mathrm{Im}\,\overline{V}_s(\boldsymbol{q}) - \frac{\mathrm{Re}\,\overline{V}_4(\boldsymbol{q})}{M+E} \quad \rightarrow \quad G_E - G_E = 0$$

- $\widehat{V}$, $\mathrm{Re}V_4$, $\mathrm{Re}V_s$ are simultaneously fitted ($a09m130W$)



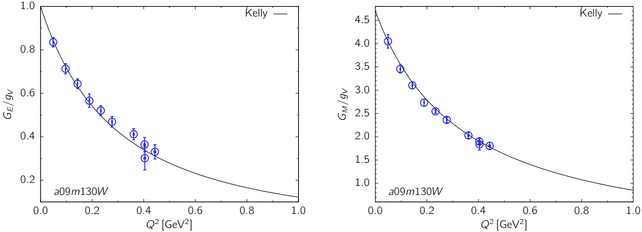


- First excited state gap in $\widehat{V}$ that coupled with N interpolating operator at source with a momentum $\boldsymbol{n} \neq 0$, $\Delta E_1^V \approx 2M_\pi$
- $N(\boldsymbol{n}) + \pi(0) + \pi(0)$
- c.f., $N(0) + \pi(\boldsymbol{n})$ for the axial channels

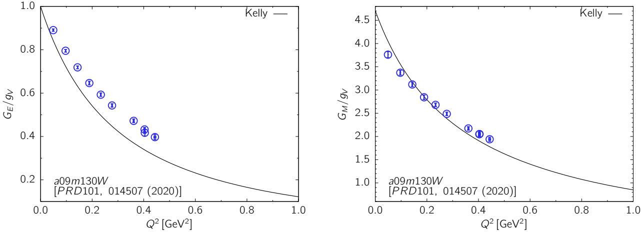
- First excited state gap in $\widehat{V}$ that coupled with the nucleon interpolating operator at sink with zero momentum, $\Delta M_1^V \approx \Delta M_{N\pi} + M_\pi$
- $N\pi(0) + \pi(0)$
- c.f., $N(-\boldsymbol{n}) + \pi(\boldsymbol{n})$ for the axial
- n.b., $M_{N\pi}$ is the rest mass converted from E_1^{A4}

G_E and G_M

- including the excited states from $\widehat{V}$

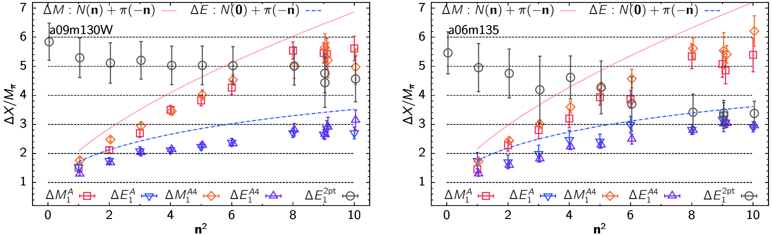


- including the excited states from $C^{2\text{pt}}$



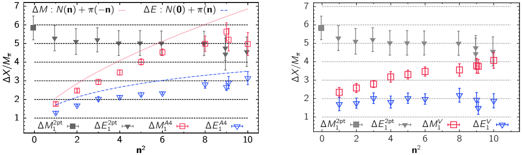
Comparison of Excited States from A_4 fit and (A_μ, P) fit

- Excited states coupled with axial and vector currents can be distinguished
- However, we cannot differentiate the excited states in A_i and P from A_4



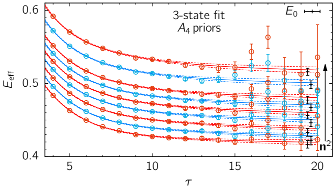
- Results from two physical pion mass ensembles
- The excited states from A_4, ΔM_1^{A4} and ΔE_1^{A4} overlaps with the results from simultaneous fit to all A_μ and P three-point correlators, ΔM_1^{A} and ΔE_1^{A}.

$C_{\boldsymbol{p}}^{\mathrm{2pt}}(\tau)$ Revisited



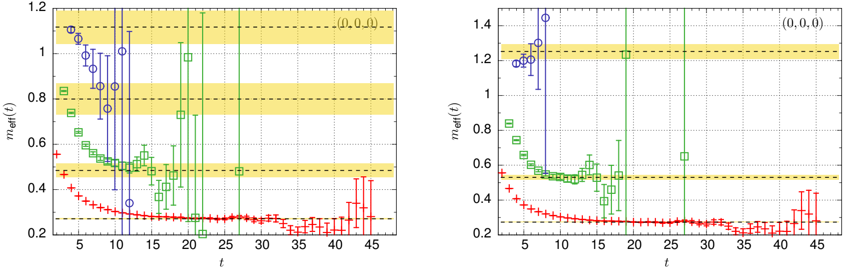
- Excited states in $C_{\boldsymbol{p}\neq 0}^{\mathrm{3pt}}[A]$ (left) $C_{\boldsymbol{p}\neq 0}^{\mathrm{3pt}}[V]$ (right)

- For $C_{\boldsymbol{p}}^{\mathrm{2pt}}$, 3-state fits with ΔE_1^{A4} as narrow priors are not discriminated by χ^2 against 4-state fits with empirical Bayesian method
- Vector correlators indicate other excited states with ΔE_1^{V} exist
- Too many states at $\boldsymbol{p} = 0$; ΔM_1^{A4} and ΔM_1^{V}
- Imposing a single $N\pi$ state prior may not be an appropriate prescription for $C_{\boldsymbol{p}}^{\mathrm{2pt}}$



[Jang. et. al., PRL 124, 072002 (2020)]

Generalized Effective Mass, $m_{\text{eff}}^{(n)}(t)$



- yellow band: E_n from the four-state (left) and three-state (right) fits

$$C^{2\text{pt}}(t) = a_0 e^{-E_0 t}\left\{1 + \sum_{k=1} b_k e^{-(E_k - E_0)t}\right\}, \quad (a_0 > 0,\ b_k > 0)$$

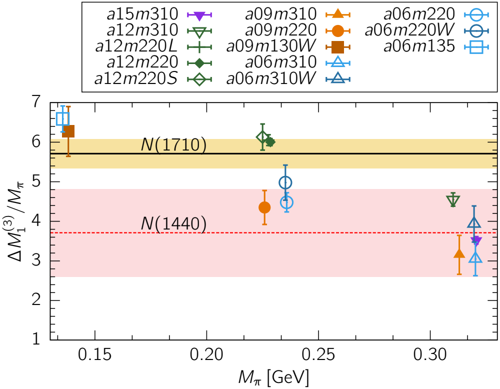
$$\Longrightarrow \quad m_{\text{eff}}^{(0)} \equiv -\frac{d}{dt}\log C^{2\text{pt}}(t)$$

$$m_{\text{eff}}^{(n)} \equiv m_{\text{eff}}^{(n-1)} - \frac{d}{dt}\log(m_{\text{eff}}^{(n-1)} - E_{n-1}), \quad (n = 1, 2, \ldots)$$

- Need to provide E_{n-1} for $m_{\text{eff}}^{(n)}$, but no information of the amplitudes are required

First Excited States in $C^{\mathbf{2pt}}_{\boldsymbol{p}=0}(t)$

- $\Delta M_1^{(3)}$: first excited state mass gap from the three-state fit, or from the two-state fit for $a \approx 0.12, 0.15\,\mathrm{fm}$
- These fits almost exhaust the time slices ($t_{\min} = 1 \sim 3$)
- No priors are needed

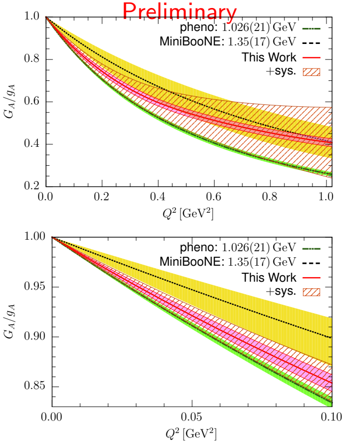


Many of possible nucleon-pion states might appear collectively ($\sim$ resonance mass)

Axial Form Factor at the Physical Limit

- Excited states are taken from $C^{3\text{pt}}_{\boldsymbol{p}\neq 0}[A]$
- Q^2 behavior of $G_A(Q^2)$ is parameterized by z^2-expansion
- G_A is normalized by $g_A = G_A(0)$
- Fixed Q^2 data sampled from z-expansion is extrapolated with

$$G_A/g_A = c_0 + c_1 a + c_2 M_\pi^2 + c_3 M_\pi^2 e^{-M_\pi L}$$



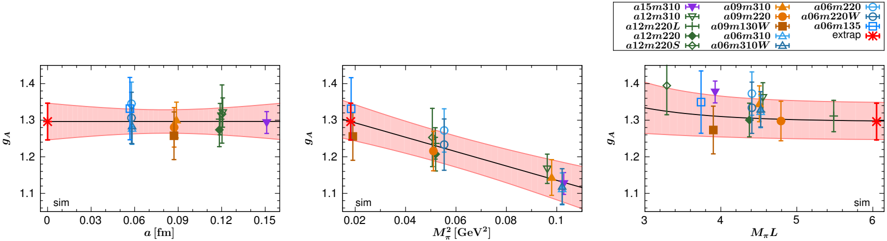
Lattice determination is consistent with the phenomenology curve at small Q^2

- No physical pion mass data for $Q^2 > 0.4\,\mathrm{GeV}^2$
- Systematic error in $\langle r_A^2\rangle$ is propagated by $\delta(G_A/g_A) = \delta\langle r_A^2\rangle \times (Q^2/6)$ and added in quadrature

g_A from the $G_A(Q^2 \neq 0)$

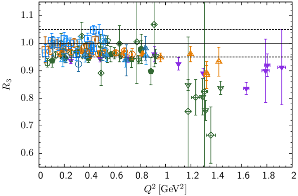
Preliminary

- $g_A = G_A(0)$ from the z^2-expansion
- Lattice determination is consistent with the experiments
- agrees with zero-momentum correlator analysis with 3-RD fit



☝ other lattice artifact terms are projected out

Pion-pole in $\widetilde{G}_P$ and G_P

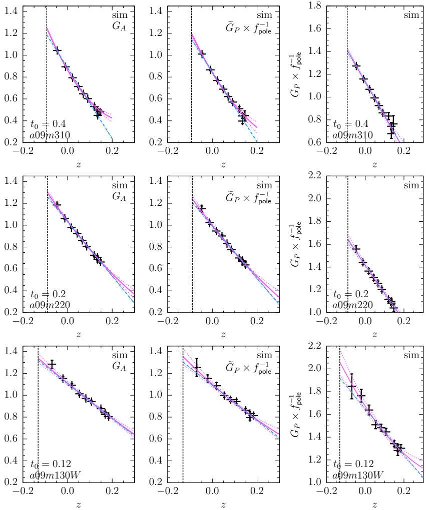


- Pion-pole dominance, $R_3 = 1$, shows up with lattice data

$$R_3 = f_{\rm pole}^{-1} \frac{\widetilde{G}_P(Q^2)}{G_A(Q^2)}\,, \qquad f_{\rm pole} = \frac{4M^2}{Q^2 + M_\pi^2}$$

- z-expansion is applied for $\widetilde{F}_P = f_{\rm pole}^{-1}\widetilde{G}_P\,, \quad F_P = f_{\rm pole}^{-1} G_P$

Comparison of G_A, $\widetilde{F}_P$, F_P



Induced Pseudoscalar Coupling g_P^*

Preliminary

- g_P^* from each ensemble is obtained by extrapolating the z-expansion

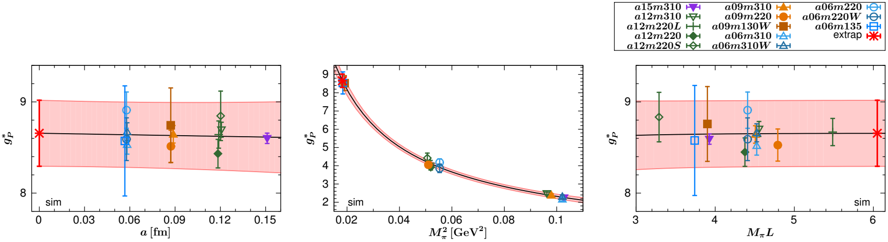
$$g_P^* = \frac{2 m_\mu M_N \widetilde{F}_P(Q^{*\,2})}{Q^{*\,2} + M_\pi^2}, \quad Q^{*\,2} = 0.88 m_\mu^2$$

- Physical limit is taken with

$$g_P^* = c_0 + c_1 a + c_2 M_\pi^2 + c_3 M_\pi^2 e^{-M_\pi L} + \frac{c_4}{Q^{*\,2} + M_\pi^2}$$

- Lattice determination is consistent with the experiments, 8.23(83)

[R. J. Hill et. al. 2019 Rep. Prog. Phys. 81 096301]



other lattice artifact terms are projected out

Nucleon-pion Coupling $g_{\pi NN}$

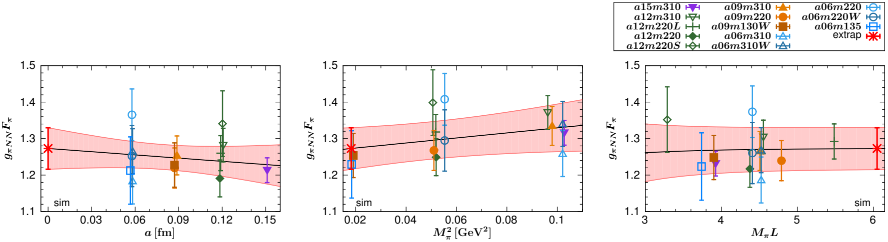
Preliminary

$$g_{\pi NN} \equiv \lim_{Q^2 \to -M_\pi^2} \frac{Q^2 + M_\pi^2}{4 M_N F_\pi} \widetilde{G}_P(Q^2)$$

- $g_{\pi NN} F_\pi = M_N \widetilde{F}_P(-M_\pi^2)$ is obtained from the z-expansion
- Physical limit is taken with

$$g_{\pi NN} F_\pi = c_0 + c_1 a + c_2 M_\pi^2 + c_3 M_\pi^2 e^{-M_\pi L}$$

- Lattice determination is consistent with the experiments, 1.219(9)



other lattice artifact terms are projected out

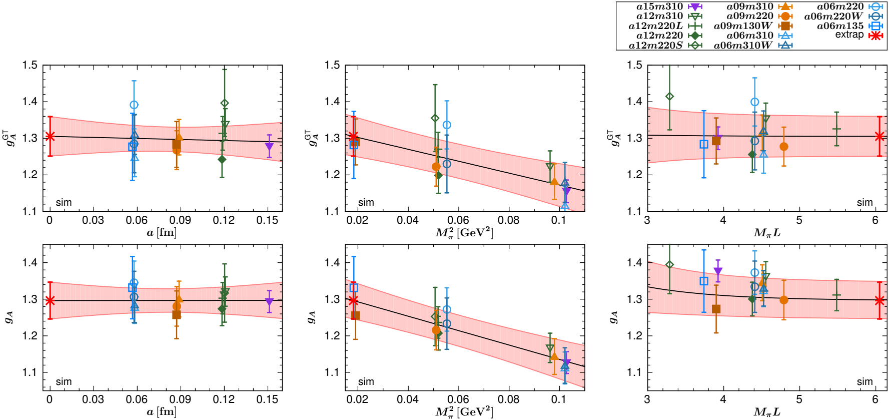
Goldberger-Treiman Relation $g_A^{\mathrm{GT}} = g_A$

Preliminary

- $g_A^{\mathrm{GT}} \equiv \widetilde{F}_P(-M_\pi^2)\,(= g_{\pi NN} F_\pi / M_N)$ is obtained from the z-expansion
- Physical limit is taken with

$$g_A^{\mathrm{GT}} = c_0 + c_1 a + c_2 M_\pi^2 + c_3 M_\pi^2 e^{-M_\pi L}$$

- $g_A^{\mathrm{GT}} = g_A$ is well satisfied with our lattice determinations

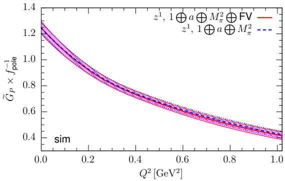


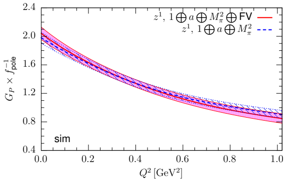
other lattice artifact terms are projected out

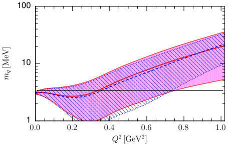
PCAC Relation $\frac{Q^2}{4M_N^2}\frac{\widetilde{G}_P(Q^2)}{G_A(Q^2)}\ (\equiv R_1) + \frac{m_q}{M_N}\frac{G_P(Q^2)}{G_A(Q^2)}\ (\equiv R_2) = 1$

Preliminary

- $\widetilde{G}_P \times f_{\rm pole}^{-1}$ and $G_P \times f_{\rm pole}^{-1}$ at the physical limit
- Finite volume effect is small

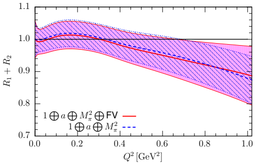


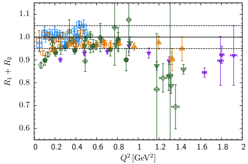




$$m_q^{\rm PCAC} = 3.1(2)\ \mathrm{MeV} \quad [Q^2 = 0]$$
$$m_{ud} = 3.410(43)\ \mathrm{MeV}\ \text{[FLAG]}$$

- For small Q^2, quark mass m_q obtained from the PCAC relation is close to the $N_f = 2+1+1$ FLAG results ($\overline{\rm MS}$ at $\mu = 2\,\mathrm{GeV}$)
- PCAC relation is satisified with the form factors at the physical limit for $Q^2 \lesssim 0.4\,\mathrm{GeV}^2$





(all errors shown here are statisitcal)

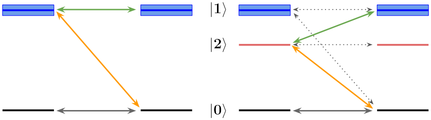
Summary

- Determine the excited state from the three-point function
- Depending on the currents - A, V, S, T - the preferred states can be different
- Differences of excited states in A_i and A_4 (or V_i and V_4) are not resolvable from the current statistics
- Two-point correlator might see the many nucleon-pion states collectively $\sim$ resonance energy
- Imposing a single $N\pi$ state prior may not be an appropriate prescription for $C_{\boldsymbol{p}}^{\text{2pt}}$

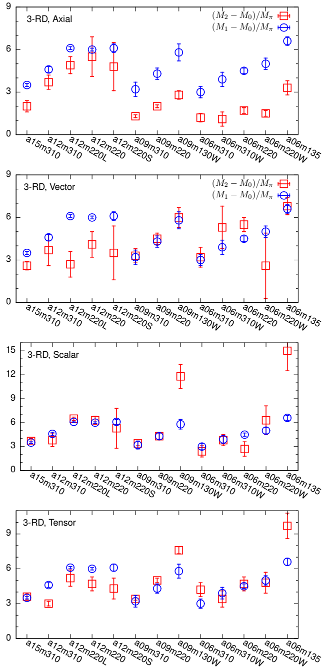
- The axial form factor agrees with phenomenology at $Q^2 \lesssim 0.4\,\mathrm{GeV}^2$
- g_P^*, $g_{\pi NN}$ also agree with experiments
- Goldberger-Treiman relation, pion-pole dominance, and PCAC hold
- We are finalizing the analysis

Thank you for your attention.

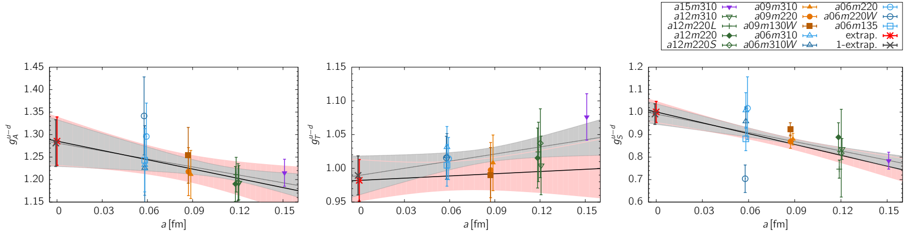
Nucleon Charges from $C^{3pt}_{\mathbf{p}=0}[J_\mu]$



- Arrows represent nonvanishing transition matrix elements $\langle i|J_\mu|j\rangle$ in 2-state fit (left), 3-RD fit (right, solid), and 3-state fit (right, dotted)
- 3-RD fit takes the nucleon mass and the first excited state mass from two-point correlator fits and has free mass gap ΔM_2 that is determined by $C^{3pt}_{\mathbf{p}=0}[J_\mu]$



Chiral-continuum-finite-volume extrapolation of 3-RD fits



$$g_X = c_0 + c_1 a + c_2 M_\pi^2 + c_3 M_\pi^2 \exp(-M_\pi L)$$

- g_T and g_S are in good agreement with the previous determinations:
  [Gupta et. al., PRD 98, 034503, 2018]

$$g_A = 1.218(25)_{\text{stat}}(30)_{\text{sys}} \qquad (3^*\text{-state})$$
$$g_T = 0.989(32)_{\text{stat}}(10)_{\text{sys}} \qquad (3^*\text{-state})$$
$$g_S = 1.022(80)_{\text{stat}}(60)_{\text{sys}} \qquad (2\text{-state})$$

- g_A becomes consistent with the experiments
- The statistical error in g_A is almost doubled
- 3-RD fit results of g_A are not combined with our form factor analysis