Relating Euclidean Correlators and Light-Cone Correlators beyond Leading Twist

(Andreas Metz, Temple University)

- Introduction

- Matching of light-cone PDFs and quasi-PDFs
  - twist-2
  - twist-3 (zero-mode contributions)
    S. Bhattacharya, K. Cichy, M. Constantinou, A.M., A. Scapellato, F. Steffens, 2005.10939, 2006.12347
  - further development for twist-3
    V. Braun, Y. Ji, A. Vladimirov, 2103.12105

- Summary and Outlook

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Light-cone PDFs, quasi-PDFs, and their relation

- Example for light-cone PDF: unpolarized quark light-cone PDF f_1 (twist-2)

$$f_1(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^+ \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^+=0, \vec{z}_\perp = \vec{0}_\perp}$$

  - correlator depends on time $t = z^0 = \frac{1}{\sqrt{2}} z^-$ $\rightarrow$ cannot be computed in LQCD

- Example for quasi-PDF: unpolarized quark quasi-PDF $f_{1,\mathrm{Q}}$ (Ji, 2013)

$$f_{1,\mathrm{Q}}(x, P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^3 \, \mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^0=0, \vec{z}_\perp = \vec{0}_\perp}$$

  - correlator depends on position z^3 $\rightarrow$ can be computed in LQCD
  - quasi-PDF depends also on hadron momentum P^3
  - light-cone PDFs and quasi-PDFs contain same non-perturbative (IR) physics $\rightarrow$ essence of quasi-PDF approach

- Several related approaches for computing light-cone PDFs through Euclidean correlators in LQCD
(Braun, Müller, 2008 / Ma, Qiu, 2014 / Radyushkin, 2017 / ...)

- Matching between light-cone PDFs and quasi-PDFs
  - light-cone PDFs and quasi-PDFs differ in UV region
  - at large P^3, difference in UV behavior is dealt with via perturbative matching
    (Xiong, Ji, Zhang, Zhao, 2013 / Stewart, Zhao, 2017 / Izubuchi, Ji, Jin, Stewart, Zhao, 2018 / ...)
  - generic structure of matching formula (scale-dependence omitted)

$$f_1(x) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C(\xi)\, f_{1,\mathrm{Q}}\Big(\frac{x}{\xi}, P^3\Big) + \mathcal{O}\Big(\frac{M^2}{(P^3)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^3)^2}\Big)$$

    * C is (perturbatively-calculable) matching coefficient
    * accuracy of approach determined by accuracy of C and size of power corrections
    * in twist-2 case, two-loop results available for C
      (Braun, Chetyrkin, Kniehl, 2021 / Chen, Wang, Zhu, 2021 / Li, Ma, Qiu, 2021)
    * minimizing power corrections requires LQCD calculation at large P^3
    * matching for twist-3 light-cone PDFs ?

Twist-3 light-cone PDFs

- Overview (Jaffe, Ji, 1991)

$$g_T(x) \qquad\qquad e(x) \qquad\qquad h_L(x)$$

  - can be as large as twist-2 PDFs
  - contain information about quark-gluon-quark (qgq) correlations
  - $g_T(x)$ and $e(x)$ related to (transverse) force acting on quarks (Burkardt, 2008)
  - difficult to extract from experiment due to kinematical suppression

- Twist-3 PDFs from experiment: current status
  - some information for $g_T(x)$ (Hall A, 2016 / Hall C, 2018 / ...)
  - hardly any information for $e(x)$ (Courtoy, 2014, based on preliminary Hall-B data / new Hall-B data on di-hadron production available)
  - no information for $h_L(x)$

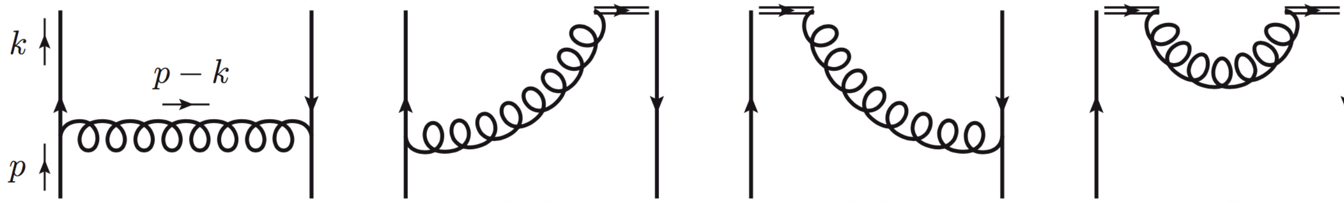
- Goal: compute twist-3 light-cone PDFs in LQCD using quasi-PDF approach; plus extension to twist-3 GPDs
  ($\rightarrow$ for numerical results, talks by M. Constantinou and K. Cichy, poster by J. Dodson)

Matching for twist-2 case

- Compute for quark target, for example, $f_1(x)$ and $f_{1,\mathrm{Q}}(x, P^3)$

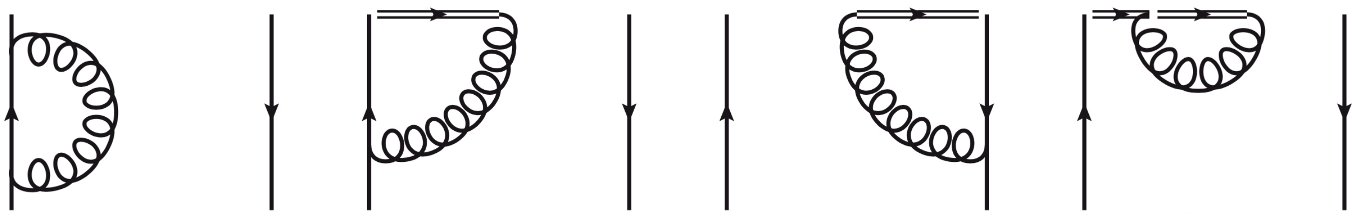
- One-loop diagrams

  – real diagrams



(1a) (1b) (1c) (1d)

  – virtual diagrams


(2a) (2b) (2c) (2d)

  – diagrams have singularity in non-perturbative region

- Sample result, with nonzero gluon mass m_g as IR regulator

$$f_1^{(1a)}(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\ln\frac{\mu^2}{x\,m_g^2} - 2\right)$$

$$f_{1,\mathrm{Q}}^{(1a)}(x,p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases}(1-x)\,\ln\dfrac{x-1}{x} - 1 & x<0\\[2ex] (1-x)\,\ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0<x<1\\[2ex] (1-x)\,\ln\dfrac{x}{x-1} + 1 & x>1\end{cases}$$

  - light-cone PDF and quasi-PDF have same $\ln m_g^2$ singularity,
    $\rightarrow$ essence of quasi-PDF approach
  - matching coefficient $C \sim f_1 - f_{1,\mathrm{Q}}$ finite for $m_g = 0$
  - light-cone PDF and quasi-PDF can be related through matching formula

Matching for twist-3 case: overview

(S. Bhattacharya, K. Cichy, M. Constantinou, A.M., A. Scapellato, F. Steffens, 2020)

- Computed twist-3 light-cone PDFs ($g_T(x), e(x), h_L(x)$) and corresponding quasi-PDFs for quark target at one loop
- Used three different IR regulators: $m_g \neq 0$, $m_q \neq 0$, $\varepsilon_{\rm IR}$
- Calculation very similar to twist-2 case
- But, complication due to singular zero-mode contributions
- Example

$$e(x) = e_{\rm can}(x) + e_{\rm sin}(x) \quad \text{with} \quad e_{\rm sin}(x) \sim \delta(x)$$

- Various studies on zero modes for twist-3

  Broadhurst, Gunion, Jaffe, 1973 / ...

  Efremov, Schweitzer, 2002 / Wakamatsu, Ohnishi, 2003 / Pasquini, Rodini, 2018 / ...

  Burkardt, 1995 / Burkardt, Koike, 2001 / Aslan, Burkardt, 2018 / Bhattacharya, A.M., 2021 / ...
- Impact of zero modes on matching for twist-3 ?

Zero modes and matching for twist-3 PDFs

- Zero modes arise from "simple" integral

$$p^+ \int dk^- \, \frac{1}{(k^2 - m_q^2 + i\varepsilon)^2} = \frac{i\pi}{\vec{k}_\perp^{\,2} + m_q^2} \, \delta(x) \qquad (k^+ = x\, p^+)$$

  - integral does not appear for twist-2 light-cone PDFs
  - integral does appear for $g_T(x), e(x), h_L(x)$
  - main challenge for $e(x), h_L(x)$, where $\delta(x)$ accompanied by IR singularity

$$e_{\text{sin}}(x) = \frac{\alpha_s C_F}{2\pi} \, \ln \frac{\mu^2}{m_q^2} \, \delta(x) + \dots$$

  - is there corresponding IR singularity for quasi-PDF ?

- Recall: IR-divergent zero mode in $e(x)$

$$e_{\rm sin}(x) = \frac{\alpha_s C_F}{2\pi}\,\ln\frac{\mu^2}{m_q^2}\,\delta(x) + \ldots$$

- Counterpart for quasi-PDF
  - naïve twist expansion

$$e_{\rm Q,sin}(x,p^3) = \frac{\alpha_s C_F}{2\pi}\,\frac{1}{\sqrt{x^2+\eta^2}} \to \frac{\alpha_s C_F}{2\pi}\,\frac{1}{|x|} \quad \text{for} \quad p^3 \to \infty \qquad \left(\eta = \frac{m_q}{p^3}\right)$$

    * where is term $\sim\ \ln m_q^2\ \delta(x)$, which is present in light-cone PDF ?
    * naïve twist expansion OK, except for $x = 0$

  - proper twist expansion cures the issue

$$e_{\rm Q,sin}(x,p^3) = \frac{\alpha_s C_F}{2\pi}\,\frac{1}{\sqrt{x^2+\eta^2}} = \frac{\alpha_s C_F}{2\pi}\,\ln\frac{p_3^2}{m_q^2}\,\delta(x) + \ldots$$

    * identity in sense of distribution (sufficient for calculation of matching coefficient)
    * $\ln m_q^2\ \delta(x)$ drops out in matching coefficient
    * generally, zero-mode contributions can be dealt with in quasi-PDF approach

- Example: one-loop matching coefficient for $h_L(x)$
  - result for modified $\overline{\mathrm{MS}}$ ($\mathrm{M}\overline{\mathrm{MS}}$) scheme, which was first used for twist-2 (C. Alexandrou et al, 2019)
  - singular term

$$C^{(\mathrm{s})}_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \delta(1-\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi > 1 \\ -\delta(\xi)\left(\ln\frac{4p_3^2}{\mu^2} + 1\right) - \mathrm{R}_0(|\xi|) & -1 < \xi < 1 \\ \delta(1+\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi < -1\,, \end{cases}$$

  - canonical term

$$C^{(\mathrm{c})}_{\mathrm{M}\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{2}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{1}{1-\xi} + \frac{1}{\xi}\right]_+ & \xi > 1 \\ \left[\frac{2}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + 2(1-\xi) - \frac{1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{2}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{1}{1-\xi} + \frac{1}{1-\xi}\right]_+ & \xi < 0\,. \end{cases}$$

- matching coefficient was used for recent calculation of $h_L(x)$ ($\rightarrow$ talk by M. Constantinou) (S. Bhattacharya, K. Cichy, M. Constantinou, A.M., A. Scapellato, F. Steffens, 2021)

Matching for twist-3 case: further development

(V. Braun, Y. Ji, A. Vladimirov, 2021)

- Full calculation requires taking into account 3-parton (qgq) correlators
- Separation for $g_T(x)$ into twist-2 and "genuine" twist-3 piece

$$g_T(x) = g_T^{\rm tw2}(x) + g_T^{\rm tw3}(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + g_T^{\rm tw3}(x)$$

- Especially qgq correlators lead to different matching coefficients for $g_T^{\rm tw2}$ and $g_T^{\rm tw3}$
- Full matching formula (symbolic)

$$g_{T,{\rm Q}} = C^{\rm tw2} \otimes g_T^{\rm tw2} + C_{\rm 2pt}^{\rm tw3} \otimes g_T^{\rm tw3} + C_{\rm 3pt}^{\rm tw3} \otimes S_{\rm 3pt} + \text{power corrections}$$

  - no mention of zero-mode contributions (?)
  - at present, too many unknowns
  - calculation of qgq quasi-correlators needed to fully solve problem
  - neglecting $S_{\rm 3pt}$ delicate, since separation between $C_{\rm 2pt}^{\rm tw3}$ and $C_{\rm 3pt}^{\rm tw3}$ not unique
  - so far, no corresponding results for $e(x)$, $h_L(x)$ available
  - currently, our approach justified, but progress is needed (work ongoing)

Summary and Outlook

- Study of twist-3 PDFs (and GPDs) well motivated

- Extraction of twist-3 PDFs from experiment very challenging

- Obtained first matching formula relating quasi-PDFs and twist-3 light-cone PDFs
  - result based on two-parton matrix elements only
  - singular zero-mode contributions cause trouble, but can be dealt with

- Full matching formula (including qgq correlations) available for $g_T(x)$
  - at present, application not straightforward though
  - nevertheless, we are exploring how results could be used for further progress
  - formalism allows one to study (x-dependent) "genuine" higher twist in LQCD

- Generally, tremendous potential for LQCD calculations at twist-3