x-dependence reconstruction of pion and kaon PDFs from Mellin moments

Colin Lauer

in collaboration with:
Constantia Alexandrou, Simone Bacchio, Ian Cloët,
Martha Constantinou, Kyriacos Hadjiyiannakou, Giannis Koutsou

The 38th International Symposium on Lattice Field Theory
July 28, 2021

Extended Twisted Mass Collaboration

Argonne NATIONAL LABORATORY

NSF

Overview

Overview

Motivation

- Pion and kaon structure is important for answering open questions in hadron structure, e.g., SU(3) flavor symmetry breaking caused by heavier strange quark mass
- Accessing x-dependence of PDFs using Lattice QCD (LQCD):
  - Novel methods: quasi-PDFs, pseudo-PDFs, current-current correlators, etc.
  - From Mellin moments:

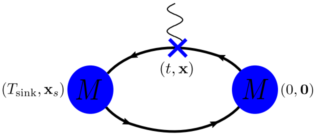
$$\langle x^n \rangle = \int_{-1}^{1} dx\, x^n f(x)$$

- Previously argued that it is unfeasible to reconstruct PDFs using lattice results for the Mellin moments, in particular, the large-x behavior cannot be reliably understood [Detmold *et al.*, arXiv:hep-lat/0108002], [Holt *et al.*, RMP **82**, 2991–3044 (2010)]
- We calculate moments directly from local operators without mixing with lower dimension operators so we attempt a reconstruction with our moment results

Overview

Meson matrix elements

- Moments under study:
  - quark momentum fraction $\langle x \rangle$
  - 2nd Mellin moment $\langle x^2 \rangle$
  - 3rd Mellin moment $\langle x^3 \rangle$



- Matrix elements in the forward limit ($Q^2 = 0$):

$$\langle M(p) | \mathcal{O} | M(p) \rangle$$

- Operators of interest:

$$\mathcal{O}_V^{\{\mu\nu\}} = \overline{q}\gamma^{\{\mu} D^{\nu\}} q$$

$$\mathcal{O}_V^{\{\mu\nu\rho\}} = \overline{q}\gamma^{\{\mu} D^{\nu} D^{\rho\}} q$$

$$\mathcal{O}_V^{\{\mu\nu\rho\tau\}} = \overline{q}\gamma^{\{\mu} D^{\nu} D^{\rho} D^{\tau\}} q$$

PDF reconstruction setup

- Standard PDF functional form:

$$q_M^f(x) = Nx^{\alpha}(1-x)^{\beta}(1+\rho\sqrt{x}+\gamma x)$$

- ρ generally assumed to be small, so we neglect $\rho\sqrt{x}$ term
- Normalization factor:

$$\langle 1 \rangle_M = \int_0^1 q_M(x) = 1 \implies N = \frac{1}{B(\alpha+1,\beta+1)+\gamma B(2+\alpha,\beta+1)}$$

- Moment integrals:

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^{n}(i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^{n}(i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}$$

Lattice details

- $N_f = 2+1+1$ twisted-clover fermions

Ensemble Parameters

a [fm]	N_f	m_π [MeV]	m_K [MeV]	volume $L^3 \times T$	L [fm]
0.093	$2+1+1$	260	530	$32^3 \times 64$	3.0

Statistics

p	p combos.	T_{sink}	confs	src pos.	Total
$(0,0,0)$	1	$12, 14, 16, 18, 20, 24$	122	16	1,920
$(\pm1, \pm1, \pm1)$	8	$12,\ 14, 16, 18$	122	72	70,272

- Boosted frame: $(\pm1, \pm1, \pm1)$ to calculate $\langle x^2 \rangle$ and $\langle x^3 \rangle$

Overview

First three non-trivial moments

- Excited states sizeable (backup slides)
- Find results for 2-state fits including up to $T_{\rm sink} = 2.2$ fm for $\langle x \rangle$ and $T_{\rm sink} = 1.7$ fm for $\langle x^2 \rangle$, $\langle x^3 \rangle$

$$\langle x \rangle_u^{\pi^+} = 0.261(3)(6)$$
$$\langle x \rangle_u^{K^+} = 0.246(2)(2)$$
$$\langle x \rangle_s^{K^+} = 0.317(2)(1)$$

$$\langle x^2 \rangle_u^{\pi^+} = 0.110(7)(12)$$
$$\langle x^2 \rangle_u^{K^+} = 0.096(2)(2)$$
$$\langle x^2 \rangle_s^{K^+} = 0.139(2)(1)$$

$$\langle x^3 \rangle_u^{\pi^+} = 0.024(18)(2)$$
$$\langle x^3 \rangle_u^{K^+} = 0.033(6)(1)$$
$$\langle x^3 \rangle_s^{K^+} = 0.073(5)(2)$$

$$\frac{\langle x^2 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.423(28)(57)$$
$$\frac{\langle x^2 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.391(10)(16)$$
$$\frac{\langle x^2 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.438(8)(11)$$

$$\frac{\langle x^3 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.092(71)(6)$$
$$\frac{\langle x^3 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.135(26)(8)$$
$$\frac{\langle x^3 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.232(16)(1)$$

- $\langle x^2 \rangle / \langle x \rangle \sim 40\%$, $\langle x^3 \rangle / \langle x \rangle \sim 10 - 20\%$
- More details in [Phys. Rev. D **103**, 014508 (2021), arXiv:2010.03495] and [arXiv:2104.02247]

SU(3) flavor symmetry breaking

$$\frac{\langle x\rangle_\pi^{u^+}}{\langle x\rangle_K^{u^+}} = 1.060(9)(7)$$
$$\frac{\langle x^2\rangle_\pi^{u^+}}{\langle x^2\rangle_K^{u^+}} = 1.148(57)(106)$$
$$\frac{\langle x^3\rangle_\pi^{u^+}}{\langle x^3\rangle_K^{u^+}} = 0.717(488)(94)$$

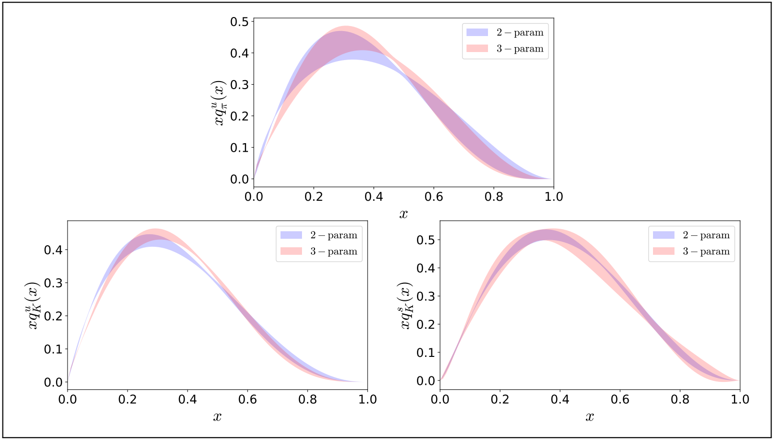
$$\frac{\langle x\rangle_\pi^{u^+}}{\langle x\rangle_K^{s^+}} = 0.823(8)(10)$$
$$\frac{\langle x^2\rangle_\pi^{u^+}}{\langle x^2\rangle_K^{s^+}} = 0.795(45)(80)$$
$$\frac{\langle x^3\rangle_\pi^{u^+}}{\langle x^3\rangle_K^{s^+}} = 0.325(244)(23)$$

- SU(3) symmetry breaking $\sim 5 - 10\%$ for $\langle x\rangle$
- $\sim 10 - 20\%$ for $\langle x^2\rangle$
- $\sim 30 - 50\%$ for $\langle x^3\rangle$
- Symmetry breaking between π and strange part of K is more pronounced in the higher moments

Overview

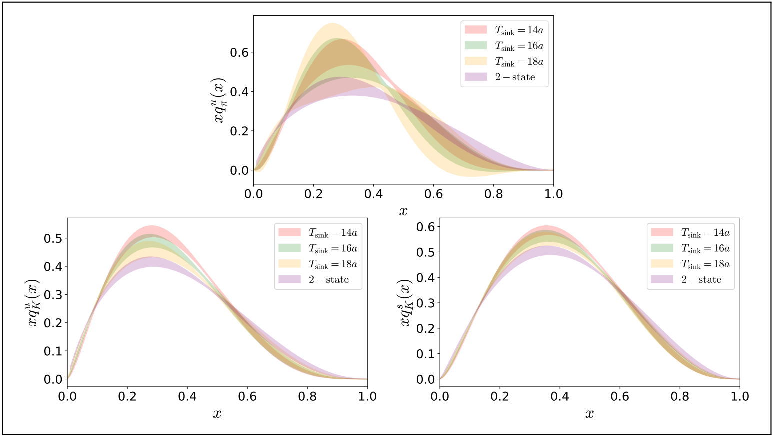
Effect of fit function

- Moments evolved to scale of 5.2 GeV
- 2-parameter fit: α, β
- 3-parameter fit: α, β, γ

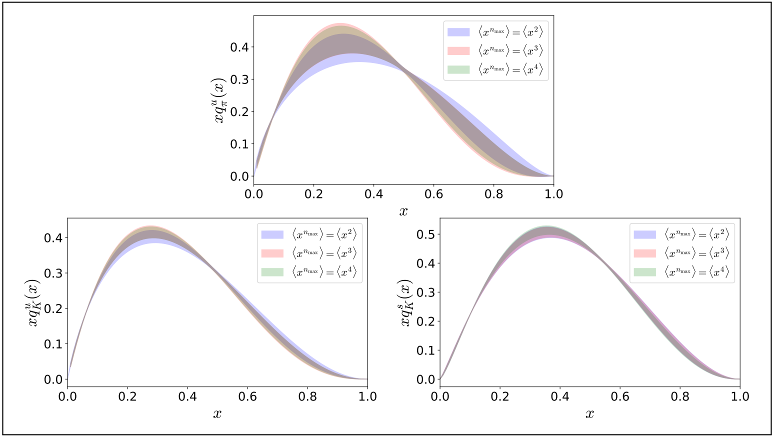


- 3-parameter fit has larger statistical uncertainty
- We find little dependence of shape on the fit function
- We proceed with the 2-parameter fits

Excited-state effects

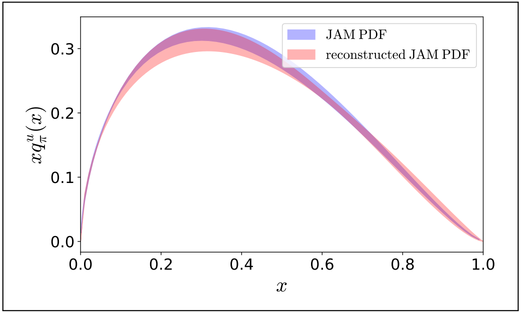


- Excited-state effects appear to raise peak
- We choose the two-state fit as our final estimates



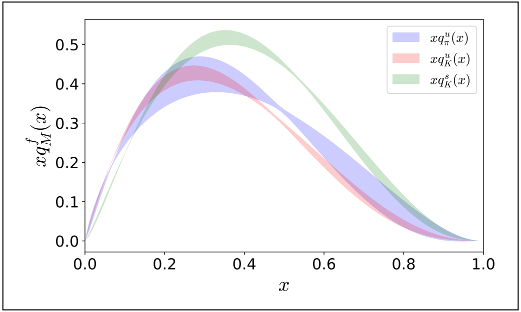
- $\langle x^{n_{\max}}\rangle = \langle x^4\rangle$: add constraint from
  - phenomenological result $\langle x^4\rangle^u_\pi = 0.027(2)$
  - model calculations $\langle x^4\rangle^s_K = 0.029^{+0.005}_{-0.004}$, $\langle x^4\rangle^u_K = 0.021^{+0.003}_{-0.003}$
- We choose $n_{\max} = 3$ as our final estimates

Can PDF be accurately reconstructed from 3 moments?



- Calculate moments from JAM global fit [P. C. Barry et. al. (JAM collaboration), arXiv:1804.01965]
- Reconstruct PDF from 1st 3 JAM moments
- Reconstructed PDF has larger errors, agrees well with actual JAM PDF
- Reconstructed $n = 4$ moment: $\langle x^4 \rangle_\pi^u = 0.026(2)$
- Actual JAM $n = 4$ moment: $\langle x^4 \rangle_\pi^u = 0.027(2)$

SU(3) flavor symmetry breaking



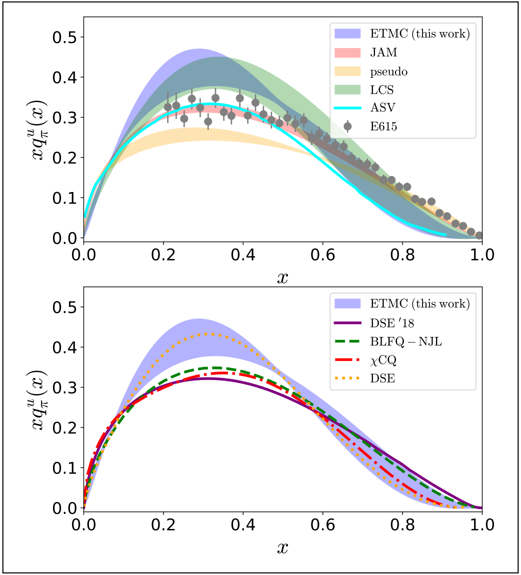
- Up quark equally prevalent in pion as in kaon for most regions of x
- Small difference between $xq^u_\pi(x)$ and $xq^u_K(x)$ around $x \approx 0.5$
- Distribution of strange quark in kaon is greater than up quark in pion for $x \approx 0.3$–0.7
- Peaks at

$$xq^u_\pi(x = 0.30) = 0.42(5)$$
$$xq^u_\pi(x = 0.27) = 0.43(2)$$
$$xq^u_\pi(x = 0.35) = 0.52(2)$$

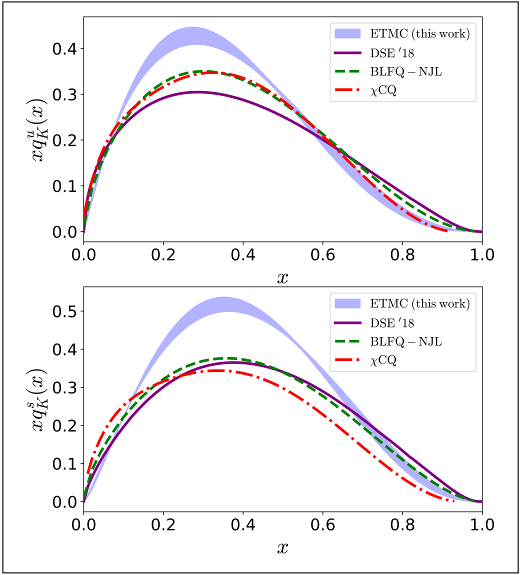
Mellin moments from reconstructed PDFs

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
q_π^u	0.229(3)(7)	0.087(5)(7)	0.042(5)(9)
q_K^u	0.217(2)(5)	0.077(2)(1)	0.035(2)(2)
q_K^s	0.279(1)(5)	0.114(2)(4)	0.057(2)(2)
q_M^f	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.023(5)(7)	0.014(4)(6)	0.009(3)(4)
q_K^u	0.019(1)(2)	0.011(1)(1)	0.007(1)(1)
q_K^s	0.032(2)(2)	0.020(1)(2)	0.013(1)(2)

- Calculate by integrating over reconstructed PDFs
- Uncertainties under control even for higher moments
- Our $\langle x^4 \rangle_\pi^u$ in agreement with moment from JAM PDF $\langle x^4 \rangle_\pi^u = 0.027(2)$
  [P. C. Barry et. al. (JAM collaboration), arXiv:1804.01965]



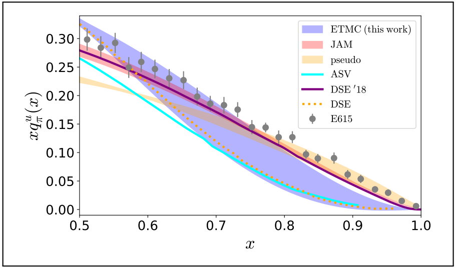
- pseudo, LCS: lattice results using non-local operators
- Qualitative comparison, studies have different systematic uncertainties which are not all quantified



- Good agreement at high- and low-x, most tension in intermediate-x region
- Qualitative comparison

Large-x behavior for pion

- There is some tension between studies of the high-x behavior of the pion PDF
- Un-quantified systematics
- Original analysis of Fermilab E615 (gray circles) experiment finds $\sim (1-x)^1$ $(\beta = 1)$



- More recent analysis of the same data (solid cyan line) finds $\sim (1-x)^2$ $(\beta = 2)$
- This study: $\beta = 2.23(65)$
- Our results favor $(1-x)^2$ large-x behavior, in agreement with ASV and DSE
- Our kaon results also favor $(1-x)^2$ large-x behavior

Overview

Summary

- Calculated first three non-trivial Mellin moments of PDFs
- Pioneering study has shown for the first time that PDFs can be reconstructed using the first three moments
- Higher order Mellin moments not included in fit can be calculated from reconstructed PDFs with well-controlled uncertainties

Summary

- Calculated first three non-trivial Mellin moments of PDFs
- Pioneering study has shown for the first time that PDFs can be reconstructed using the first three moments
- Higher order Mellin moments not included in fit can be calculated from reconstructed PDFs with well-controlled uncertainties

Thank you

Backup Slides

Meson decomposition for $\langle x \rangle$, $\langle x^2 \rangle$, and $\langle x^3 \rangle$

- Lattice breaks Euclidean Lorentz group O(4) symmetry to discreet hyber cubic group H(4) $\Longrightarrow$ mixing among operators
- We only use operators that are free of mixing with lower dimension operators, i.e., all indices are taken different for the 2- and 3-derivative operators
- This leads to decomposition in forward limit for general frame:

$$\Pi^{\{00\}} = \frac{1}{2E}\left(\frac{m^2}{2} - 2E^2\right)\langle x \rangle$$

$$\Pi^{\{0ij\}} = -p_i p_j \langle x^2 \rangle$$

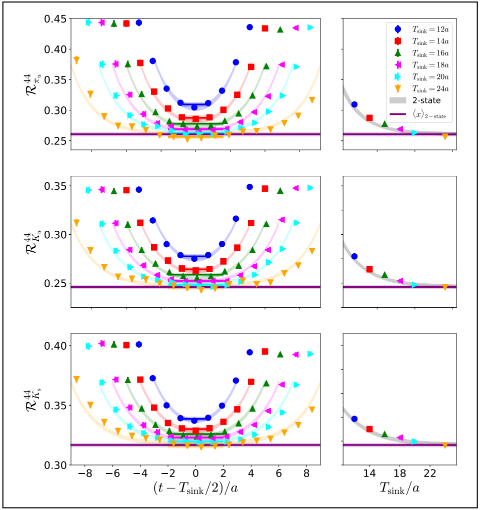
$$\Pi^{\{0ijk\}} = -i p_i p_j p_k \langle x^3 \rangle$$

$$E = \sqrt{m^2 + p^2}$$

p : hadron momentum

- Due to p in kinematic factor, $\langle x^n \rangle$ with $n > 1$ requires boosted frame to calculate $\langle x^n \rangle$
- Since indices i, j, and k are different, we need a boosted frame with at least:
  - $p = (\pm 1, \pm 1, 0)\frac{2\pi}{L}$ for $\langle x^2 \rangle$
  - $p = (\pm 1, \pm 1, \pm 1)\frac{2\pi}{L}$ for $\langle x^3 \rangle$

First Mellin moment $\langle x \rangle$, rest frame



- Plateau: $\dfrac{C^{\rm 3pt}_{\mathcal{O}}(t, T_{\rm sink})}{c_0 e^{-E_0 T_{\rm sink}}}$

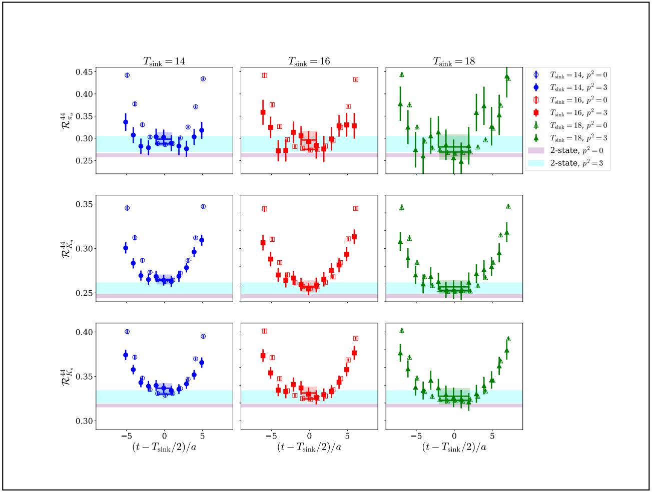
  from two-state fit

- Two-state fit consistent with plateau for $T_{sink} \geq 18a$ (1.6 fm)
- $\overline{MS}(2GeV)$

$$\langle x \rangle_u^{\pi^+} = 0.261(3)_{\rm stat}(6)_{\rm syst}$$
$$\langle x \rangle_u^{K^+} = 0.246(2)_{\rm stat}(2)_{\rm syst}$$
$$\langle x \rangle_s^{K^+} = 0.317(2)_{\rm stat}(1)_{\rm syst}$$

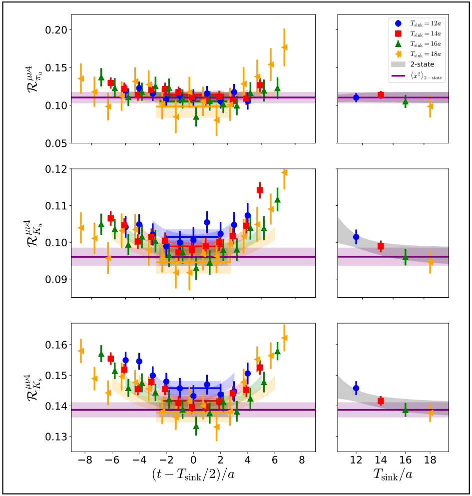
- Phenomonological results:
  $2\langle x \rangle_u^{\pi^+} = 0.48(1)$
  [Barry et. al., arXiv:1804:01965]
- Compatible with other lattice calculations at similar m_π. Comparisons in [Alexandrou et. al., arXiv:2010.03495]
- $\langle x \rangle_u^{K^+} < \langle x \rangle_u^{\pi^+} < \langle x \rangle_s^{K^+}$

First Mellin moment $\langle x \rangle$, momentum frame comparison



- Serves to test signal of $\langle x \rangle$ in boosted frame
- Useful for selecting $T_{\rm sink}$ to optimize computer resources
- Agreement between two frames in plateau and two-state fit
- More details in [Alexandrou et. al., arXiv:2010.03495]

Second Mellin moment $\langle x^2 \rangle$



$$\langle x^2 \rangle_u^{\pi^+} = 0.110(7)(12)$$
$$\langle x^2 \rangle_u^{K^+} = 0.096(2)(2)$$
$$\langle x^2 \rangle_s^{K^+} = 0.139(2)(1)$$

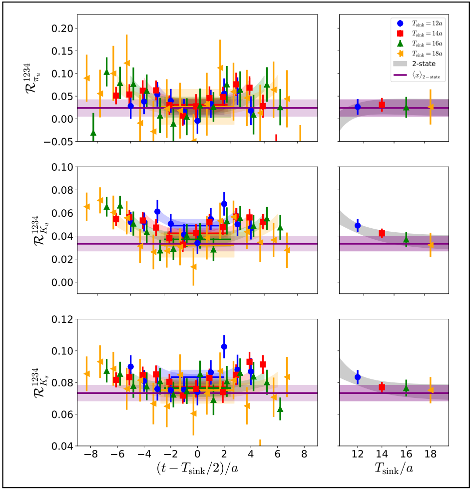
- Phenomonological results: $2\langle x^2 \rangle_u^{\pi^+} = 0.210(5)$

  [Barry et. al., arXiv:1804:01965]

- Ratio $\langle x^2 \rangle / \langle x \rangle$ is an indication of how quickly the PDFs lose support at large x

$$\frac{\langle x^2 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.423(28)(57)$$
$$\frac{\langle x^2 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.391(10)(16)$$
$$\frac{\langle x^2 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.438(8)(11)$$

Third Mellin moment $\langle x^3 \rangle$



$$\langle x^3 \rangle_u^{\pi^+} = 0.024(18)(2)$$
$$\langle x^3 \rangle_u^{K^+} = 0.033(6)(1)$$
$$\langle x^3 \rangle_s^{K^+} = 0.073(5)(2)$$

- π near zero due to high uncertainties
- Clear signal for both flavors of K
- $\langle x^3 \rangle < \langle x^2 \rangle < \langle x \rangle$

$$\frac{\langle x^3 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.092(71)(6)$$
$$\frac{\langle x^3 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.135(26)(8)$$
$$\frac{\langle x^3 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.232(16)(1)$$

Effect of fit function

fit type	α^u_π	β^u_π	γ^u_π	χ^2/d.o.f.
2-parameter	-0.05(19)	2.20(64)	0	1.50
3-parameter	-0.57(15)	2.72(61)	24.86(1.93)	—
fit type	α^u_K	β^u_K	γ^u_K	χ^2/d.o.f.
2-parameter	-0.005(81)	2.59(28)	0	1.95
3-parameter	-0.52(6)	3.17(27)	24.01(91)	—
fit type	α^s_K	β^s_K	γ^s_K	χ^2/d.o.f.
2-parameter	0.26(9)	2.27(22)	0	0.012
3-parameter	0.29(37)	1.85(2.21)	-0.58(5.32)	—