Hadronic vacuum polarization from step scaling in the Schwinger model

Lattice 2021

Fabian Frech for the BMWc

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BERGISCHE UNIVERSITÄT WUPPERTAL

Contents

HVP in the M_Z^2 range

The Adler function

- HVP: Observable for calculating hadronic contributions to α_{em}

$$\Pi(Q^2) - \Pi(Q^2_{\min}) = \sum_t \left(\frac{e^{iQt} - 1}{Q^2} + \frac{t^2}{2} \right) c(t)$$

$$c(t) = \sum_{\vec{x}} \langle j_i(\vec{0}, 0) j_i(\vec{x}, t) \rangle \quad (i \text{ fixed})$$

- Unlike the HVP the Adler function does not depend on the lower energy scale

$$D(Q^2) = 12\pi^2 Q^2 \frac{d\Pi}{dQ^2}$$

Discrete Adler function

- Adler function uses continous momenta
  $\Rightarrow$ Replace with $\Delta(Q^2) = \Pi(4Q^2) - \Pi(Q^2)$
- Zero momentum subtraction in the HVP not necessary

$$\Pi(Q^2) = \sum_t \frac{e^{iQt}}{Q^2} c(t)$$

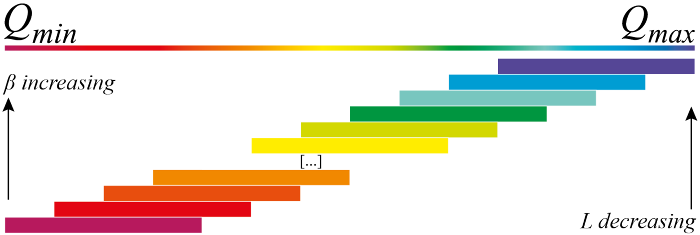
- Use diagonal momenta to get rid of zero modes and increase the energy range

$$c(t,x) = \frac{1}{2}(c_{\hat{t}\hat{t}}(t,x) - c_{\hat{t}\hat{x}}(t,x) - c_{\hat{x}\hat{t}}(t,x) + c_{\hat{x}\hat{x}}(t,x))$$

$$\Pi(Q^2) = \sum_{t,x} \frac{e^{iQ(t+x)}}{2Q^2} c(t,x)$$

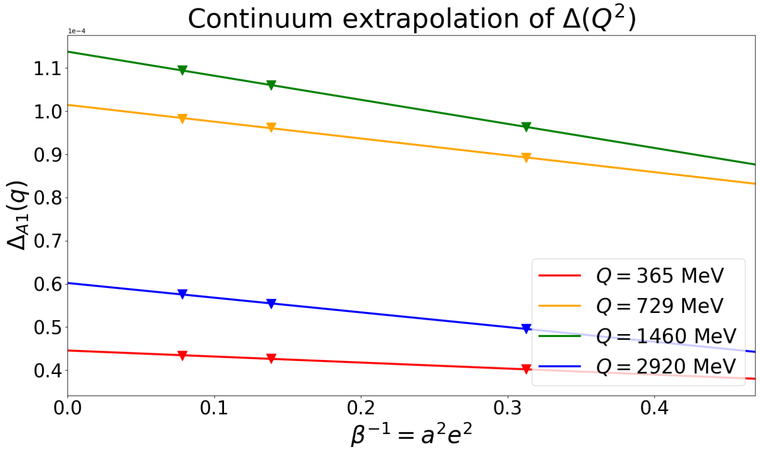
Strategy

- Cover the whole energy range $[0, M_Z]$ with different lattices
- Compute the Adler function on every lattice
- Reconstruct the HVP from the Adler function

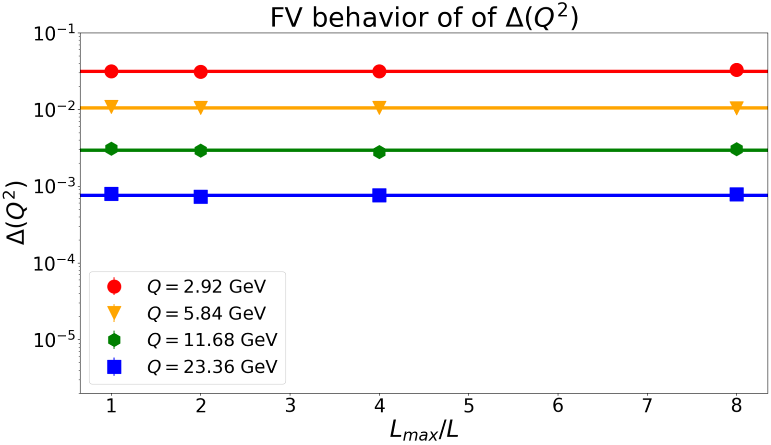


$$\Pi(Q^2_{max}) - \Pi(Q^2_{min}) =$$
$$[\Pi(Q^2_{max}) - \Pi(Q^4_{max}/2)] + [\Pi(Q^2_{max}/4) - \Pi(Q^2_{max}/16)] + \ldots + [\Pi(_{Qmax}/2^{2n}) - \Pi(Q^2_{min})]$$

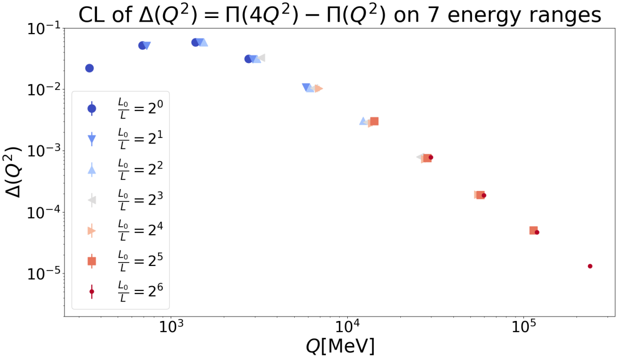
Discretization and FV effects



Discretization and FV effects



$\Delta(Q^2)$ on different scales and HVP value preliminary



Schwinger model results:
$\Delta\alpha_{had}(M_Z = 39.2M_\rho) \simeq 0.18$ with a statistical error of below 1%

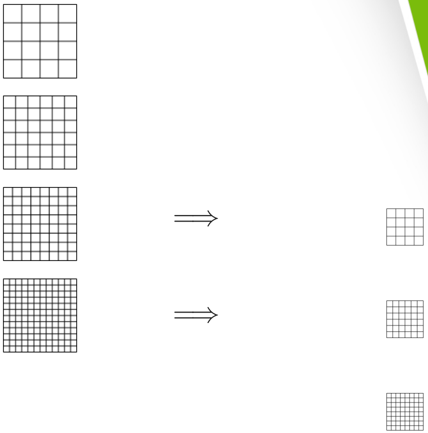
Estimation of LCP with the Adler function

Step scaling in the Schwinger model

- The parameters for the next energy range are set with step scaling
- Computations are relatively cheap and the exact values for the parameters are known:

$$\beta a^2 \sim const. \quad m \sim const.$$

- We can test a strategy for e.g. QCD

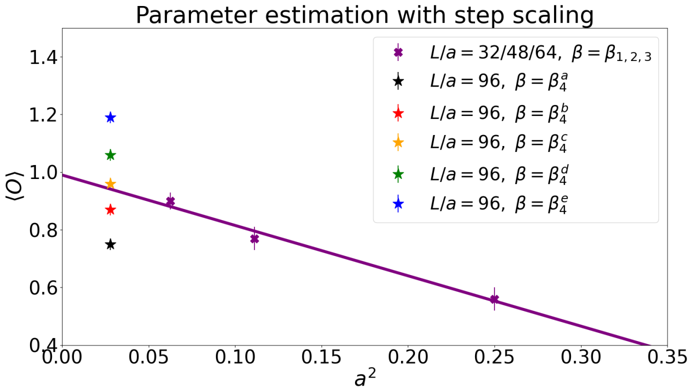


Step scaling in the Schwinger model

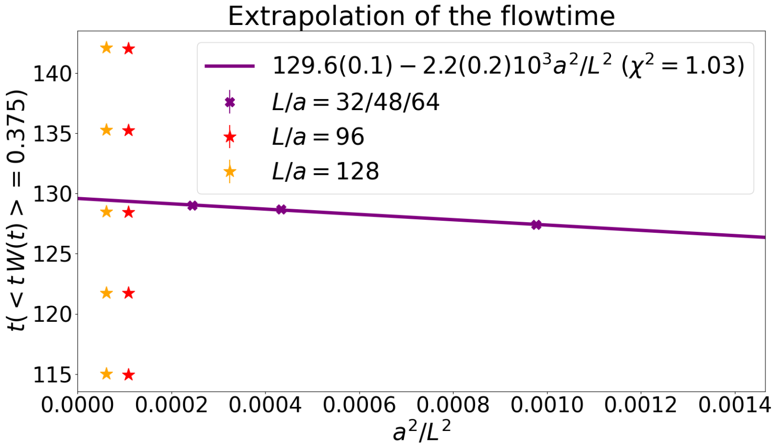
β_i	ma_i	$L_1/a = L/a$	$L_2/a = 2^{-1}L/a$	$L_3/a = 2^{-2}L/a$	...
$\beta_1 = 3.2$	$ma_1 = 0.5$	32			
$\beta_2 = 7.2$	$ma_2 = 0.33$	48			
$\beta_3 = 12.8$	$ma_3 = 0.25$	64	32		
β_4	ma_4	96	48		
β_5	ma_5	128	64	32	
β_6	ma_6		96	48	
β_7	ma_7		128	64	32
...	...				

Strategy

- $\beta_{1,2,3}$ are known are known, $\beta_4^{a,b,c,d,e}$ are estimators for β_4
- Estimation of β_4 with a fit using β_4^a, ...,β_4^e

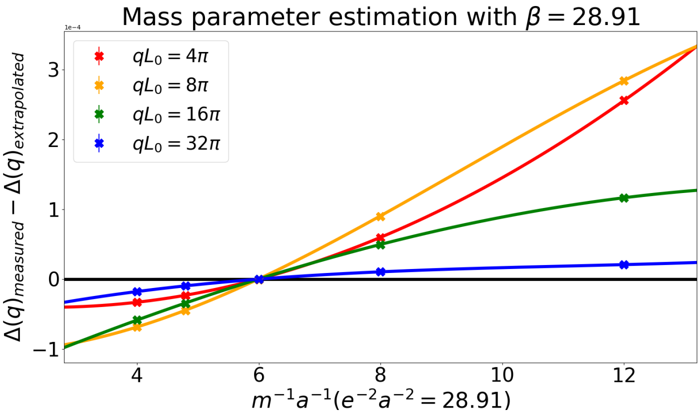


Setting of the gauge coupling parameter



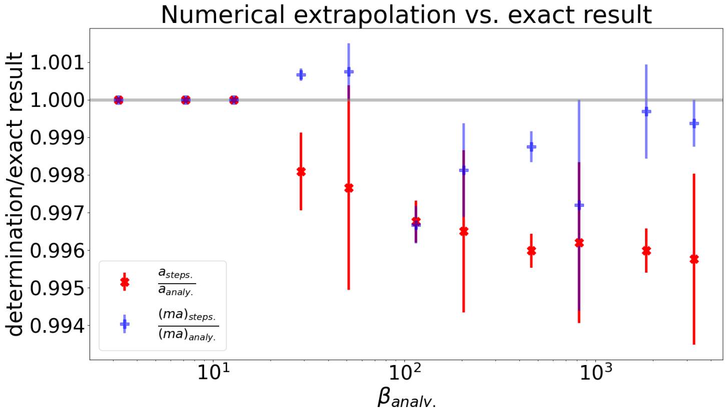
Mass parameter estimation

- Pion mass does not fit in small lattices $\Rightarrow$ Discrete Adler function can be measured for arbitrary large Q^2



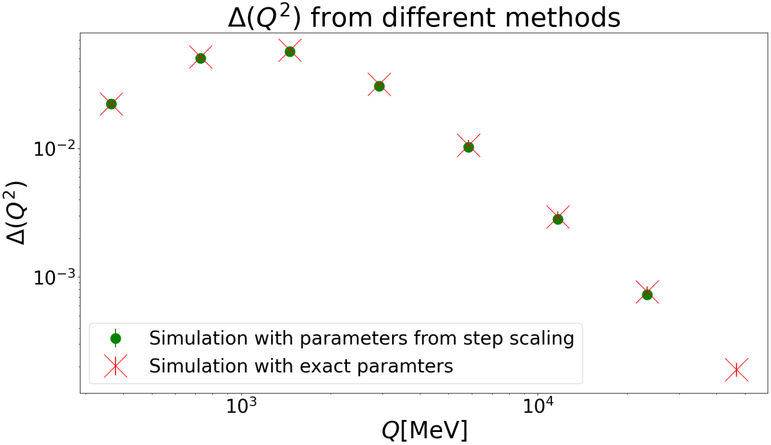
Results

Results for the LCP



(only statistical errors are shown)

$\Delta(Q^2)$ from different methods



Thank you for your attention!

Any questions?

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