Pion Distribution Amplitudes in the Continuum Limit

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This work is a collaboration with Huey-Wen Lin (MSU), Rui Zhang (MSU), and Carson Honkala (MSU)

Meson Distribution Amplitudes

Meson DA: probability of finding the meson in $q_1 - \bar{q}_2$ fock state.

$$\phi_M(x,\mu) = \frac{i}{f_M}\int \frac{d\mathcal{E}}{2\pi} e^{i(x-1)P\cdot\mathcal{E}n} \langle M(P)|\bar{\psi}(0) n\cdot\gamma\gamma_5 U(0,\mathcal{E}n)\psi(\mathcal{E}n)|0\rangle$$

Meson DAs important for understanding how light-quark hadron masses emerge from QCD

Like parton distribution functions:

- Universal quantity
- Non-perturbative

Unlike parton distribution functions:

- Less constrained by experiments
- Model-dependent calculations
- No global-fitting result to compare with

Lattice Setup

- 2+1+1 highly improved staggered quarks (HISQ) in the sea
- Clover fermion action for the valence quarks
- $M_\pi \approx$ 310 MeV, 220 MeV

Ensemble	a (fm)	M_π (MeV)	$M_\pi L$	$L^3 \times T$	P_z (GeV)	N_{src}	N_{conf}	N_{meas}
a15m310	0.1510	320(5)	3.93	$16^3 \times 48$	{1.02, 1.54, 2.05}	24	452	10,848
a12m310	0.1207	310(3)	4.55	$24^3 \times 64$	{1.28, 1.71, 2.14}	192	1013	194,496
a12m220	0.1184	228(2)	4.38	$32^3 \times 64$	{1.31, 1.63, 1.96}	384	959	368,256
a09m310	0.0888	313(3)	4.51	$32^3 \times 96$	{1.31, 1.74, 2.18}	48	889	39,648
a06m310	0.0582	320(2)	3.90	$48^3 \times 144$	{1.33, 1.77, 2.22}	4	593	2,372

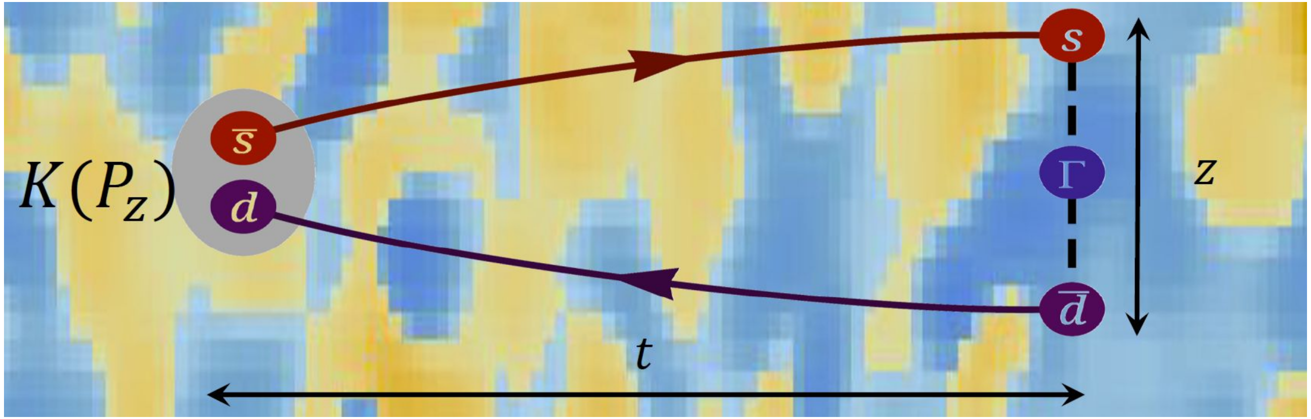
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Matrix Elements

DA two-point correlator:

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\,\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$

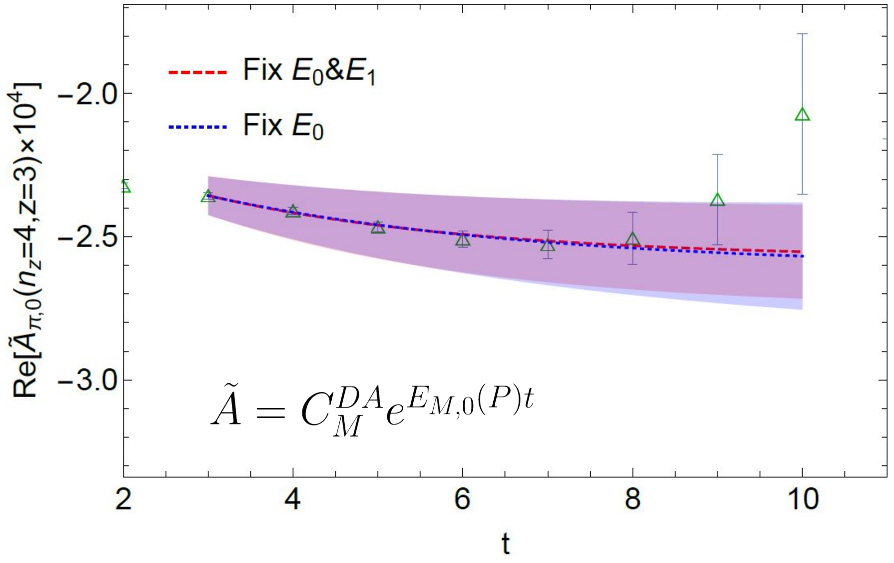
Matrix elements in coordinate space: $\tilde{h}_M(z,P_z) = \langle M(P)|\bar{\psi}(0)\gamma^z\gamma_5 U(0,z)\psi(z)|0\rangle$

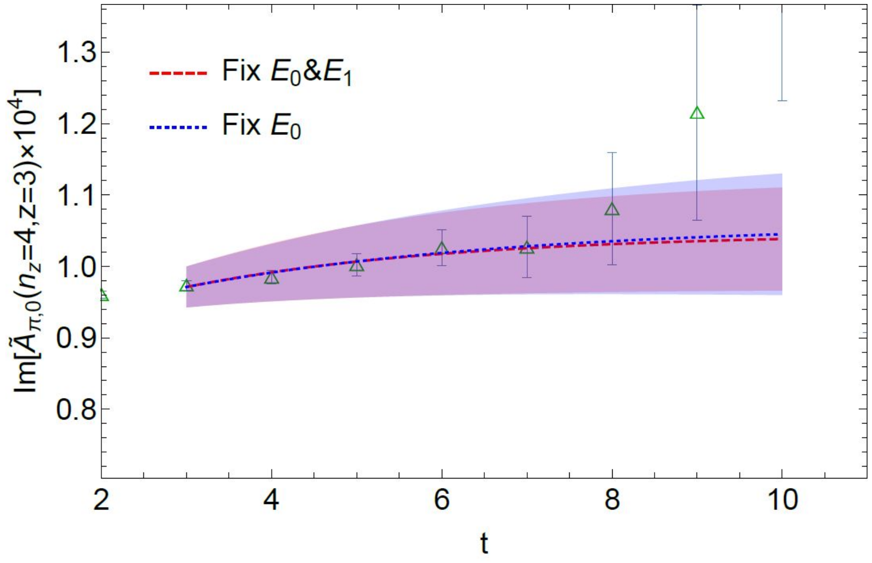


Matrix Elements

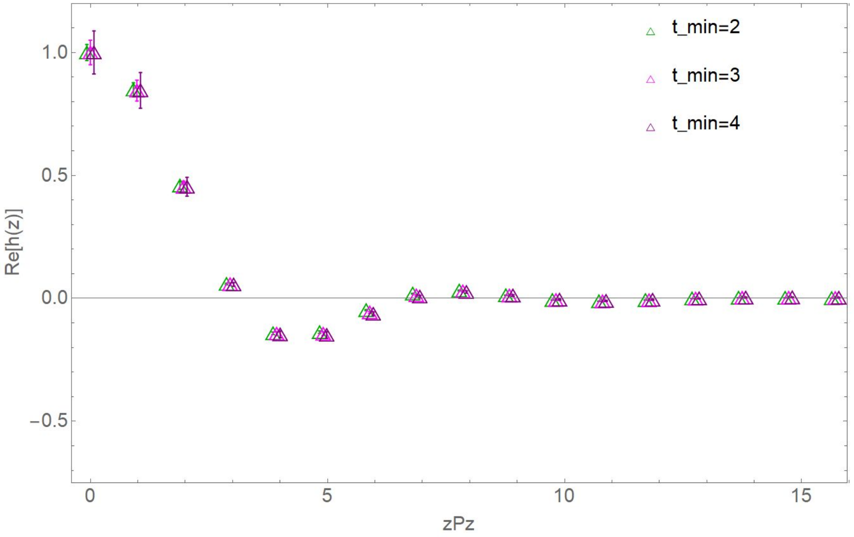
The DA matrix elements can be extracted by a two-point correlator fit to the form:

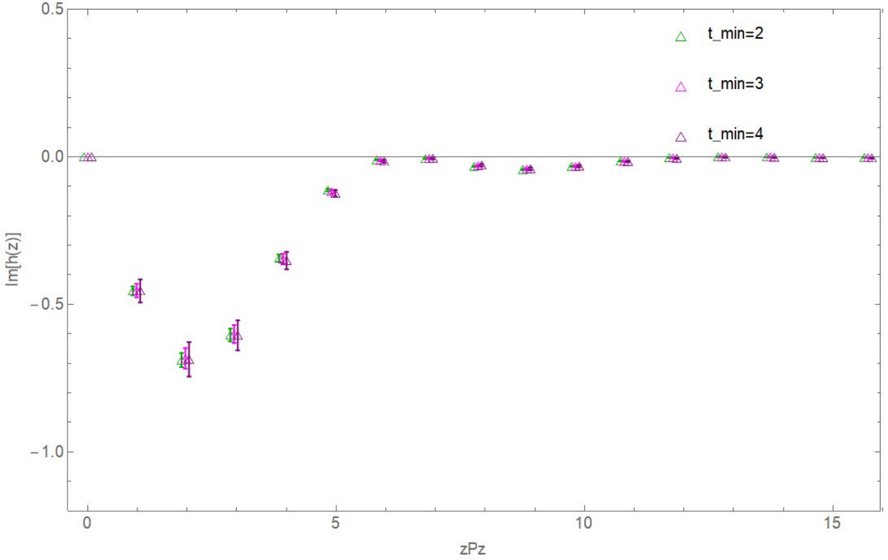
$$C_M^{DA}(z,P,t) = A_{M,0}^{DA}(P,z)e^{-E_{M,0}(P)t} + A_{M,1}^{DA}(P,z)e^{-E_{M,1}(P)t} + \ldots$$





Bare Matrix Elements: t_{min} Dependence

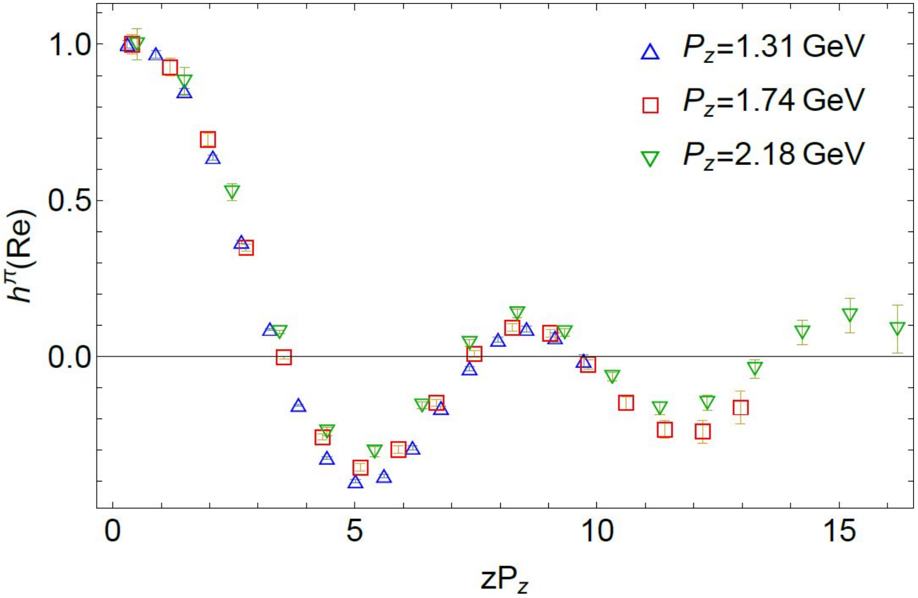


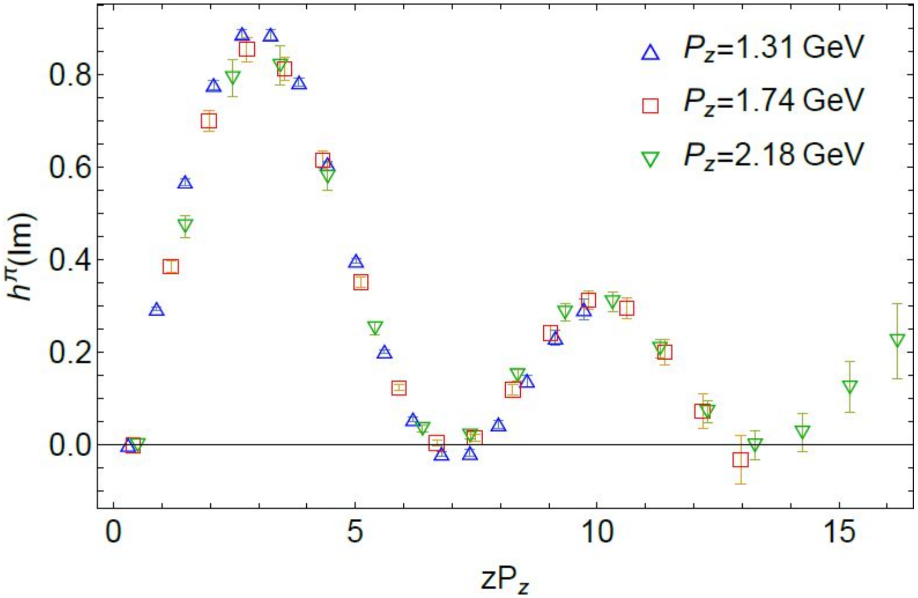


- a09m310, P_z=2.18 GeV
- t_{min} = {2,3,4} → t dependence

P_z Dependence of Renormalized Matrix Elements

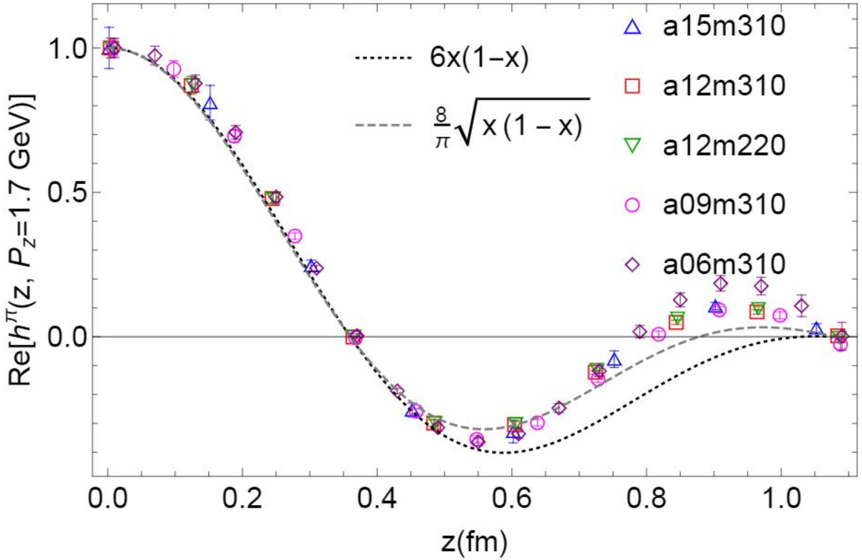
- RI/MOM [see R. *Zhang, C. Honkala, H.-W. Lin, and J.-W. Chen (2020)*]
  - $\mu^R = 3.8$ GeV, $p_z^R = 0$
- For example: a09m310

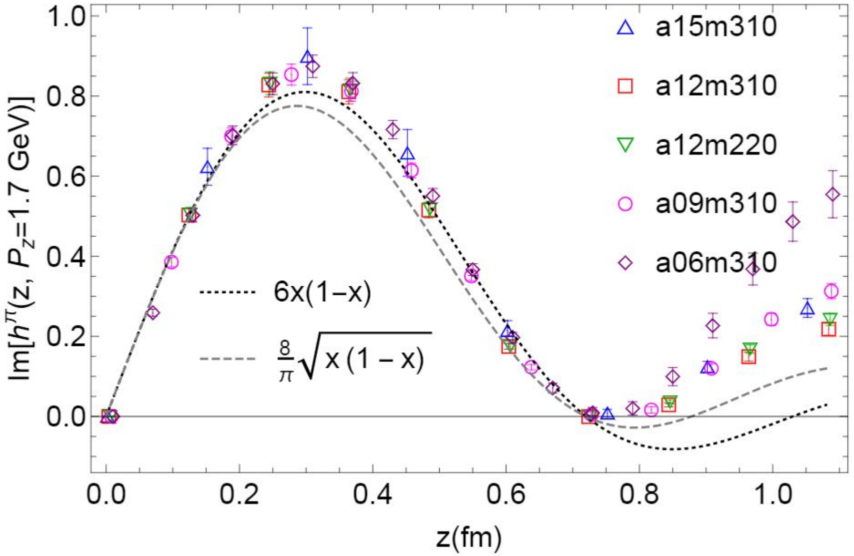




M_π and a Dependence of Renormalized Matrix Elements

- RI/MOM
  - μ^R = 3.8 GeV, p_z^R=0
- P_z≈1.7 GeV

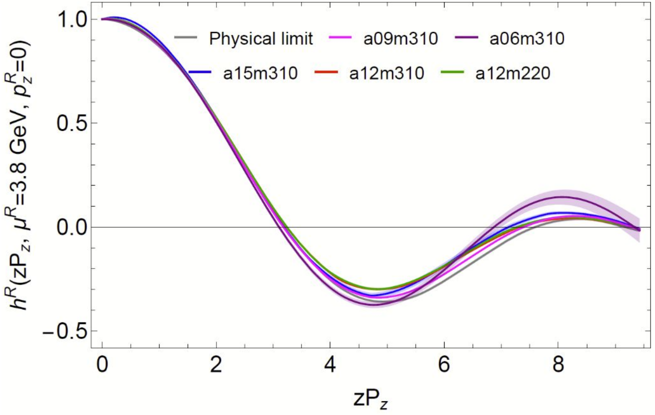


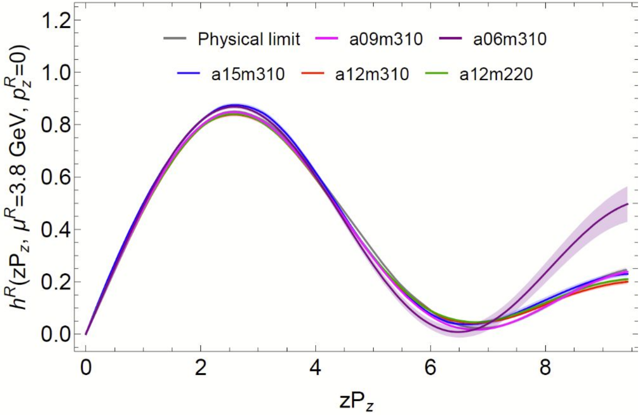


Continuum Matrix Elements and Extracting Pion DA

$$h = h_0(1 + c_2 a^2 + d_2 M_\pi^2)$$

P_z=1.7 GeV





$$h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy \;\; C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

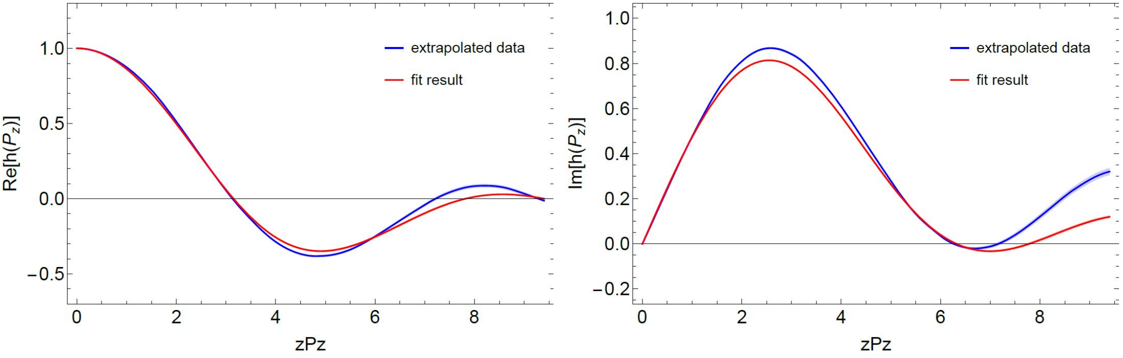
- *C* is the matching kernel (calculated perturbatively) [*Y.-S. Liu et al. (2019)*]
- We seek the lightcone DA, *f(y)*

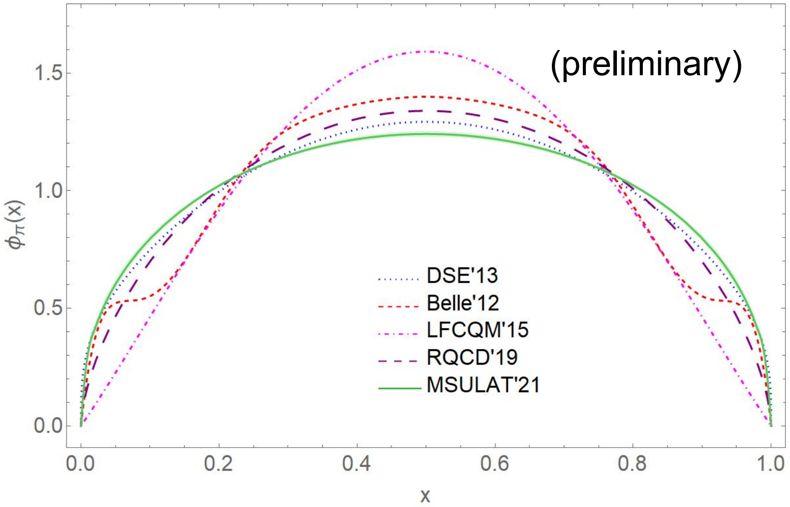
Fitting to Analytical Form

- Fit *h(z)* to continuum-physical matrix elements with the functional form:

$$f_{m,n} = x^m(1-x)^n / B(m+1, n+1)$$

1.74 GeV

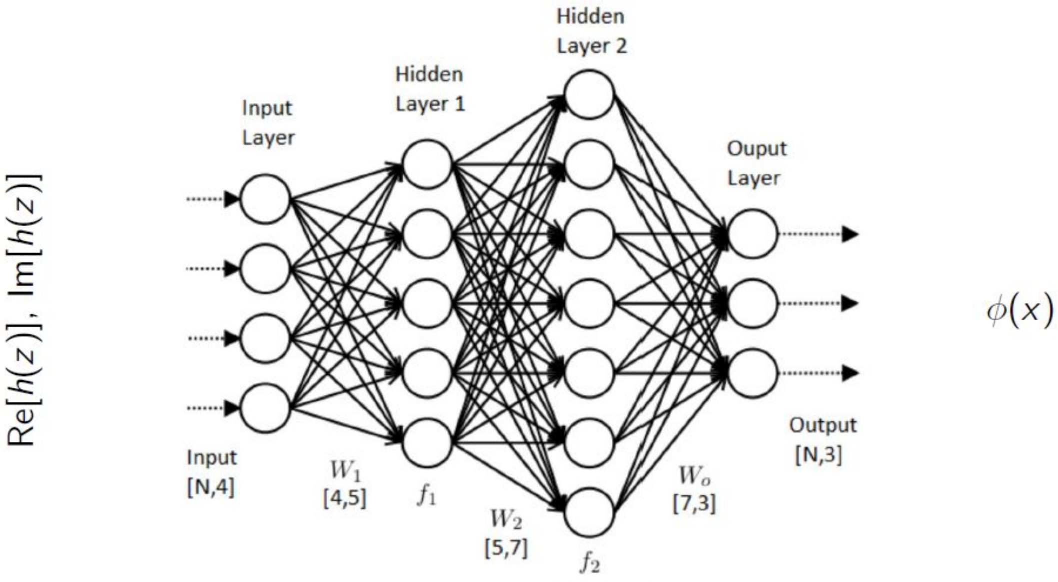




Machine Learning Approach?

Alternatively, we use ML to predict meson DA

- MLP regressor (scikit-learn)
- 3 hidden layers, 20 nodes each
- Default loss function (MSE)
- activation function
  g(x) = max(x,0)



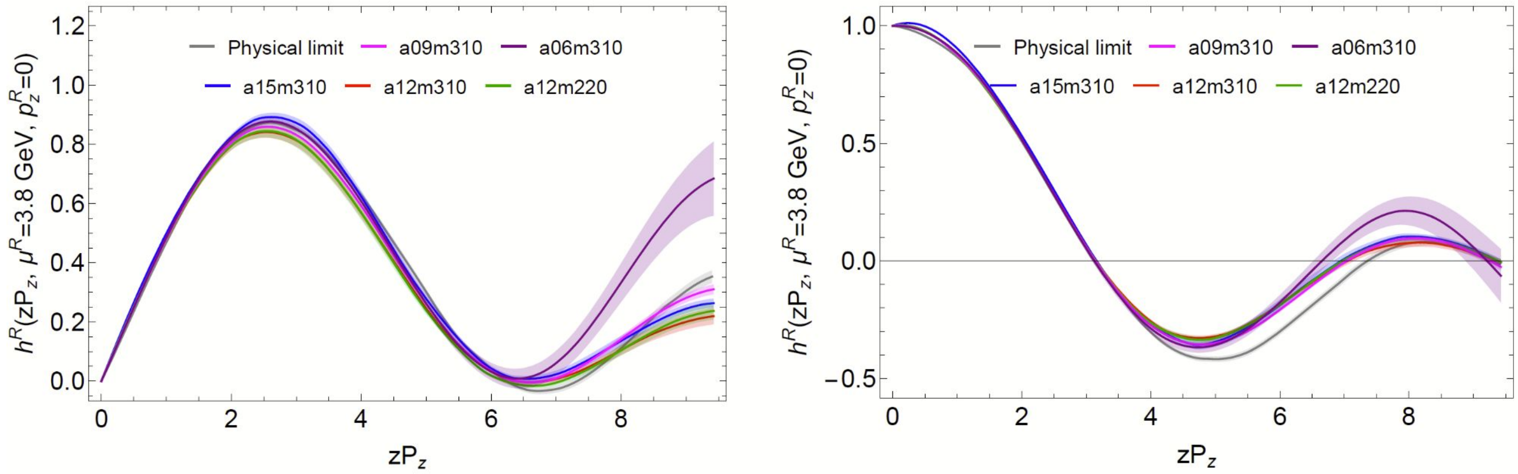
Credit: *VIASAT*

[cf. *R. Zhang et al. (2020)*]

Additional Slides

Continuum Matrix Elements

$$h = h_0(1 + c_1 a^1 + d_2 M_\pi^2)$$



Machine Learning Approach

Testing model predictions on known sine function