Nucleon isovector momentum fraction, helicity and transversity moment using Lattice QCD

Santanu Mondal*

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Collaborators: Rajan Gupta*, Sungwoo Park*, Boram Yoon*, Tanmoy Bhattacharya*, Huey-Wen Lin†, Balint Joo**, and Frank Winter††

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*Los Alamos National Laboratory, USA † Michigan State University, USA
** Oak Ridge National Laboratory, USA, †† Jefferson Lab, USA

Outline

- The three moments are obtained from matrix elements of twist-two operators
- The lattice two- and three-point correlators
- The lattice set-up
- Controlling the excited state contaminations in fits to correlators
- Chiral, continuum and finite volume (CCFV) extrapolations
  $\Longrightarrow$ results
  $\Longrightarrow$ comparison with other lattice results and global fits
- Summary

The three moments studied

- momentum fraction:

$\langle x \rangle_q = \int_0^1 \; dx \; x \; q(x)$, $q(x) \rightarrow$ unploarized distribution

$$\langle 0 | \overline{q} \gamma^4 \overleftrightarrow{D}^4 q | 0 \rangle \;\; = \;\; -M_N \langle x \rangle_q$$

- helicity moment:

$\langle x \rangle_{\Delta q} = \int_0^1 \; dx \; x \; \Delta q(x)$, $\Delta q(x) \rightarrow$ ploarized distribution

$$\langle 0 | \overline{q} \gamma^{\{3} \gamma^5 \overleftrightarrow{D}^{4\}} q | 0 \rangle \;\; = \;\; -\frac{iM_N}{2} \langle x \rangle_{\Delta q}$$

- transversity moment:

$\langle x \rangle_{\delta q} = \int_0^1 \; dx \; x \; \delta q(x)$, $\delta q(x) \rightarrow$ transversity distribution

$$\langle 0 | \overline{q} \sigma^{1\{2} \gamma^5 \overleftrightarrow{D}^{4\}} q | 0 \rangle \;\; = \;\; -\frac{iM_N}{2} \langle x \rangle_{\delta q}$$

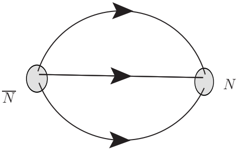
Lattice Correlators

The lattice correlators needed:

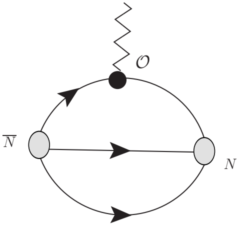
$$C^{2pt}(\tau) = \sum_{\boldsymbol{x}} \langle\Omega| T\Big(\mathcal{N}(\tau,\boldsymbol{x})\, \overline{\mathcal{N}}(0,\boldsymbol{0}) \Big)|\Omega\rangle$$

$$C^{3pt}_{\mathcal{O}}(\tau,t) = \sum_{\boldsymbol{x}',\boldsymbol{x}} \langle\Omega| T\Big(\mathcal{N}(\tau,\boldsymbol{x})\, \mathcal{O}(t,\boldsymbol{x}')\, \overline{\mathcal{N}}(0,\boldsymbol{0}) \Big)|\Omega\rangle$$

$$\mathcal{N} = \epsilon^{abc}\left[\psi^{aT}(x) C\gamma^5 \psi^b(x)\right] \psi^c(x), \quad \mathcal{O}(t,\boldsymbol{x}') = \overline{\psi}(t,\boldsymbol{x}')\Gamma D^{\mu}\psi(t,\boldsymbol{x}')$$

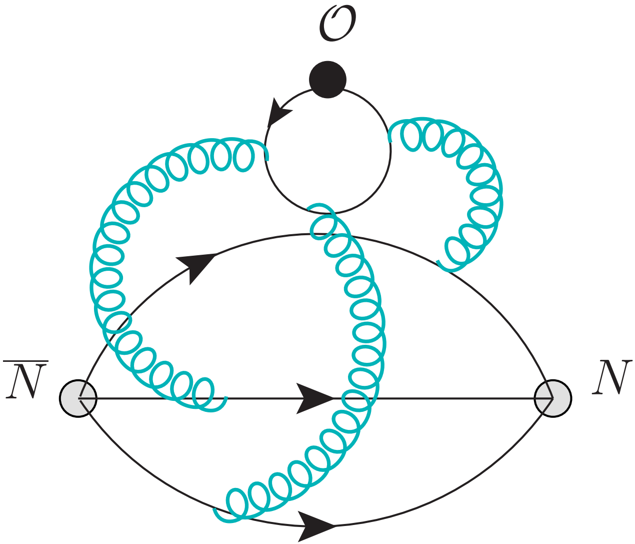


C^{2pt}



C^{3pt}

In iso-vector $(u-d)$ combination, disconnected contributions cancel



Lattice set up: Clover-on-HISQ (C-on-H)

- HISQ action for sea quarks and Clover action for the valance
- $2+1+1$ flavour QCD–physical strange and charm quark mass
- Large parameter space covered by 9 ensembles:
  $0.057 \leq a \leq 0.15$ fm; $135 \leq M_\pi \leq 310$ MeV; $3.7 \leq M_\pi L \leq 5.5$

Ensemble ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	τ/a	N_{conf}
$a15m310$	0.1510(20)	320.6(4.3)	$16^3 \times 48$	3.93	$\{5, 6, 7, 8, 9\}$	1917
$a12m310$	0.1207(11)	310.2(2.8)	$24^3 \times 64$	4.55	$\{8, 10, 12, 14\}$	1013
$a12m220$	0.1184(09)	227.9(1.9)	$32^3 \times 64$	4.38	$\{8, 10, 12, 14\}$	1156
$a12m220L$	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	$\{8, 10, 12, 14\}$	1000
$a09m310$	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	$\{10, 12, 14, 16\}$	2263
$a09m220$	0.0872(07)	225.9(1.8)	$48^3 \times 96$	4.79	$\{10, 12, 14, 16\}$	960
$a09m130$	0.0871(06)	138.1(1.0)	$64^3 \times 96$	3.90	$\{10, 12, 14, 16\}$	1041
$a06m310W$	0.0582(04)	319.6(2.2)	$48^3 \times 144$	4.52	$\{18, 20, 22, 24\}$	500
$a06m135$	0.0570(01)	135.6(1.4)	$96^3 \times 192$	3.7	$\{16, 18, 20, 22\}$	751

PNDME 20, Phys.Rev. D102 (2020) no.5, 054512, Thanks to MILC collaboration for the HISQ ensembles

Lattice set up: Clover-on-Clover (C-on-C)

- Clover action for both sea and valance quarks
- $2 + 1$ flavour QCD with physical strange quark mass
- Large parameter space covered by 9 ensembles:
  $0.056 \leq a \leq 0.127$ fm; $127 \leq M_\pi \leq 285$ MeV; $3.7 \leq M_\pi L \leq 6.2$

Ensemble ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	τ/a	N_{conf}
$a127m285$	0.127(2)	285(3)	$32^3 \times 96$	5.85	$\{8, 10, 12, 14\}$	2001
$a094m270$	0.094(1)	270(3)	$32^3 \times 64$	4.11	$\{10, 12, 14, 16, 18\}$	1464
$a094m270L$	0.094(1)	269(3)	$48^3 \times 128$	6.16	$\{10, 12, 14, 16\}$	4501
$a091m170$	0.091(1)	169(2)	$48^3 \times 96$	3.74	$\{8, 10, 12, 14, 16\}$	4015
$a091m170L$	0.091(1)	169(2)	$64^3 \times 128$	5.08	$\{8, 10, 12, 14, 16\}$	1533
$a073m270$	0.0728(8)	272(3)	$48^3 \times 128$	4.8	$\{11, 13, 15, 17, 19\}$	4477
$a071m170$	0.0707(8)	167(2)	$72^3 \times 192$	4.26	$\{15, 17, 19, 21\}$	2100
$a071m130$	0.0707(8)	127(1)	$96^3 \times 192$	4.36	$\{13, 15, 17, 19, 21\}$	440
$a056m280$	0.056(1)	280(5)	$64^3 \times 192$	5.09	$\{18, 21, 24, 27, 30\}$	1723

NME 20, JHEP 2104 (2021) 044

Spectral Decomposition

$$C_{2pt}(\tau) = |A_0|^2 e^{-M_0\tau} + |A_1|^2 e^{-M_1\tau} + |A_2|^2 e^{-M_2\tau} + |A_3|^2 e^{-M_3\tau} + \cdots$$

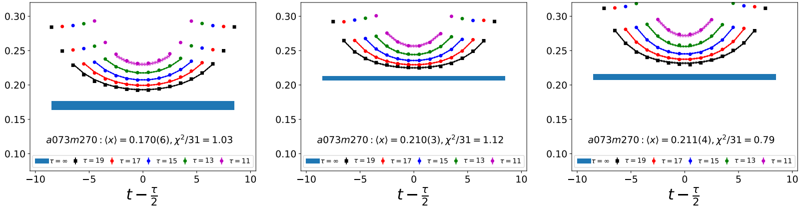
$$\begin{aligned} C_{3pt}(\tau, t) &= |A_0|^2 \langle 0|\mathcal{O}|0\rangle e^{-M_0\tau} + A_0^* A_1 \langle 1|\mathcal{O}|0\rangle e^{-M_1(\tau-t)} e^{-M_0 t} \\ &+ A_0 A_1^* \langle 0|\mathcal{O}|1\rangle e^{-M_0(\tau-t)} e^{-M_1 t} + |A_1|^2 \langle 1|\mathcal{O}|1\rangle e^{-M_1\tau} + \cdots , \end{aligned}$$

Data displayed using the ratio

$$\frac{C_{3pt}(\tau, t)}{C_{2pt}(\tau)} = \langle 0|\mathcal{O}|0\rangle + \text{excited state contaminations}$$

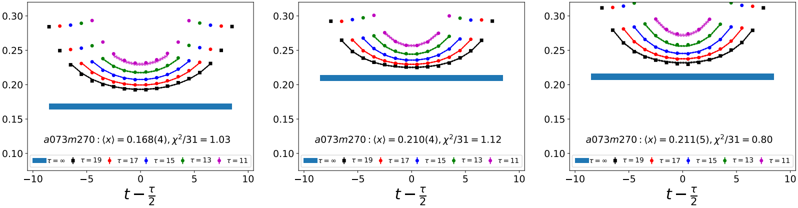
Three fit strategies: strategy $\{4, 3^*\}$

- 4-state fit of C_{2pt} ; output $M_1 \sim N(1440)$ MeV
- Use output M_0, M_1, M_2 and A_0 as inputs in 3-state fit to $C_{3pt}(\tau)$ with 3–4 largest values of τ (set $\langle 2|\mathcal{O}|2\rangle = 0$)



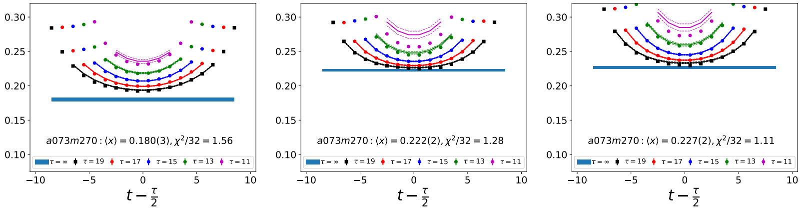
Three fit strategies: strategy $\{4^{N\pi}, 3^*\}$

- 4-state fit to C_{2pt} with a narrow prior for M_1 about the non-interacting $N\pi$ (or $N\pi\pi$) state mass
- Use output M_0, M_1, M_2 and A_0 as inputs in 3-state fit to $C_{3pt}(\tau)$ with 3–4 largest values of τ (set $\langle 2|\mathcal{O}|2\rangle = 0$)



Three fit strategies: strategy $\{4, 2^{\text{free}}\}$

- Ground state M_0 and A_0 from 4-state fit to C_{2pt}
- M_1 is a free parameter in 2-state fits to $C_{3pt}(\tau)$



Renormalization

- Non-perturbative renormalization factors determined in RI'-MOM scheme
- Converted to $\overline{MS}$ scheme using 3-loop perturbative factors
- These numbers are then run in the continuum $\overline{MS}$ scheme from lattice scale to 2 GeV using 3-loop anomalous dimensions

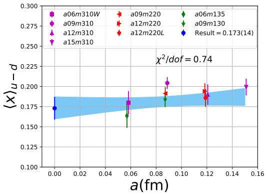
PNDME 20, Phys.Rev. D102 (2020) no.5, 054512

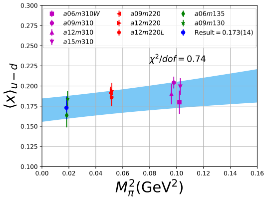
NME 20, JHEP 2104 (2021) 044

NME 21: NME 20 + two ensembles

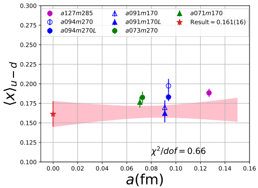
CCFV extrapolation for $\langle x \rangle_{u-d}$ ($\{4, 3^*\}$)

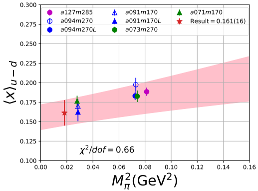
C-on-H (PNDME 20): $\langle x \rangle (M_\pi; a; L) = c_1 + c_2 a + c_3 M_\pi^2$ (FV term not resolved)

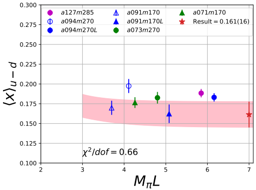




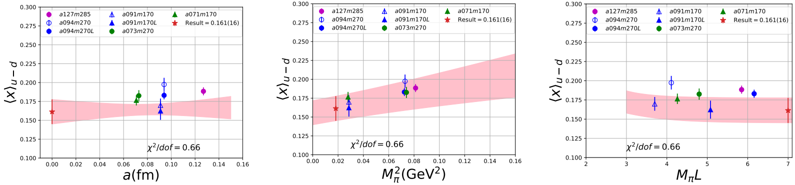
C-on-C (NME 20): $\langle x \rangle (M_\pi; a; L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 \frac{M_\pi^2 \; e^{-M_\pi L}}{\sqrt{M_\pi L}}$



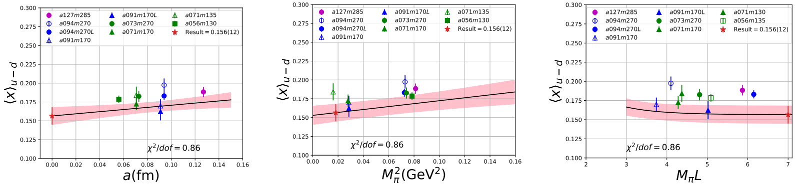




C-on-C (NME 20) 7 ensembles:



C-on-C (NME 21 (Preliminary)): 9 ensembles



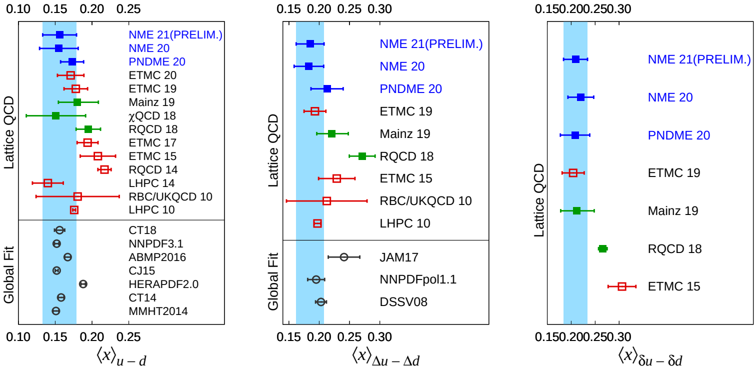
Final results

- Data converge from above $\longrightarrow$ estimates of moments increase as $\{4^{N\pi}, 3^*\} \longrightarrow \{4, 3^*\} \longrightarrow \{4, 2^{\mathrm{free}}\}$ because
  $M_1^{4^{N\pi}} \quad < \quad M_1^4 \quad < \quad M_1^{4,2free}$
- Since $M_1^{4,2free} > M_1^4 \approx 1440$ MeV $\longrightarrow$ we choose $\{4, 3^*\}$ to quote the final result
- To account for possible bias due to this choice, we add a second, systematic, error which is half the difference between $\{4^{N\pi}, 3^*\}$ and $\{4, 2^{\mathrm{free}}\}$ values.

Final results

Moment	PNDME 20	NME 20	NME 21
$\langle x \rangle_{u-d}$	0.173(14)(07)	0.155(17)(20)	0.156(12)(20)
$\langle x \rangle_{\Delta u-\Delta d}$	0.213(15)(22)	0.183(14)(20)	0.185(12)(20)
$\langle x \rangle_{\delta u-\delta d}$	0.208(19)(24)	0.220(18)(20)	0.209(15)(20)

Comparison with World data



Summary

- Presented lattice results for the iso-vector momentum fraction, helicity and transversity moments of a proton
- High statistics data, use of quark source smearings and three-point data at multiple source-sink separations allow us to study the effect of excited state contaminations, specially the effect of the first excited state
- 7–9 ensembles spanning resonable ranges of $\{a, M_\pi, M_\pi L\}$ allow us to perform reliable chiral-continuum-finite-volume extrapolations
- The overall consistency of results suggests that lattice QCD calculations of these isovector moments are now mature and future calculations will steadily reduce the statistical and systematic uncertainties in them

Thank You for your attention!

Comparison with World data

Martha Constantinou et al., arXiv:2006.08636 (2020)

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value
$\langle x \rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)
	ETMC 19	(Alexandrou *et al.*, 2020c)	2+1+1	■	★	○	★	★	**	0.178(16)
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)
	ETMC 19	(Alexandrou *et al.*, 2020c)	2	■	★	○	★	★	**	0.189(23)
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)
$\langle x \rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)
$\langle x \rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)
$\langle x \rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)
$\langle x \rangle_{g}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)

** No quenching effects are seen.

TABLE III Lattice QCD values of the benchmark moments of unpolarized PDFs $\langle x \rangle_{u^+-d^+}$, $\langle x \rangle_{u^+}$, $\langle x \rangle_{d^+}$, $\langle x \rangle_{s^+}$ and $\langle x \rangle_{g}$, rated according to the criteria in Tab. II. The numbers in parentheses refer to the statistical and systematic uncertainties, respectively, or to the combination of the two, if a single value is provided. All values are obtained at $\mu = 2$ GeV.