Unpolarized and polarized gluon pseudo-distributions at short distances: Forward case

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Motivation

- Lattice calculations of parton distribution functions (PDFs) are a subject of considerable interest and efforts
- PDFs not directly calculable on the lattice, $z^2 = 0$ doesn't work in Euclidean space
- X. Ji's ground-breaking proposal to consider equal-time versions of nonlocal operators: quasi-PDFs [Ji, 2013]. Taking $z = (0, 0, 0, z_3)$:

$$\tilde{q}\left(x, \mu^2, P_3\right) = \int \frac{\mathrm{d}z}{4\pi} e^{-ixzP_3} \langle P| \bar{\psi}(z) \gamma^3 \exp\left(-ig \int_0^z \mathrm{d}z' A^z(z')\right) \psi(0) |P\rangle \quad (1)$$

- PDFs are obtained from the large-momentum $P_3 \to \infty$ limit of quasi-PDFs
- A. Radyushkin introduced a coordinate-space oriented approach [Radyushkin, 2017]

$$\mathcal{P}\left(x, z_3^2\right) = \int_{-\infty}^{\infty} \mathrm{d}\nu e^{-ix\nu} \langle p| \phi(z) \phi(0) |p\rangle = \int_{-\infty}^{\infty} \mathrm{d}\nu e^{-ix\nu} \mathcal{M}\left(p_3 z_3, z_3^2\right), \quad (2)$$

$$\mathcal{P}(x, 0) = f(x)$$

- Ioffe-time distribution (ITD) $\mathcal{M}\left(\nu, z_3^2\right)$, with $\nu = -(pz) = p_3 z_3$ [Braun, et al, 1995]
- PDFs are obtained from $z_3 \to 0$ limit of psuedo-PDFs

Pseudo-PDFs

- We would like, small z_3, to have $1/z_3$ analogous to the renormalization parameter μ of scale-dependent PDFs $f\left(x, \mu^2\right)$ of the standard OPE approach
- z_3^2 dependence comes not only from evolution logarithms: $\log\left(z_3^2 \mu_{IR}^2\right)$, but UV logarithms: $\log\left(z_3^2 \mu_{UV}^2\right)$ as well
- Since UV divergences have no ν dependence at leading log, and if $\mathcal{M}\left(\nu, z_3^2\right)$ is multiplicatively renormalizable, can define reduced ITD [Orginos, et al, 2017]:

$$\mathfrak{M}\left(\nu, z_3^3\right) = \left(\frac{\mathcal{M}\left(\nu, z_3^2\right)}{\mathcal{M}\left(\nu, 0\right)\big|_{z=0}}\right) / \left(\frac{\mathcal{M}\left(0, z_3^2\right)\big|_{p=0}}{\mathcal{M}\left(0, 0\right)\big|_{p=0, z=0}}\right) \quad (3)$$

- This leads to the evolution equation:

$$\frac{\mathrm{d}}{\mathrm{d}\log z_3^2}\mathfrak{M}\left(\nu, z_3^3\right) = -\frac{\alpha_s}{2\pi} C \int_0^1 \mathrm{d}u \; B\left(u\right) \mathfrak{M}\left(u\nu, z_3^3\right) \quad (4)$$

- Taking $z_3 \to 0$ to extract light-cone PDF is singular, and one needs to use matching relations to go from Euclidean lattice data to PDFs

- The gluon distribution calculation is complicated by gauge-invariance
- Effective to use external field method along with the Schwinger representation for the propagator via the QCD heat kernel [Balitsky, Braun, 1988]
- External field method involves separating fields into a fluctuating quantum field with virtualities between μ_2^2 and μ_1^2 and a slowly varying "classical" field with virtualities below μ_1^2 ($A_\mu = A_\mu^q + A_\mu^{cl}$ and $\psi = \psi_q + \psi_{cl}$)

$$\mathcal{L} = -\frac{1}{4g^2}\left(G_{\mu\nu}^{cl,a} + D_\mu A_\nu^{q,a} - D_\nu A_\mu^{q,a} + f^{abc} A_\mu^{q,b} A_\nu^{q,c}\right)^2 + \left(\bar{\psi}_q + \bar{\psi}_{cl}\right)\left(i\not{D} + A_\mu^{q,a}\gamma^\mu t^a\right)\left(\psi_q + \psi_{cl}\right) + \mathcal{L}_{GF} + \mathcal{L}_g \tag{5}$$

$$gA_\mu \to A_\mu, \quad D_\mu = \partial_\mu - iA_\mu^{cl}, \quad G_{\mu\nu}^{q,a} = D_\mu A_\nu^{a,q} - D_\nu A_\mu^{a,q} + f^{abc} A_\mu^{q,b} A_\nu^{q,c}$$

Method of calculation

- The background field gauge: $D^\mu A^q_\mu = 0$ is used for quantum fields
- And the Fock-Schwinger gauge: $z^\mu A^{cl}_\mu(z) = 0$ is used for "classical" fields

$$\implies A^{cl}_\nu(z) = \int_0^1 \mathrm{d}v v z^\mu G^{cl}_{\mu\nu}(vz) \tag{6}$$

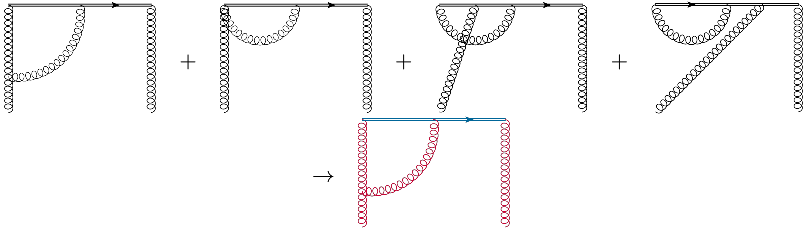
- Schwinger representation for the propagator

$$\frac{i}{P^2 + i\epsilon} = \int_0^\infty \mathrm{d}s \exp\left[is\left(P^2 + i\epsilon\right)\right] \tag{7}$$

- Gluon propagator in terms of external gluon fields (omitting ϵ):

$$g^{-2} i A^a_\mu(z) A^b_\nu(0) = \langle z| \left(\frac{1}{P^2 g_{\mu\nu} + 2iG_{\mu\nu}}\right)^{ab} |0\rangle = -i \int_0^\infty \mathrm{d}s \, \langle z| \, e^{is\left(P^2 g_{\mu\nu} + 2iG_{\mu\nu}\right)} |0\rangle$$
$$= -ig_{\alpha\beta}\frac{\Gamma(d/2-1)}{4\pi^2\left(-z^2\right)^{d/2-1}} + \frac{\Gamma(d/2-2)}{16\pi^2\left(-z^2\right)^{d/2-2}} \int_0^1 \mathrm{d}u \left\{2G_{\alpha\beta}(uz) - \bar{u}uD_\sigma G^{\sigma\rho}(uz) z_\rho g_{\alpha\beta}\right.$$
$$\left. -2ig_{\alpha\beta}\int_0^u \mathrm{d}v \bar{u} v z^\lambda G_{\lambda\xi}(uz) z^\rho G_\rho^{\ \xi}(vz)\right\} - \frac{i\Gamma(d/2-3)}{16\pi^2\left(-z^2\right)^{d/2-3}}$$
$$\times \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}v \left[G_{\alpha\xi}(uz) G_\beta^{\ \xi}(vz) - \tfrac{1}{2} i\bar{u} D^2 G_{\alpha\beta}(uz)\right]$$
$$+ \mathcal{O}(\text{twist 3}) \tag{8}$$

Vertex diagrams:



Linear divergences are 'hidden' inside the vertex diagram:

$$\mathcal{O}^V_{\mu\alpha;\nu\beta}(z)$$
$$\to \frac{g^2 N_c \Gamma(d/2-1)}{4\pi^2(-z^2)^{d/2-1}} \int_0^1 \mathrm{d}u \int_0^{\bar{u}} \mathrm{d}v \left\{ \delta(u) \left(\frac{v^{3-d}-v}{d-2} \right) G_{\mu\alpha}(\bar{u}z) \left(z_\beta G_{x\nu}(vz) - z_\nu G_{z\beta}(vz) \right) \right.$$
$$\left. + \delta(v) \left(\frac{u^{3-d}-u}{d-2} \right) \left(z_\alpha G_{x\mu}(\bar{u}z) - z_\mu G_{z\alpha}(\bar{u}z) \right) G_{\nu\beta}(vz) \right\}$$
$$+ \frac{N_c \Gamma(d/2-2)}{8\pi^2(-z^2)^{d/2-2}} \int_0^1 \mathrm{d}u \int_0^{\bar{u}} \mathrm{d}v \left\{ \delta(u) \left[\frac{v^{3-d}-1}{d-3} \right]_+ + \delta(v) \left[\frac{u^{3-d}-1}{d-3} \right]_+ \right\} G_{\mu\alpha}(\bar{u}z) G_{\nu\beta}(vz) \tag{9}$$

Matrix elements

Nucleon spin-averaged matrix elements with non-contracted indices

$$M_{\mu\alpha;\nu\beta}(z,p) \equiv \langle p| \, G_{\mu\alpha}(z) \, [z,0] \, G_{\nu\beta}(0) \, |p\rangle \tag{10}$$

with straight-line gauge link in the adjoint representation

$$[x,y] \equiv \mathcal{P} \exp\left[ig \int_0^1 \mathrm{d}u (x-y)^\mu A_\mu \left(ux + (1-u)y\right)\right] \tag{11}$$

with Lorentz decomposition

$$\begin{aligned}
&M_{\mu\alpha;\nu\beta}(z,p) \\
&= \left(g_{\mu\nu}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\nu - g_{\alpha\nu}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\nu\right) \mathcal{M}_{pp}(\nu, z^2) \\
&+ \left(g_{\mu\nu}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\nu - g_{\alpha\nu}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\nu\right) \mathcal{M}_{zz}(\nu, z^2) \\
&+ \left(g_{\mu\nu}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\nu - g_{\alpha\nu}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\nu\right) \mathcal{M}_{zp}(\nu, z^2) \\
&+ \left(g_{\mu\nu}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\nu - g_{\alpha\nu}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\nu\right) \mathcal{M}_{pz}(\nu, z^2) \\
&+ \left(p_\mu z_\alpha p_\nu z_\beta - p_\alpha z_\mu p_\nu z_\beta - p_\mu z_\alpha p_\beta z_\nu + p_\alpha z_\mu p_\beta z_\nu\right) \mathcal{M}_{ppzz}(\nu, z^2) \\
&+ \left(g_{\mu\nu}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\nu}\right) \mathcal{M}_g(\nu, z^2)
\end{aligned} \tag{12}$$

Matrix elements

The light-cone distribution is obtained from

$$g^{\alpha\beta} M_{+\alpha,\beta+}(z_-, p) = -2p_+^2 \mathcal{M}_{pp}(\nu, 0) \tag{13}$$

so

$$-\mathcal{M}_{pp}(\nu, 0) = \frac{1}{2} \int_{-1}^{1} \mathrm{d}x e^{-ix\nu} x f_g(x) \tag{14}$$

Taking other projections, there are three multiplicatively renormalizable [Zhang et. al, 2018] quantities

$$\langle p| G_{3i}(z) G_{i3}(0) |p\rangle = -2\mathcal{M}_g + 2p_3^2 \mathcal{M}_{pp} + 2z_3^2 \mathcal{M}_{zz} + 2z_3 p_3 \left(\mathcal{M}_{zp} + \mathcal{M}_{pz}\right) \tag{15}$$

$$\langle p| G_{0i}(z) G_{i0}(0) |p\rangle = 2\mathcal{M}_g + 2p_0^2 \mathcal{M}_{pp} \tag{16}$$

$$\langle p| G_{0i}(z) G_{i3}(0) + G_{3i}(z) G_{i0}(0) |p\rangle = 4p_0 p_3 \mathcal{M}_{pp} + 2p_0 z_3 \left(\mathcal{M}_{pz} + \mathcal{M}_{zp}\right) \tag{17}$$

There are higher twist contaminations, but can isolate $\mathcal{M}_{pp}$ through:

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2 \mathcal{M}_{pp} \tag{18}$$

where $M_{ji;ij} = -2\mathcal{M}_g$ shares the same anomalous dimension as $M_{0i;i0}$.

Leading twist one-loop gluon calculation for $M_{0i;i0} + M_{ji;ij}$:

$$\begin{aligned}
&\mathcal{M}_{pp}(\nu, z_3^2) \\
&\to \frac{\alpha_s N_c}{2\pi} \left\{ \left(\frac{1}{\epsilon_{\text{UV}}} + \log\left(z_3^2 \mu_{UV}^2 e^{2\gamma}/4\right) + 2 \right) \delta(\bar{u}) \right. \\
&\quad - \int_0^1 \mathrm{d}u \left[\frac{2}{3}\left(1 - u^3\right) + \frac{4u + 4\log(\bar{u})}{\bar{u}} \right]_+ \\
&\quad \left. + \left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 \mu_{IR}^2 e^{2\gamma}/4\right) \right) \int_0^1 \mathrm{d}u \, 2 \left[\frac{(1 - \bar{u}u)^2}{\bar{u}} \right]_+ \right\} \mathcal{M}_{pp}(u\nu, 0)
\end{aligned} \tag{19}$$

The evolution kernel is:

$$B_{gg}(u) = 2 \left[\frac{(1 - \bar{u}u)^2}{\bar{u}} \right]_+ \tag{20}$$

Gluon-quark mixing

$$2p_0^2\mathcal{M}_{pp}(\nu, z_3^2)$$
$$\to \frac{g^2 C_F}{8\pi^2 z_3}\left(\frac{1}{\epsilon_{\text{IR}}} - \log\left(z_3^2 e^{\gamma_E}\right)\right)\frac{p^0}{p_3}\int_0^1 \mathrm{d}u\,(2\bar{u}+\delta(\bar{u}))\,\langle p|\,\mathcal{O}_q(uz_3)\,|p\rangle \tag{21}$$

with singlet combination:

$$\mathcal{O}_q(z_3) = \frac{i}{2}\sum_f \left(\bar{\psi}_f(0)\gamma^0\psi_f(z_3) - \bar{\psi}_f(z_3)\gamma^0\psi_f(0)\right) \tag{22}$$

Evolution kernel: $B_{gq}(u) = 2\bar{u} + \delta(\bar{u})$

Related to ITD through parametrization of the matrix element and oddness in z_3:

$$\frac{1}{z_3}\int_0^1 \mathrm{d}u B(u)\,\langle p|\,\mathcal{O}(uz_3)\,|p\rangle = p^0 p_3 \int_0^1 \mathrm{d}w \mathcal{B}_{gq}(w)\mathcal{I}(w\nu) \tag{23}$$

where

$$\mathcal{B}_{gq}(w) = \int_w^1 \mathrm{d}u B_{gq}(u) \implies \mathcal{B}_{gq}(w) = 1 + (1-w)^2 \tag{24}$$

Matching Relation

Relating reduced Ioffe-time pseudo-distribution to light-cone Ioffe time distribution

$$\begin{aligned}
&\mathfrak{M}(\nu, z_3^2)\mathcal{I}_g(0,\mu^2) \\
&= \mathcal{I}_g(\nu,\mu^2) - \frac{\alpha_s N_c}{2\pi}\int_0^1 \mathrm{d}u \mathcal{I}_g(u\nu,\mu^2)\left\{\ln\left(z_3^2\mu^2 e^{2\gamma_E}/4\right) B_{gg}(u)\right. \\
&\quad \left. +4\left[\frac{u+\log(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+\right\} \\
&\quad - \frac{\alpha_s C_F}{2\pi}\ln\left(z_3^2\mu^2 e^{2\gamma_E}/4\right)\int_0^1 \mathrm{d}w \mathcal{I}_S(w\nu,\mu^2)\mathcal{B}_{gq}(w)
\end{aligned} \tag{25}$$

$\mathcal{B}_{gq}(w)$ has been given the plus-prescription here: $1+(1-w)^2 \to \left[1+(1-w)^2\right]_+$

Can be directly related to light-cone PDFs using:

$$\mathcal{I}_g\left(\nu,\mu^2\right) = \frac{1}{2}\int_{-1}^1 \mathrm{d}x e^{ix\nu} x f_g\left(x,\mu^2\right), \quad \mathcal{I}_g\left(0,\mu^2\right) = \langle x\rangle_{\mu^2} \tag{26}$$

New kernel form:

$$\mathfrak{M}(\nu, z_3^2) = \int_0^1 \mathrm{d}x \frac{x f_g\left(x,\mu^2\right)}{\langle x\rangle_{\mu^2}} R_{gg}\left(x\nu, z_3^2\mu^2\right) + \int_0^1 \mathrm{d}x \frac{x f_S\left(x,\mu^2\right)}{\langle x\rangle_{\mu^2}} R_{gq}\left(x\nu, z_3^2\mu^2\right) \tag{27}$$

Need to independently calculate $\langle x\rangle_{\mu^2}$, and calculate or estimate singlet quark function $\mathcal{I}_S(w\nu,\mu^2)$.

Matching Relation

New kernels found by cosine transformation only, because sine part integrates to zero due to evenness of $xf_g(x,\mu^2)$:

$$R(y) = \int_0^1 \mathrm{d}u B(u)\cos(uy) \tag{28}$$

Gluon kernel given by:

$$R_{gg}\left(y, z_3^2\mu^2\right) = \cos y - \frac{\alpha_s N_c}{2\pi}\left\{\ln\left(z_3^2\mu^2 e^{2\gamma_E}/4\right) \underset{\text{Evolution}}{R_B(y)} + \underset{\text{log}}{R_L(y)} + \underset{\text{Constant}}{R_C(y)}\right\} \tag{29}$$

Mixing kernel given by:

$$R_{gq}\left(y, z_3^2\mu^2\right) = -\frac{\alpha_s N_c}{2\pi}\ln\left(z_3^2\mu^2 e^{2\gamma_E}/4\right) R_{\mathcal{B}}(y) \tag{30}$$

$$R_L(y) = 4\mathrm{Re}\left[iye^{iy}\,{}_3F_3(1,1,1;2,2,2;-iy)\right]$$

$$R_B(y) = -\frac{12}{y^4} + \frac{4}{y^2} + \cos(y)\left(2\mathrm{Ci}(y) + \frac{12}{y^4} - \frac{6}{y^2} + \frac{11}{6} - 2\gamma - 2\log(y)\right) + \sin(y)\left(2\mathrm{Si}(y) + \frac{8}{y^3} - \frac{4}{y}\right)$$

$$R_C(y) = -\frac{4}{y^4} + \cos(y)\left(4\mathrm{Ci}(y) + \frac{4}{y^4} - \frac{2}{y^2} + \frac{7}{2} - 4\gamma - 4\log(y)\right) + \sin(y)\left(4\mathrm{Si}(y) + \frac{4}{y^3} - \frac{4}{y}\right)$$

$$R_{\mathcal{B}}(y) = \frac{2}{y^2} - \frac{4\cos(y)}{3} - \sin(y)\left(\frac{2}{y^3} - \frac{1}{y}\right)$$

Conclusions and ongoing work

Important points:

- $R(y, z_3^2, \mu^2)$ kernels are given by explicit perturbatively calculable expressions
- Lattice data and LC PDFs directly relatable
- Taking some parametrization of $f_g(x, \mu^2)$ and $f_S(x, \mu^2)$ distributions, one can fit parameters and α_s from the lattice data for $\mathfrak{M}(\nu, z_3^2)$
- Essentially same procedure as that used in the "good lattice cross sections" approach [Ma, Qiu, 2018]

Ongoing work:

- Paper on gluon helicity pseudo-distribution coming soon
- Currently working on transverse quark pseudo-distribution
- Also working on gluon "condensate" calculation: $\langle p| G^{\mu\nu}(z) G_{\mu\nu}(0) |p\rangle$

Helicity distribution (preliminary)

The gluon helicity distribution can be obtained from projections of the matrix element:

$$m_{\mu\alpha;\nu\beta}\left(z,p\right) \equiv \langle p,s|\, G_{\mu\alpha}\left(z\right)[z,0]\tilde{G}_{\mu\beta}\left(0\right)|p,s\rangle \tag{31}$$

where the dual tensor is defined as $\tilde{G}_{\mu\beta} \equiv \frac{1}{2}\epsilon_{\mu\beta\rho\lambda}G^{\rho\lambda}$.

A promising combination is:

$$\begin{aligned}
&m_{0i;i0}(z,p) + m_{ji;ij}(z,p) \\
&\to \frac{g^2 N_c}{8\pi^2}\frac{16}{6}\left(\frac{1}{\epsilon_{\text{UV}}} + \log\left(z_3^2 e^{\gamma}\right)\right)\left(m_{0i;i0}(z,p) + m_{ji;ij}(z,p)\right) \\
&\quad + \frac{g^2 N_c}{8\pi^2}\int_0^1 \mathrm{d}u \left\{4\delta(\bar{u}) - 2\bar{u}u + 2\left(\frac{1}{\bar{u}} - \bar{u}\right)_+ - \left[\frac{4u}{\bar{u}} + \frac{4\log(1-u)}{\bar{u}}\right]_+\right. \\
&\quad \left. + \left(\frac{1}{\epsilon_{IR}} - \log\left(z_3^2 e^{\gamma}\right)\right)\left[\left\{4u\bar{u} + 2\left[u^2/\bar{u}\right]_+\right\} - \frac{1}{2}\left(\frac{\beta_0}{N_c} + 6\right)\delta(\bar{u})\right]\right\} \\
&\quad \times \left(m_{0i;i0}(uz,p) + m_{ji;ij}(uz,p)\right)
\end{aligned} \tag{32}$$

$m_{0i;i0}(z,p) + m_{ji;ij}(z,p) = 2p_3p_0\left[\mathcal{M}_{sp}^{(+)}\left(\nu,z^2\right) - \left(1 + m^2/p_3^2\right)\nu\mathcal{M}_{pp}\left(\nu,z^2\right)\right]$, which

approaches the LC amplitude, $\mathcal{M}_{sp}^{(+)}\left(\nu,z^2\right) - \nu\mathcal{M}_{pp}\left(\nu,z^2\right)$, as $p_3 \to \infty$.

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