Towards determining polarized gluon distribution from Lattice QCD

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(For the HadStruc Collaboration)

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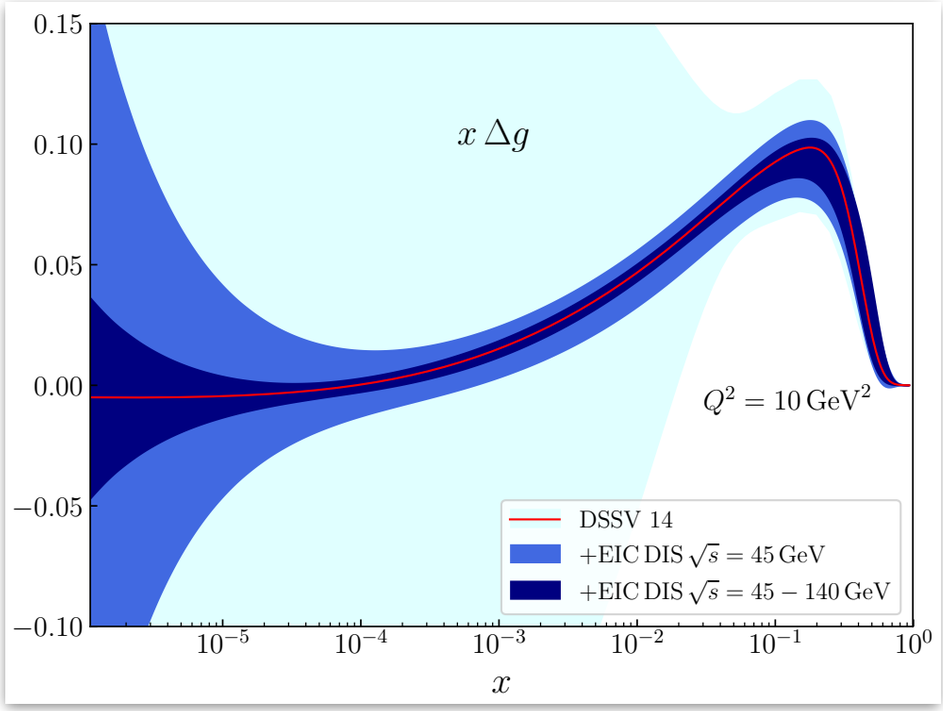
Gluon distributions and lattice QCD

- Origin of proton spin : A BIG question

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu) + \Delta G(\mu) + L_{Q+G}(\mu)$$

- Gluon contribution to proton spin is not well-constrained from experiment



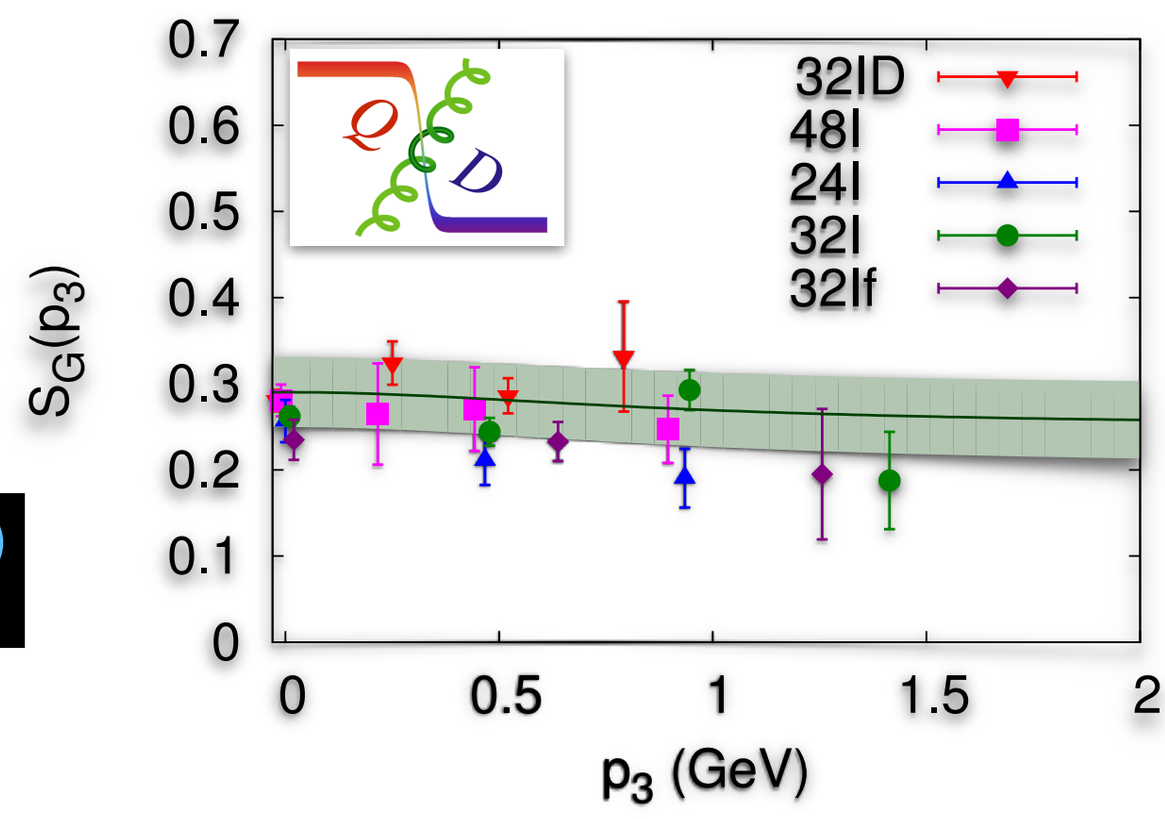
Impact of projected EIC data (EIC Yellow Report)

- Direct LQCD determination of gluon spin

Ji, Zhang, Zhao [PRL 2013]

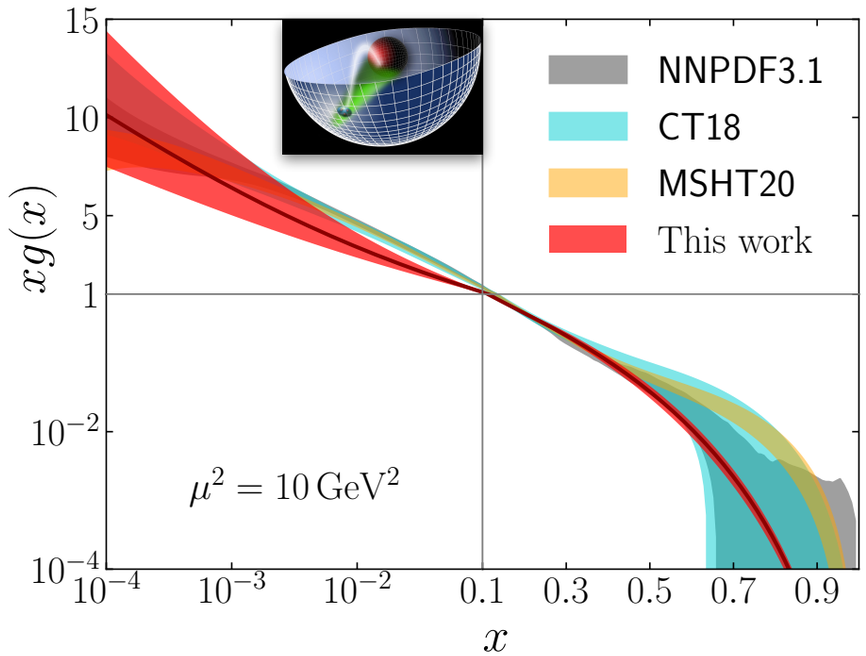
$$\Delta G(\mu^2 = 10\,\mathrm{GeV}^2) = 0.251(47)(16)$$

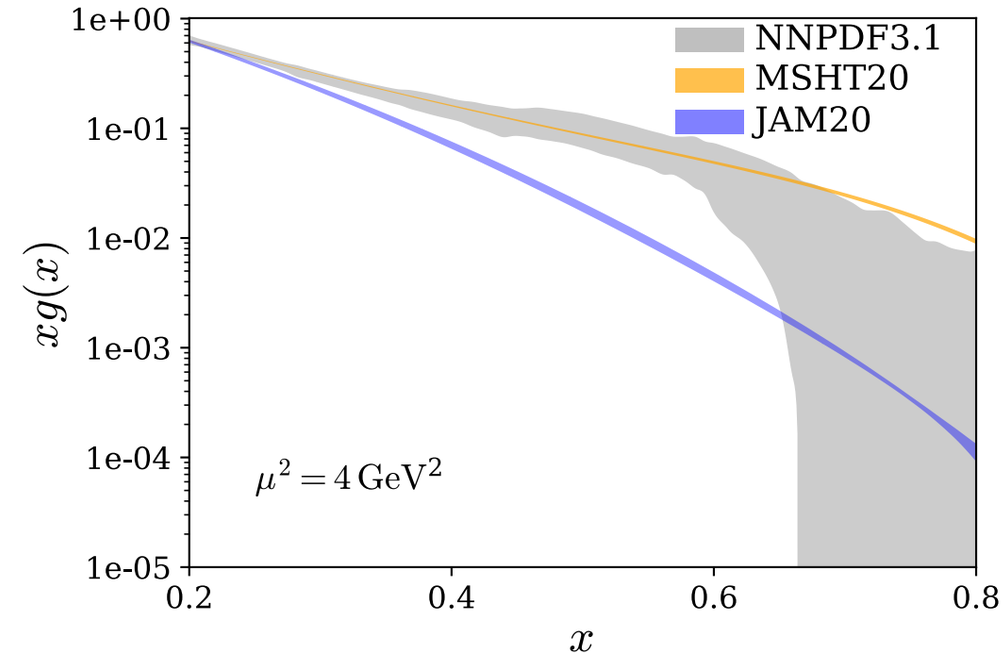
RSS, Glatzmaier, Yang, Liu, Sun *PoS* LATTICE(2014)
Yang, ***RSS***, et al (PRL 2017)



Gluon distributions and lattice QCD

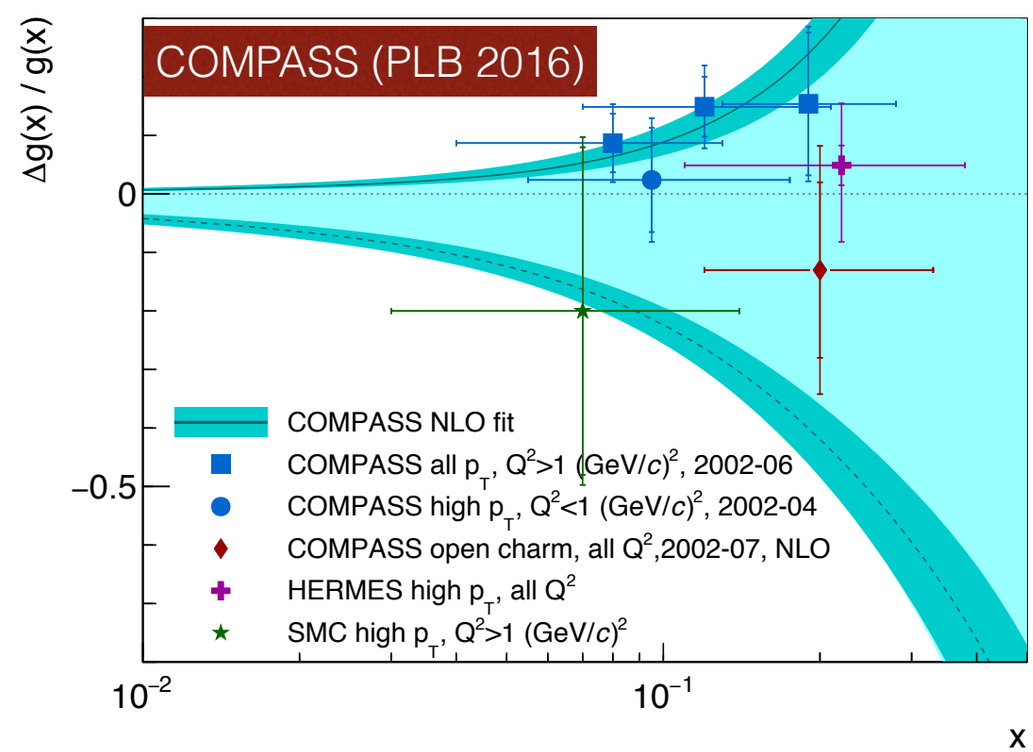
- Gluon PDF at large-x is phenomenologically interesting and there is tension between model calculations and global fits





de Teramond, Dosch, Liu, ***RSS***, Brodsky, Deur
Light-front AdS/QCD
arXiv: 2107.012231

- Small-x gluon distributions from experiment (e.g. EIC) and moderate to large-x PDF from LQCD can be complementary



x-dependent gluon distribution from lattice QCD

- On the lattice, calculate spatial correlation in coordinate space

$$\Delta\mathcal{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv < p,s|G_{\mu\alpha}(z)[z,0]\widetilde{G}_{\lambda\beta}(0)|p,s >$$

Quasi-PDFs/LaMET
Ji [PRL 2013]
[Sci. China Phys 2014]

  - Extra linear UV divergence, z/a $(a \to 0)$ from Wilson line
  - Multiplicative renormalizability (coordinate space)

Ishikawa, Ma, Qiu, Yoshida [PRD 2017]

- "Pseudo-PDF" approach: Based on QCD short-distance factorization

Radyushkin [PLB 2017]

- **Renormalization**: $\mathfrak{M}(\nu, z^2) = \dfrac{\mathcal{M}_{pp}(\nu, z^2)}{\mathcal{M}_{pp}(0, z^2)}$

Orginos, et al [PRD 2017]

Reduced Ioffe-time distribution

  - Ioffe time, $\nu = p_z z$ (convention from Braun, et al [PRD 1995])

Gluon distribution in Pseudo-PDF approach

- To determine unpolarized gluon Ioffe-time distribution

$$\mathcal{M}_{0i;i0}(\nu, z^2) + \mathcal{M}_{ji;ij}(\nu, z^2) = 2p_0^2 \mathcal{M}_{pp}(\nu, z^2)$$

Balitsky, Morris, Radyushkin [PLB 2020]

$$\mathcal{M}_{pp}(\nu, \mu^2) = \int_0^1 dx \; x\, g(x) \cos(x\nu)$$

- To determine **polarized** gluon Ioffe-time distribution

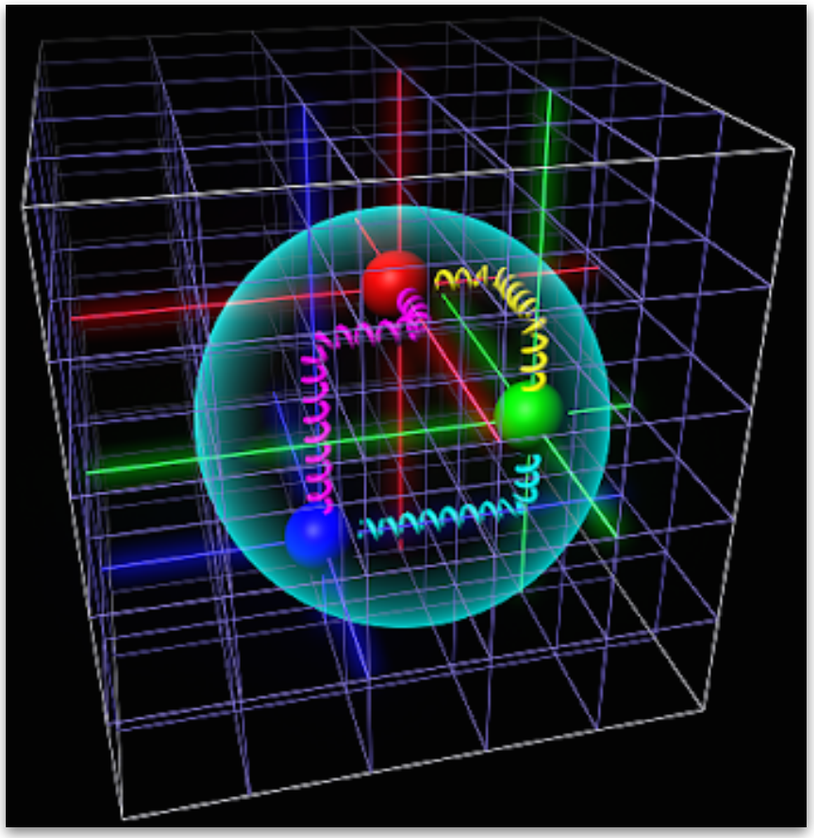
Lattice $$\Delta\mathcal{M}_{0i;0i} + \Delta\mathcal{M}_{ij;ij} = -2p_0 p_z \left[\Delta\mathcal{M}_{ps} - (1 + m_N^2/p_z^2)\nu\Delta\mathcal{M}_{pp}\right]$$

$$[\Delta\mathcal{M}_{ps} - \nu\,\Delta\mathcal{M}_{pp}](\nu, \mu^2) = \int_0^1 dx \; x\,\Delta g(x) \sin(x\nu)$$

- Large momentum required to suppress unwanted contribution

Lattice QCD calculation

- 2+1 flavor clover Wilson fermions
- Lattice size, $L \times T = 32^3 \times 64$
- Lattice spacing, $a \approx 0.094$ fm
- Pion mass, $m_\pi = 358$ MeV
- 349 configurations
- $z = [0, 0.56]$ fm



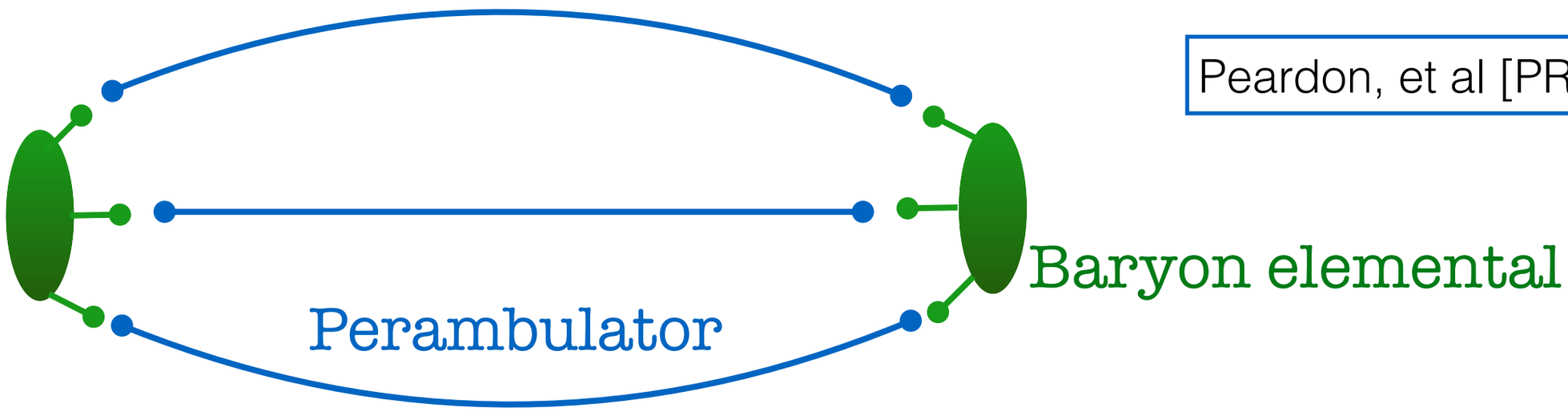
Some features of this calculation

- Gluonic operator using "Gradient flow"
  - Flow of gauge field, $B_\mu(\tau, x_\mu)$ so that $B_\mu|_{\tau=0} = A_\mu$
  - Diffusion length in x is $\sqrt{8\tau}$ $\quad (\tau \sim a^2)$

M. Luscher, JHEP 2010

- Nucleon correlation function using "Distillation"

Peardon, et al [PRD 2009]



  - Basis of operators (positive/negative parity, hybrid, higher spin)
  - Optimized operators reduce excited-state contaminations
  - Perambulators are independent of baryon elementals

Some features of this calculation

- Momentum smearing for enhanced overlap of the nucleon interpolating operators onto the lowest lying states of the boosted hadron

Bali, et al(PRD 2016)

Distillation at high momentum (Egerer, et al PRD 2021)

- Correlation matrix analysis using variational technique (summed generalized eigenvalue problem, sGEVP)

Bulava, et al(JHEP 2012)

  - System of generalized eigenvalue equations for correlation matrix

$$C(t)v^n(t) = \lambda_n(t)C(t_0)v^n(t)$$

  - Orthogonality conditions on the eigenvectors of different states

$$v^{n'\dagger}C(t_0)v^n = \delta_{n,n'}$$

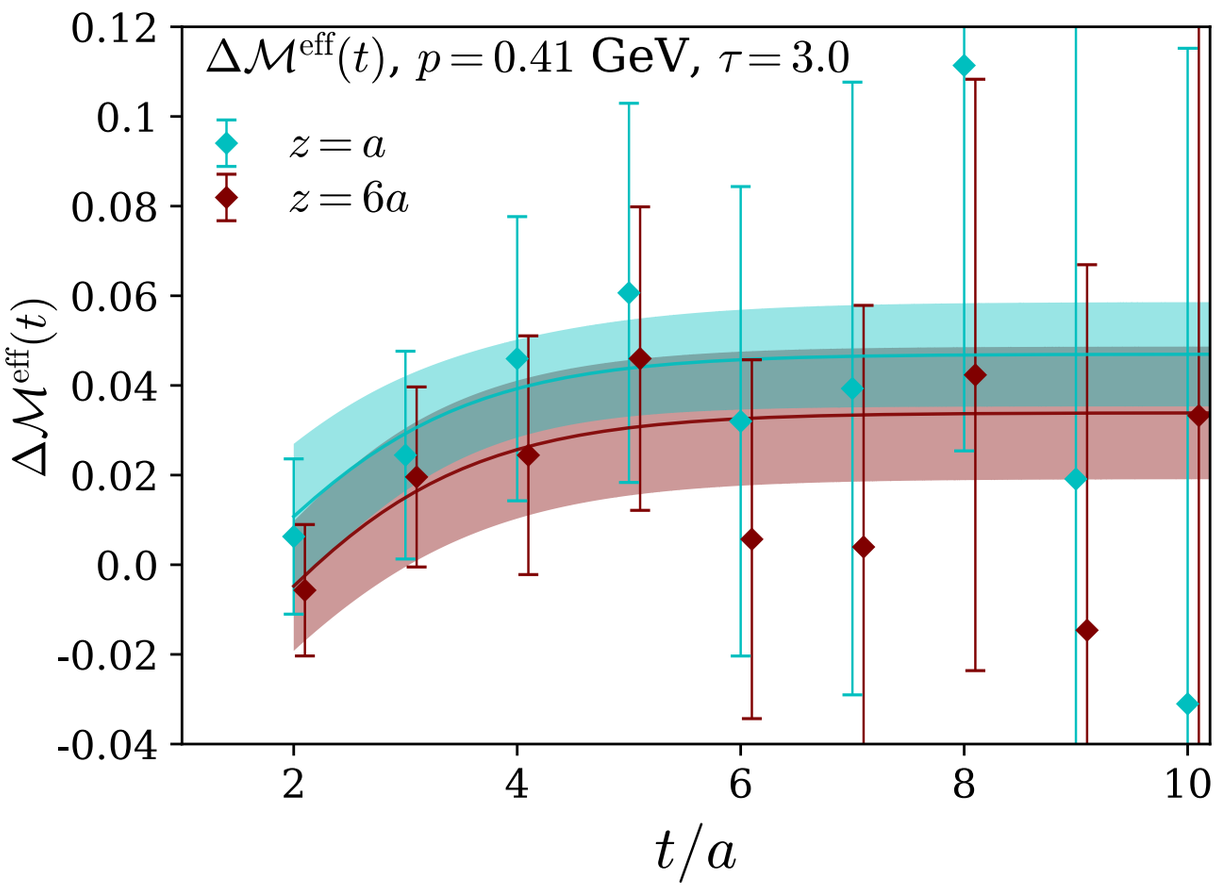
Difficult to distinguish degenerate states by their time-dependence alone

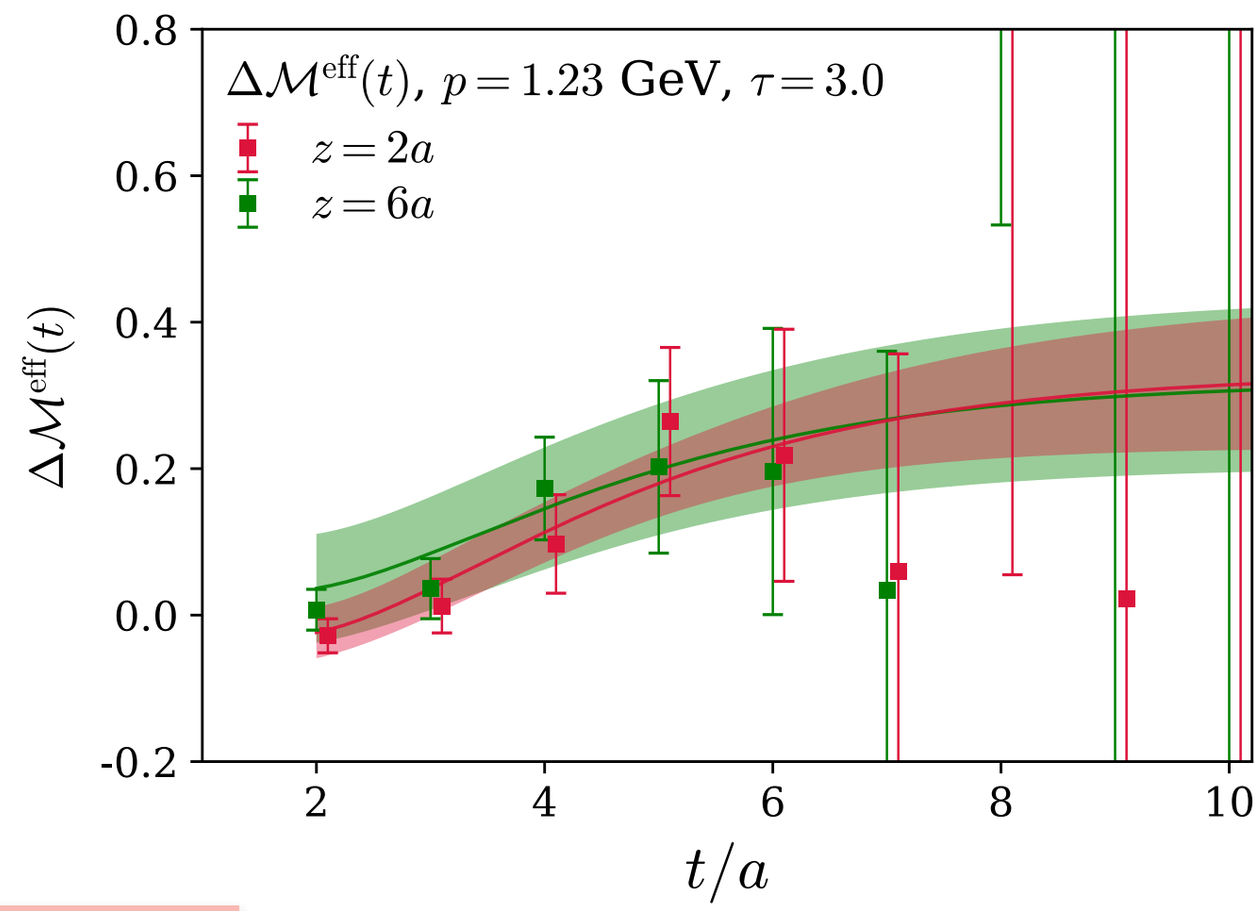
Lattice QCD matrix elements

- Simultaneous correlated fit to matrix elements for all z (fixed momentum & gradient flow)

$$\Delta\mathcal{M}^{\text{eff}}(t)_i = A_i + B_i\, t \exp(-\Delta E\, t)$$

Bulava, et al(JHEP 2012)



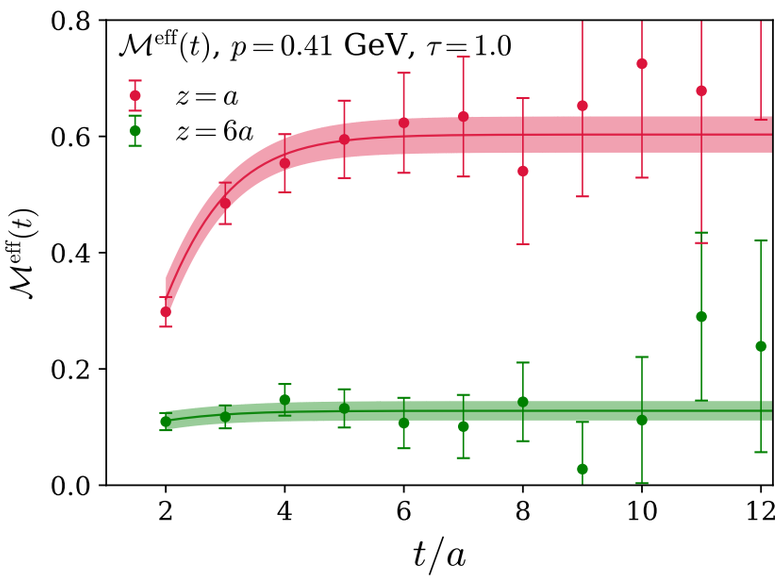


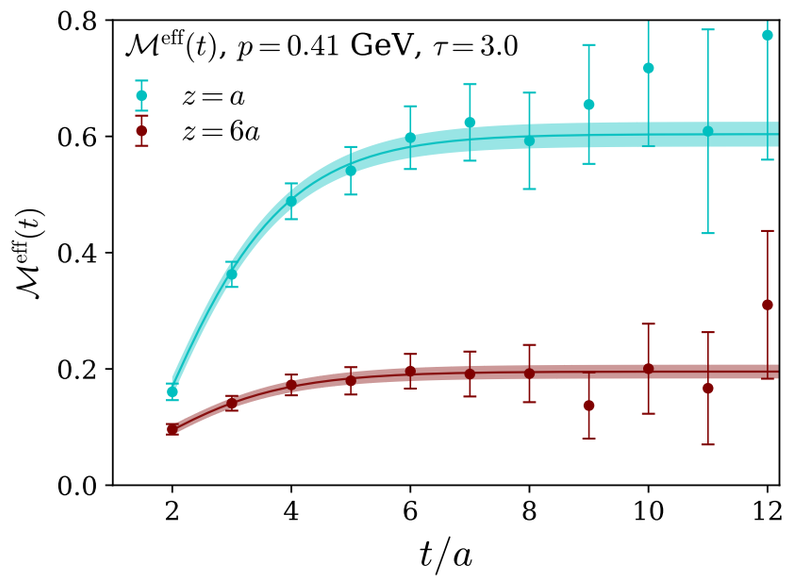
$$\tau = 3.0$$

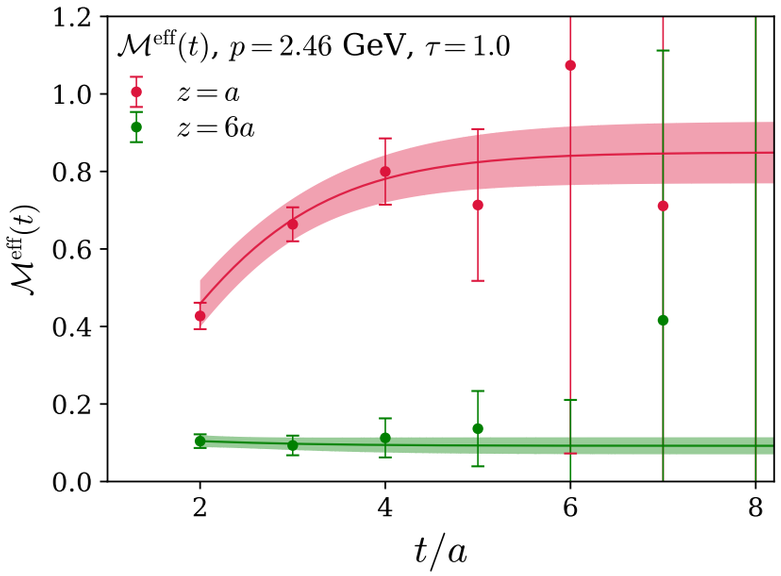
- Signal-to-noise ratio much smaller compared to unpolarized gluon distribution matrix elements

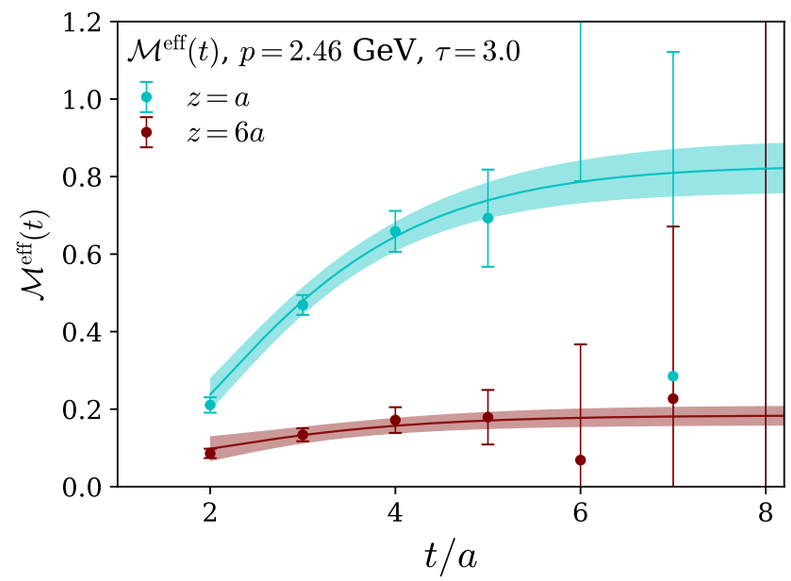
Lattice QCD matrix elements

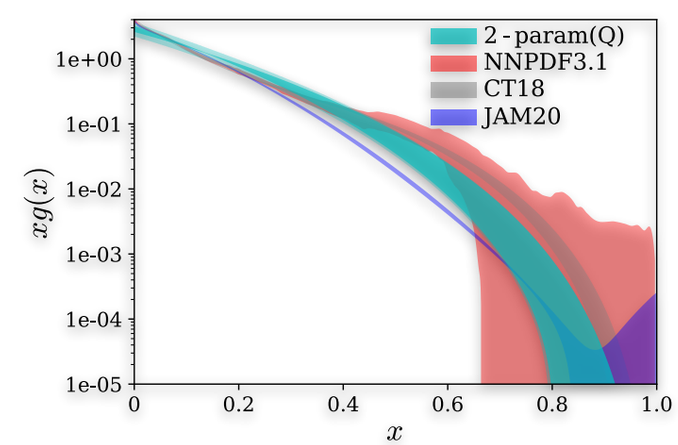
- Compare with unpolarized gluon matrix elements







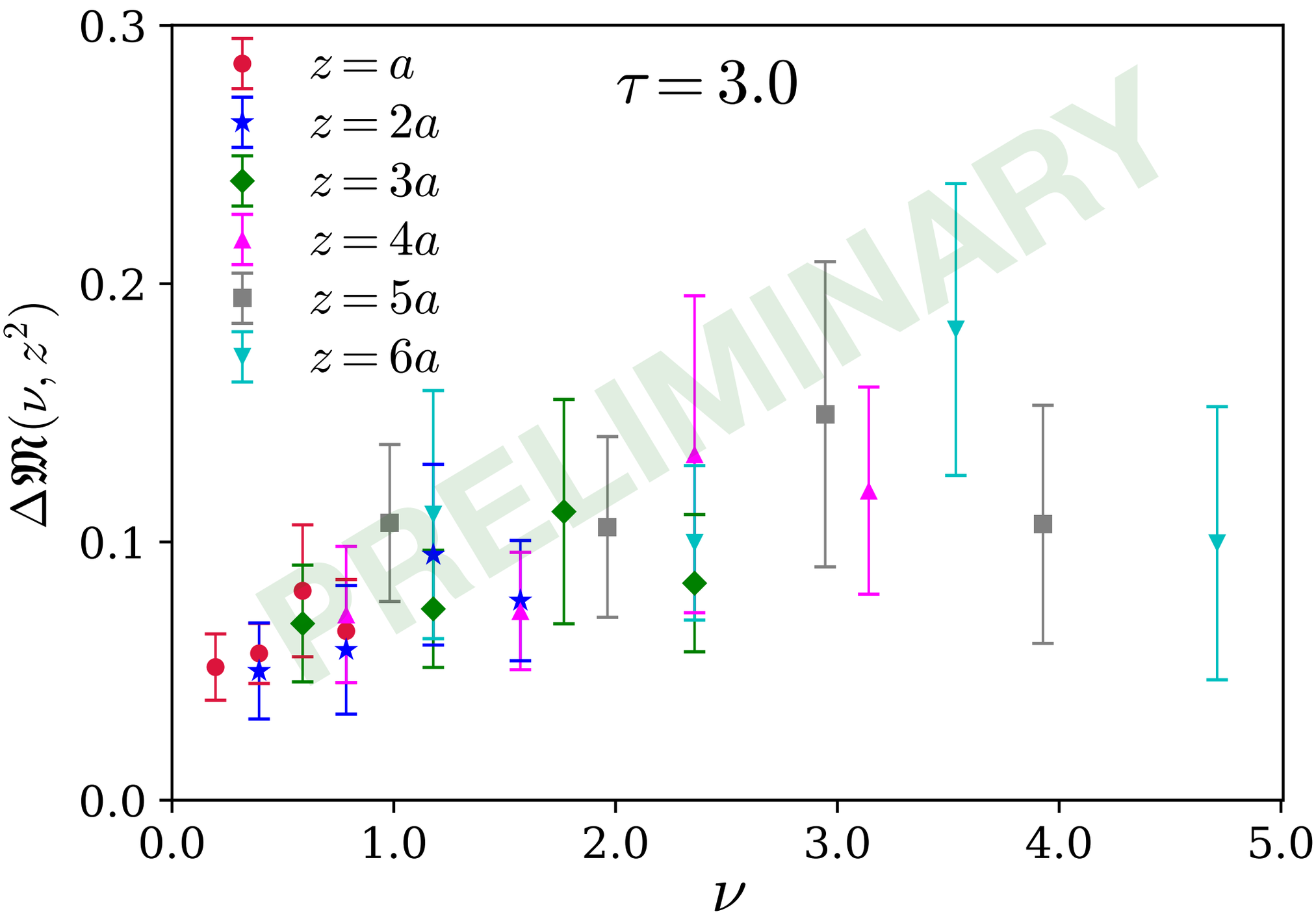




Khan, ***RSS***, Karpie, Monahan, et al
(HadStruc Collaboration)
arXiv: 2107.08960

More in Tanjib Khan's talk
Jul 27, 2021, 9:00 PM

Reduced Ioffe-time distribution of polarized gluon distribution

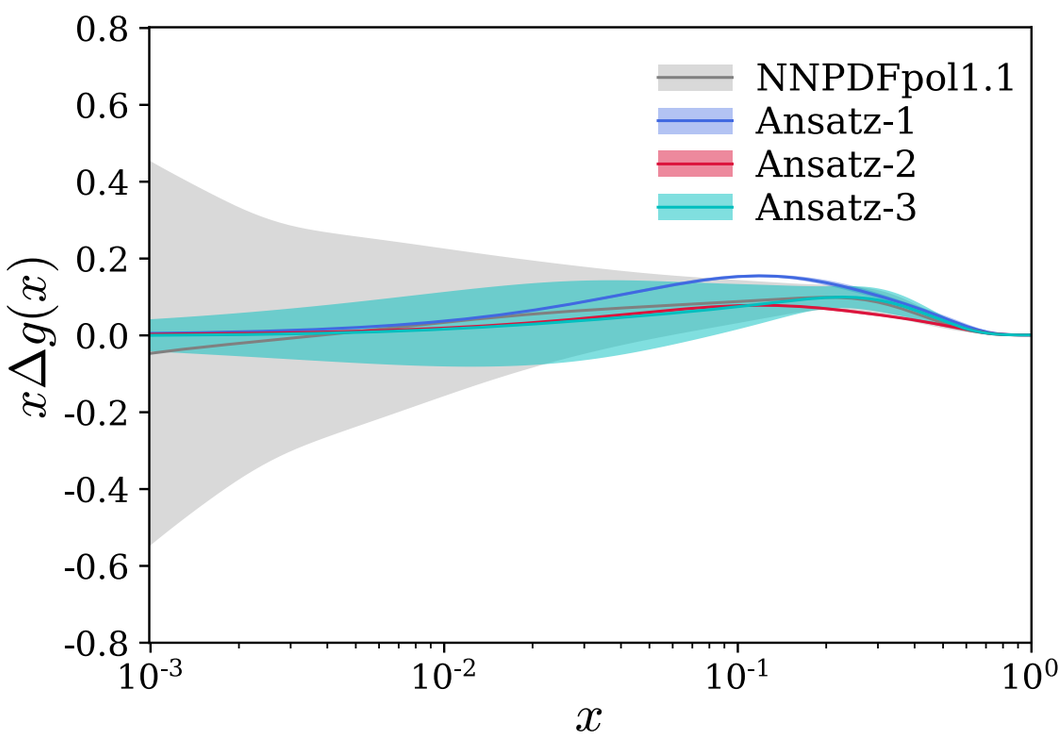


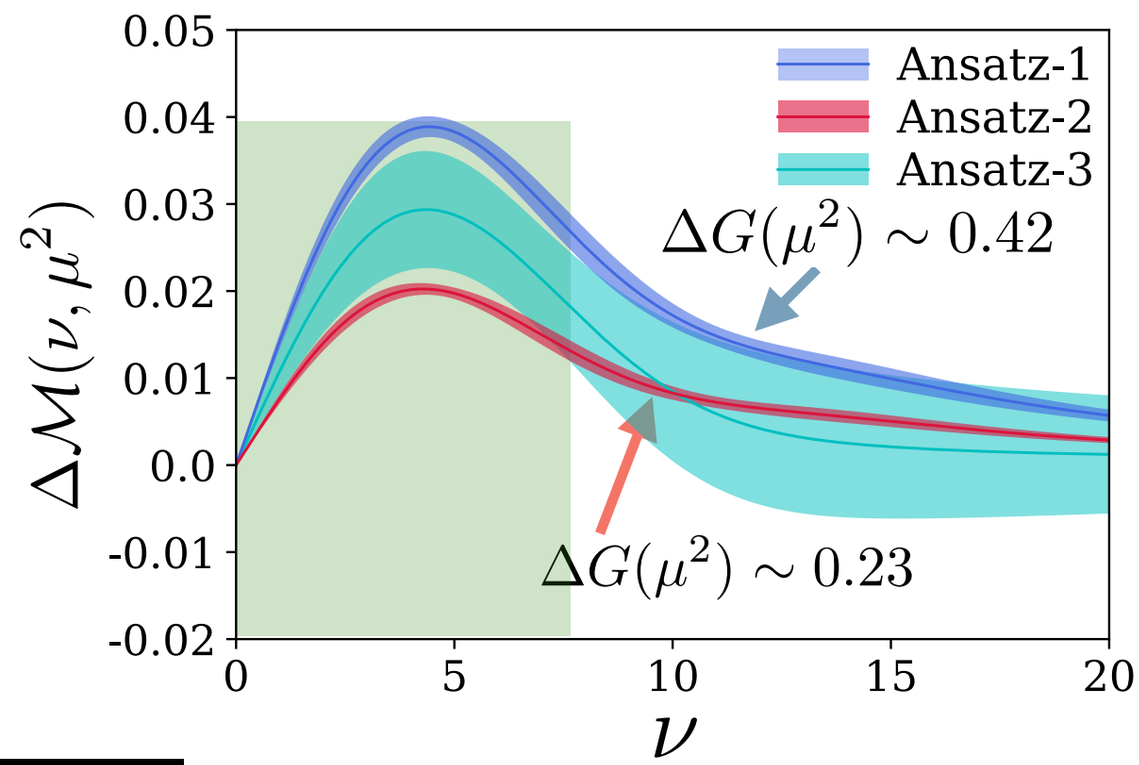
- Caution: More careful study required for the ratio (reduced ITD)

Prospect of Lattice QCD on gluon helicity distribution

- Gluon helicity from light cone Ioff-time distribution

$$\Delta G(\mu^2) = \int_0^\infty d\nu \; \Delta\mathcal{M}_{\text{light-cone}}(\nu, \mu^2)$$





RSS, Liu, Paul
PRD 2021

- LQCD determination of polarized gluon ITD, even at small Ioffe-time window can have important impact

Summary & Outlook

- First LQCD attempt to determine gluon helicity distribution
- In progress: increase statistics by 4 times
- Resolve theoretical issues of calculating reduced ITD
- Polarized gluon distribution from LQCD

  challenging but very important

The Big Picture

Contribute to 3D imaging of the nucleon from LQCD

Thank you!