Isovector Axial Vector Form Factors of the Nucleon from Lattice QCD with $N_f = 2 + 1$ $\mathcal{O}(a)$-improved Wilson Fermions

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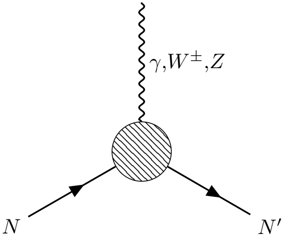
- **Motivation**: Size of the nucleon as seen from electroweak (EW) current interactions

$$\mathcal{L}_{\text{int}}^{\text{EW}} = eJ^{\mu}_{\text{em}}(x)A_{\mu}(x) + \frac{g}{2\sqrt{2}}\left(J^{\mu}_{\text{CC}}(x)W^{+}_{\mu}(x) + \text{h.c.}\right) + \frac{g}{4\cos\theta_W}J^{\mu}_{\text{NC}}(x)Z_{\mu}(x)$$



- Inelastic (elastic) νN scattering mediated by the charged (neutral) weak current, ordinary and radiative muon capture

$$\nu_\mu + n \to \mu^- + p, \quad \bar{\nu}_\mu + p \to \mu^+ + n, \quad \nu + N \to \nu + N$$

$$\mu^- + p \to \nu_\mu + n, \quad \mu^- + p \to \nu_\mu + n + \gamma$$

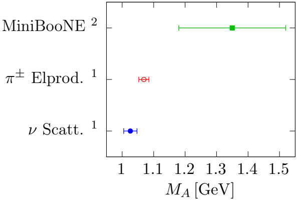
- Isovector axial vector current matrix element parametrization ($Q^2 = -(p' - p)^2 \geq 0$)

$$\langle N(\vec{p}', s')|A^a_\mu(0)|N(\vec{p}, s)\rangle = \bar{u}(\vec{p}', s')\left[\gamma_\mu G_A(q^2) - i\frac{Q_\mu}{2m_N}G_P(q^2)\right]\gamma_5 T^a u(\vec{p}, s)$$

in terms of the **axial** and the **induced pseudoscalar** form factor

- **Dipole parametrization** and **axial radius**

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad \langle r_A^2\rangle = \frac{12}{M_A^2}$$



[1] Bernard et al. J. Phys. G: Nucl. Part. Phys. 28 R1, 2001

[2] Aguilar-Arevalo et al. (MiniBooNE Collaboration) Phys. Rev. D 81, 092005, 2010

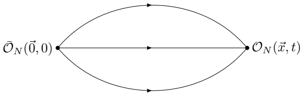
- Interpolating operator coupling to the nucleon, $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

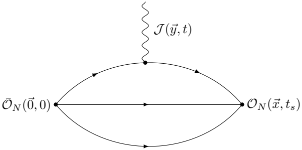
$$\mathcal{O}_N(x) = \epsilon_{abc} u(x)^a \left(u^T(x)^b C\gamma_5 d(x)^c \right), \quad \langle 0|\mathcal{O}_N(\vec{x},0)|N(\vec{p},s)\rangle = e^{i\vec{p}\cdot\vec{x}} Z(\vec{p}) u(\vec{p},s)$$

- Lattice **nucleon 2-point** and **3-point correlation functions** (with $\Gamma = (1+\gamma_0)(1+i\gamma_5\gamma_3)/2$)

$$C_2^N(\Gamma;\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\, \Gamma_{\alpha\beta} \langle \mathcal{O}_N(\vec{x},t)_\beta \bar{\mathcal{O}}_N(\vec{0},0)_\alpha \rangle$$

$$C_{3,\mathcal{J}}^N(\Gamma;\vec{p}\,',\vec{p},t_s,t) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\,'\cdot(\vec{x}-\vec{y}) - i\vec{p}\cdot\vec{y}} \Gamma_{\alpha\beta} \langle \mathcal{O}_N(\vec{x},t_s)_\beta \mathcal{J}(\vec{y},t) \bar{\mathcal{O}}_N(\vec{0},0)_\alpha \rangle$$





- Inversions using the fixed-sink method for the sequential propagator of C_3^N, $\vec{p}\,' = 0 \rightarrow \vec{q} = -\vec{p}$
- Exact isospin symmetry: Disconnected contributions cancelled for $C_{3,u-d}^N$ (isovector channel)

- Non-perturbative operator improvement and renormalization

$$(A_\mu^a)_R^I(x) = Z_A(\tilde{g}_0^2)(1 + b_A(g_0^2)am_q)(A_\mu^a(x) + ac_A(g_0^2)\tilde{\partial}_\mu P^a(x))$$

- **Ratio of correlation functions** [1]

$$R_{A_\mu}(\Gamma;\vec{q},t_s,t) \ = \ \frac{C^N_{3,A_\mu}(\Gamma;\vec{0},-\vec{q},t_s,t)}{C_2^N(\Gamma;\vec{0},t_s)}\sqrt{\frac{C_2^N(\Gamma;-\vec{q},t_s-t)C_2^N(\Gamma;\vec{0},t)C_2^N(\Gamma;-\vec{q}',t_s)}{C_2^N(\Gamma;\vec{0},t_s-t)C_2^N(\Gamma;-\vec{q},t)C_2^N(\Gamma;-\vec{q},t_s)}}$$

$$\xrightarrow{t,t_s\gg 1} R_{A_\mu}(\Gamma;\vec{q}) + \ldots$$

- Solve overdetermined system of equations for spatial ratio components R_{A_j} by linear regression

$$\chi^2 = (\vec{R} - M\vec{G})^T \mathrm{Cov}^{-1}(\vec{R} - M\vec{G}), \quad \vec{G} = (G_A, G_P)^T$$

  to obtain **effective form factors** $G^{\mathrm{eff}}_{A,P}(Q^2, t_s, t)$

- **Summation Method** [2]

$$S_{G_A}(Q^2, t_s) = \sum_{t=1}^{t_s-1} G_A^{\mathrm{eff}}(Q^2, t_s, t) \to c + t_s G_A(Q^2) + \ldots$$

- **Two-State Fits** (including priors for energy gaps $\Delta_1(\vec{0})$, $\Delta_1(\vec{q})$ and overlaps $\tilde{a}_1(\vec{0})$, $\tilde{a}_1(\vec{q})$)

$$G_A^{\mathrm{eff}}(Q^2, t_s, t) = G_A(Q^2)\Big[1 + k_{01}(\vec{q})e^{-\Delta_1(\vec{q})t} + k_{10}(\vec{q})e^{-\Delta_1(\vec{0})(t_s-t)} + k_{11}(\vec{q})e^{-\Delta_1(\vec{0})(t_s-t)-\Delta_1(\vec{q})t} +$$
$$+ \frac{1}{2}\tilde{a}_1(\vec{q})\left(e^{-\Delta_1(\vec{q})(t_s-t)} - e^{-\Delta_1(\vec{q})t_s}\right) + \frac{1}{2}\tilde{a}_1(\vec{0})\left(e^{-\Delta_1(\vec{0})t} - e^{-\Delta_1(\vec{0})t_s}\right)\Big] + \ldots$$

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[1] C. Alexandrou et al. Phys. Rev. D 74 (2006)

[2] L. Maiani et al. Nuclear Physics B 293 (1987)

Gauge Ensembles

Ensemble	β	L/a	T/a	M_π [MeV]	$M_\pi L$	m_N [MeV]	t_s [fm]
H105	3.40	32	96	287	3.89	1020	1.0, 1.2, 1.4
C101	3.40	48	96	223	4.68	984	1.0, 1.2, 1.4
S400	3.46	32	128	350	4.33	1123	1.1, 1.2, 1.4, 1.5, 1.7
N451*	3.46	48	128	280	5.20	1049	1.2, 1.4, 1.5
D450*	3.46	64	128	216	5.36	966	1.1, 1.2, 1.4, 1.5
S201	3.55	32	128	293	3.05	1098	1.0, 1.2, 1.3, 1.4
N203	3.55	48	128	347	5.42	1105	1.0, 1.2, 1.3, 1.4, 1.5
N200	3.55	48	128	283	4.42	1053	1.0, 1.2, 1.3, 1.4
D200	3.55	64	128	203	4.23	960	1.0, 1.2, 1.3, 1.4
E250*	3.55	96	192	130	4.06	938	1.0, 1.2, 1.3, 1.4
N302	3.70	48	128	353	4.28	1117	1.0, 1.1, 1.2, 1.3, 1.4
J303	3.70	64	192	262	4.23	1052	1.0, 1.1, 1.2, 1.3

- Ensembles created by the Coordinated Lattice Simulations (**CLS**) initiative [3] [4]
- $N_f = 2 + 1$-flavor **Wilson-Clover fermions**, tree-level improved **Lüscher-Weisz gauge action**
- Truncated Solver Method
- Open [5] (periodic *) boundary conditions in the time direction
- Increased number of sources for increased t_s on periodic ensembles
- **Gaussian smeared** quark fields ($r_G \sim 0.5$ fm), **APE-smeared** link variables

[3] M. Bruno et al. Journal of High Energy Physics 2015.2 (2015)

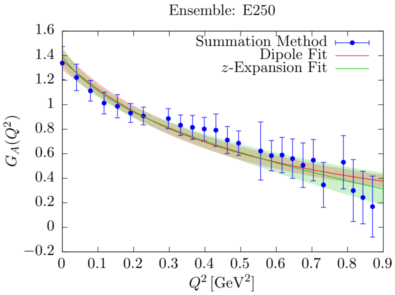
[4] M. Bruno et al. Phys. Rev. D 95, 074504 (2017)

[5] M. Lüscher and S. Schaefer. Comp. Phys. Comm. 184.3 (2013)

- Perform (model-independent) z-Expansion parametrization [6] of the Q^2-dependence ($t_0 = 0$)

$$G_A(Q^2) = \sum_{k=0}^{N} a_k z^k(Q^2), \quad z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}, \quad t_{\text{cut}} = (3M_\pi)^2$$

- Use Gaussian priors to restrict higher order parameters $a_{k>1}$ from reaching unphysically large values



[6] R. J. Hill and G. Paz. Phys. Rev. D 82 (11 2010)

Chiral and Continuum Extrapolation

- Extrapolation to the continuum limit [7]

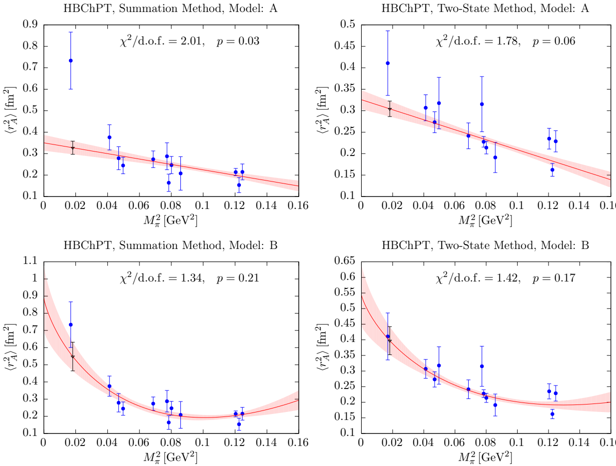
$$M_\pi \to M_\pi^{\text{phys}} = 134.8(3)\,\text{MeV}, \quad a \to 0, \quad L \to \infty$$

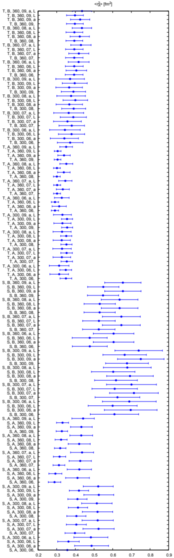
- **HBChPT**-inspired fit models (including/excluding chiral logarithm) given by

$$\text{Fit A} : \langle r_A^2 \rangle (M_\pi, a, L) = A + BM_\pi^2 + Da^2 + EM_\pi^2 e^{-M_\pi L}$$
$$\text{Fit B} : \langle r_A^2 \rangle (M_\pi, a, L) = A + BM_\pi^2 + CM_\pi^2 \log(M_\pi) + Da^2 + EM_\pi^2 e^{-M_\pi L}$$

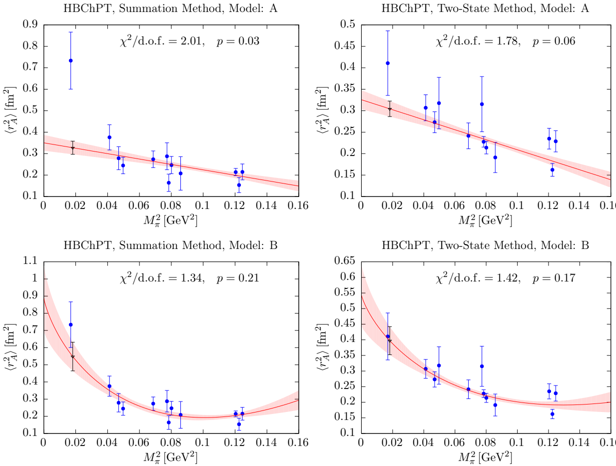
- **Variations:**
  - Fit Models A, B
  - Neglect cutoff and finite size effects $D = 0$, $E = 0$
  - Pionmass Cuts $M_\pi^{\text{cut}} \in \{300, 360\}$ MeV
  - Momentum Cuts (for the z-Expansion fits) $Q^2_{\text{cut}} \in \{0.6, 0.7, 0.8, 0.9\}$ GeV^2

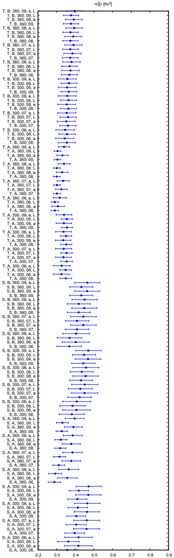
[7] S. Aoki et al., Eur. Phys. J. C 77, 112 (2017)





- **Observations:**
  - large cutoff effects (for the summation method): use prior for a^2-term coefficient (restrict to 10%)
  - Cancellations between BM_π^2 and $CM_\pi^2 \log(M_\pi)$ terms: use prior for the coefficient C





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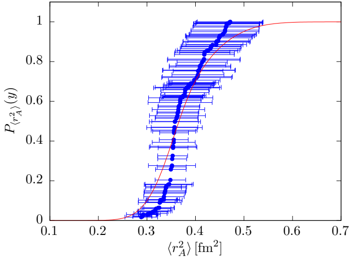
- Perform a **model average** ($\{$Summation, Two-State, A, B, Q^2_{cut}, $\mathcal{O}(a^2)$, $\ldots\}$) based on the **Akaike information criterion** (following [8])
- Define (normalized) AIC weights for the i-th model estimate given by

$$\text{AIC} := \chi^2_{\text{min}} + 2n_F + 2n_C, \quad w_i^{\text{AIC}} = \frac{e^{-\text{AIC}_i/2}}{\sum_k e^{-\text{AIC}_k/2}}$$

n_F : number of fit parameters, n_C : number of cut data points

- **Cumulative distribution function (CDF)**

$$P_{\langle r_A^2\rangle}(y) = \sum_i w_i^{\text{AIC}} F_{\mathcal{N}}(y; x_i, \sigma_i), \quad \sigma^2_{\text{tot}} = ((y_{84} - y_{16})/2)^2 = \sigma^2_{\text{stat}} + \sigma^2_{\text{sys}}, \quad \text{s.t.: } P_{\langle r_A^2\rangle}(y_{16}) = 0.16$$



FINAL RESULT: $\langle r_A^2\rangle = 0.366(46)_{\text{stat}}(42)_{\text{sys}}\ \text{fm}^2$

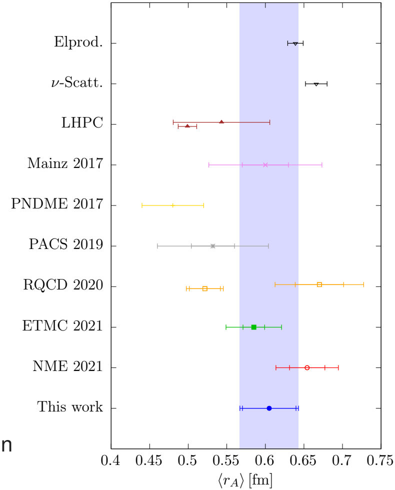
[8] Borsanyi, et al. Nature 593, 51–55 (2021).

Summary

- Explicit inclusion of excited state contributions
- Complete chiral, continuum and finite size extrapolations
- Successful extraction of $\langle r_A^2 \rangle$ with 13% (11%) statistical (systematic) error

Outlook

- Increase number of configurations for E250
- Compare with Covariant Baryon ChPT Extrapolation
- Analyze PCAC relation, extract g_P



$$\langle r_A^2 \rangle^{1/2} = 0.605(38)_{\text{stat}}(35)_{\text{sys}}\ \text{fm}$$