Nucleon Form Factors From N_f=2+1+1 Twisted Mass QCD at the Physical Point

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Nucleon form factors

Nucleon EM and Axial form factors from twisted mass fermions at physical point

- Lattice setup
  - Correlation functions (connected & disconnected)
  - Ensembles used and statistics
  - Excited state analysis
- Electromagnetic form factors
  - Proton and neutron, incl. disconnected contributions
  - Strange EM form factors
- Axial form factors
  - Isovector, plus combinations requiring disconnected

Extended Twisted Mass Collaboration

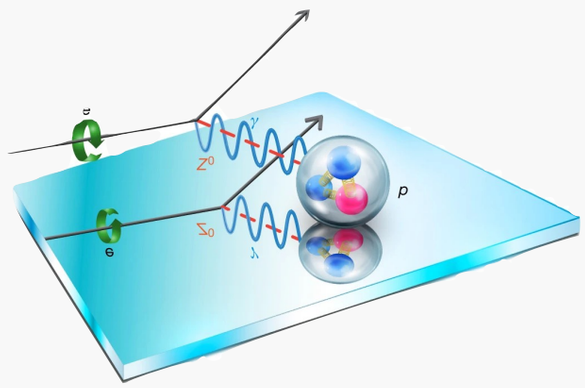
Motivation

- Proton and neutron EM form factors

  ➜ *radii and magnetic moments*
- Strange EM form factors

  ➜ *strangeness, connect e.g. to QWeak*
- Axial form factors

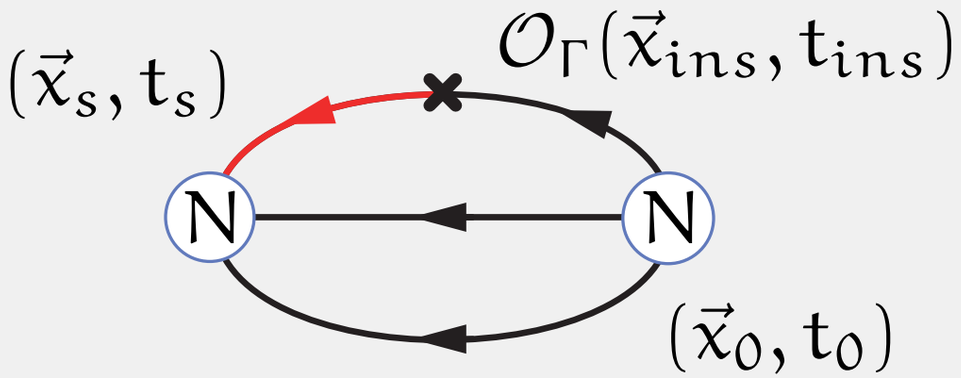
  ➜ *weak interactions, check of GT relations, pseudo-scalar couplings*



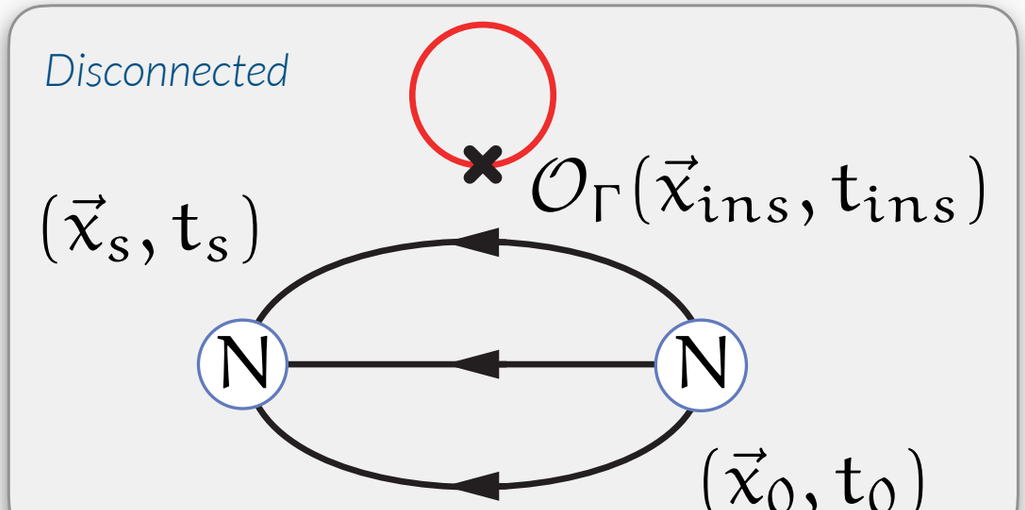
Evaluation of matrix elements

$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}}\, P^{\alpha\beta}\, \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s)|\mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins})|\chi_N^\alpha(\vec{0};0)\rangle$$

Connected



Disconnected



Sequential source through sink

- Inversions for each of the 4 projections

$$P: \begin{cases} P_0 = \frac{1+\gamma_0}{4} \\ P_k = i\gamma_5\gamma_k P_0,\ k = 1,\ 2,\ 3 \end{cases}$$

- Additional inversions for each t_s

Disconnected

- Two-point functions from connected three-point + additional
- Combination of deflation, hierarchical probing, and dilution for fermion loop (details follow)

Ensembles

N_f	Ens. ID	Vol.	a [fm]
2	cA2.090.48	48×96	0.093
2	cA2.090.64	64×128	0.093
2+1+1	cB211.072.64	64×128	0.081
2+1+1	cC211.060.80	80×160	0.070

- N_f=2 results reported previously; here for comparison
- Larger volume N_f=2 for assessing volume effects (only connected)
- Results from a=0.07 fm N_f=2+1+1 are preliminary

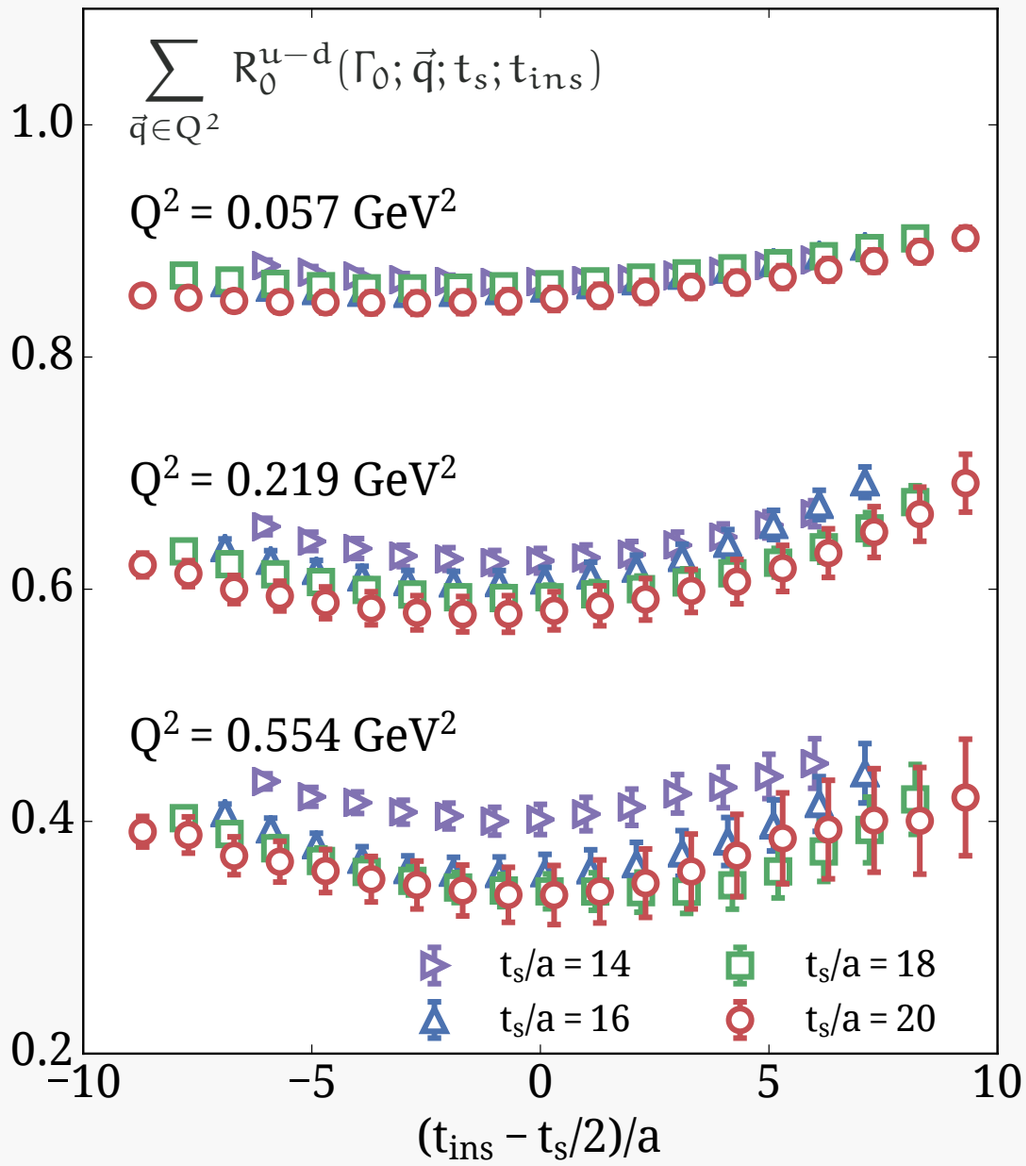
More on ETMC physical point simulations by J. Finkenrath, Friday @ 05:00 EDT

Statistics

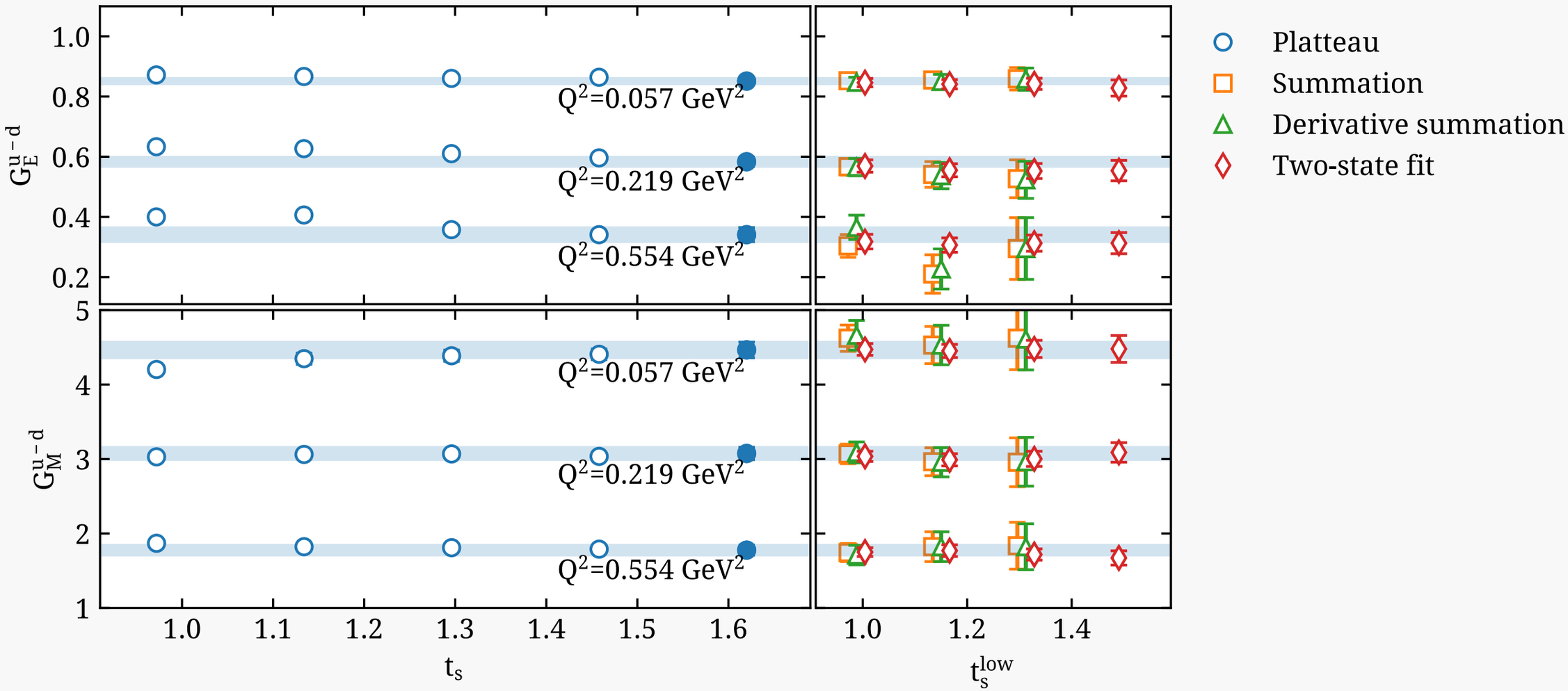
$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)} \sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

Increased N_{src} to achieve approximately constant errors with t_s

t_s/a	$N_{src} \times N_{conf}$ cB211.072.64
12	4×750
14	6×750
16	16×750
18	48×750
20	64×750
2-point	264×750

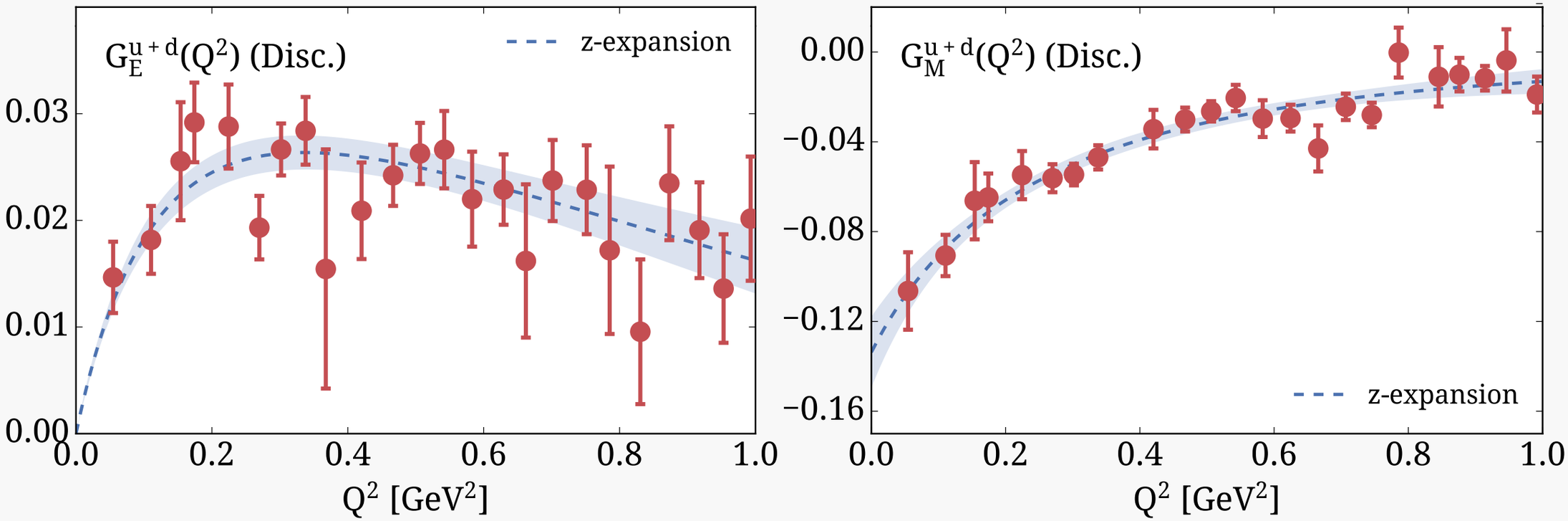


Treatment of excited states



- **Plateau**: fit to constant w.r.t. t_{ins} for multiple values of t_s
- **Summation**: sum ratio over t_{ins}, linear fit w.r.t t_s
- **Derivative summation**: fit constant to finite-difference of summed ratio
- **Two-state fit**: fit including 1st excited state

Disconnected EM form factors

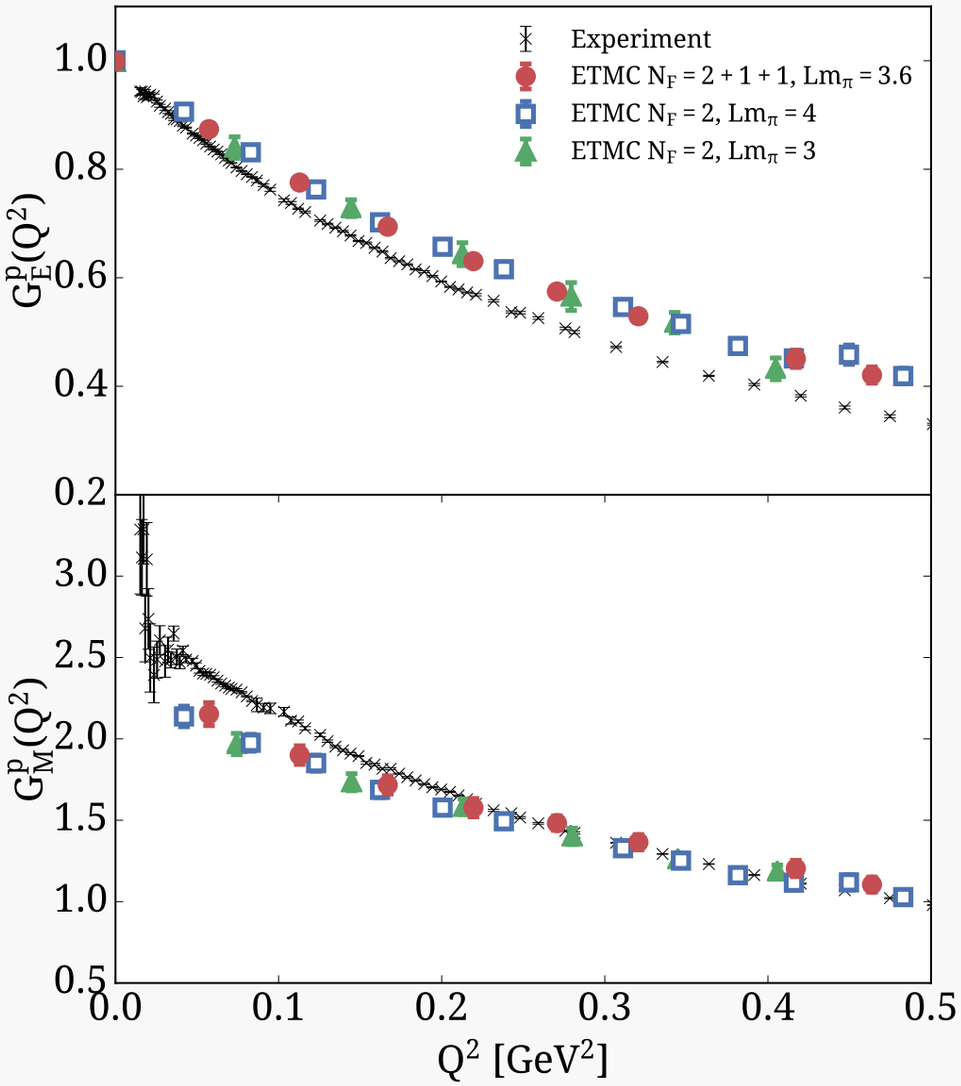


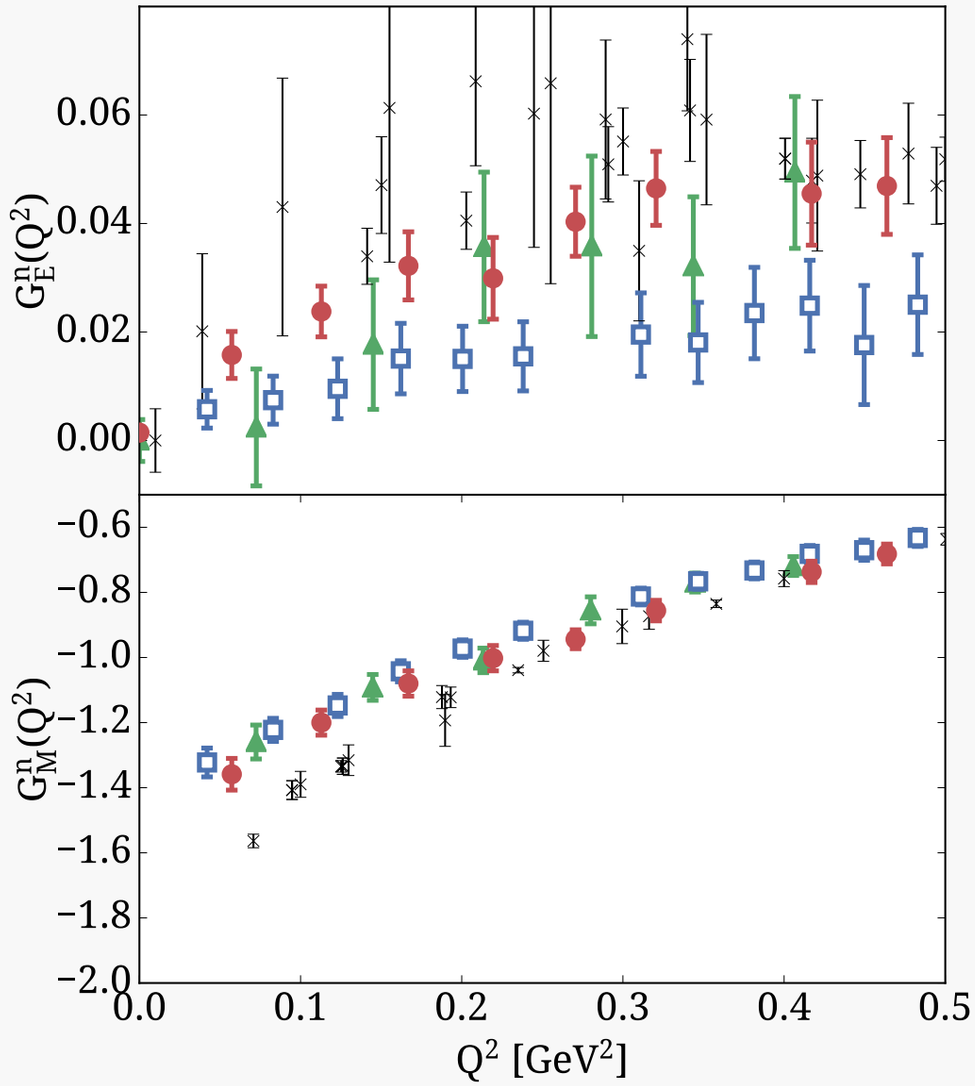
Disconnected isovector EM form factors ($N_f=2+1+1,\ a=0.081\ \mathrm{fm}$)

- High accuracy determination at physical point:
  - Color-spin dilution
  - Hierarchical probing (2^3-distance) → 512 Hadamard vectors
  - High-statistics two-point functions
- For light quarks combine deflation of 200 lowest EVs

Phys. Rev. **D101** (2020) 031501, arXiv:1909.10744 [hep-lat]

Proton and Neutron EM form factors

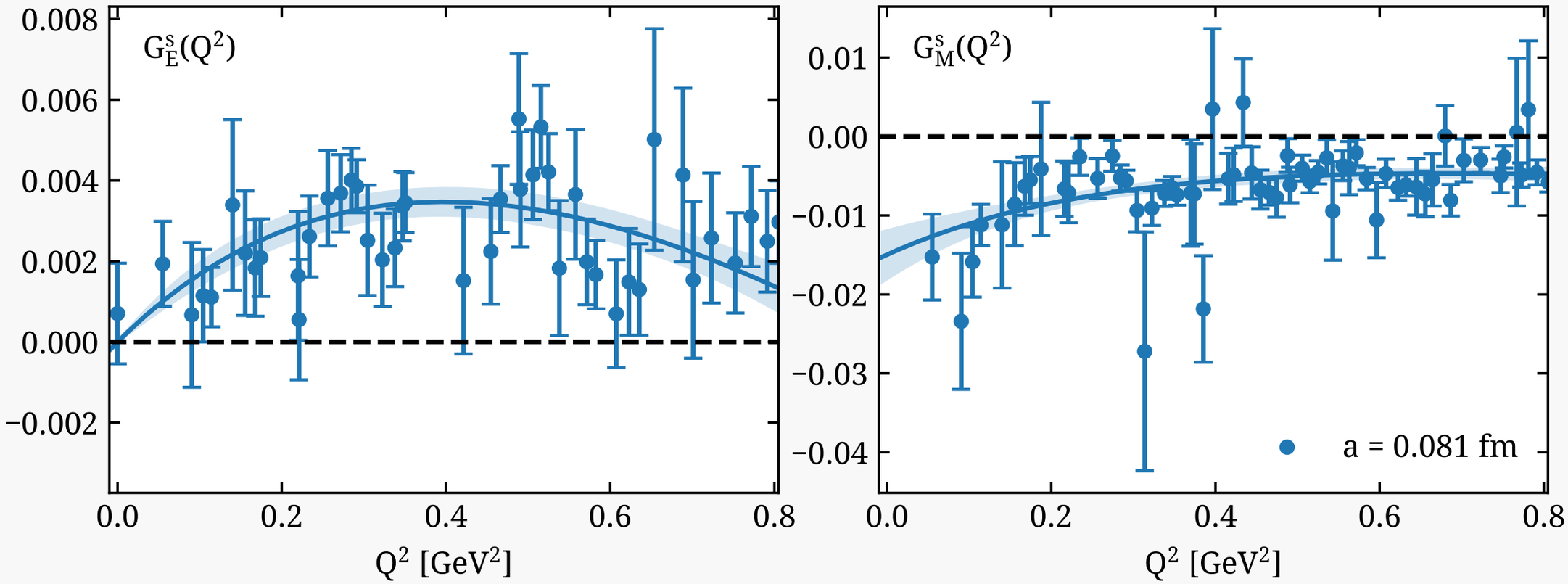




- Disconnected at the few percent level
- Two physical point ensembles with disconnected, $a = 0.093$ & 0.081 fm
- Two volumes with N_f=2

Phys. Rev. **D101** (2020) 031501, arXiv:1909.10744 [hep-lat]

Strange EM form factors

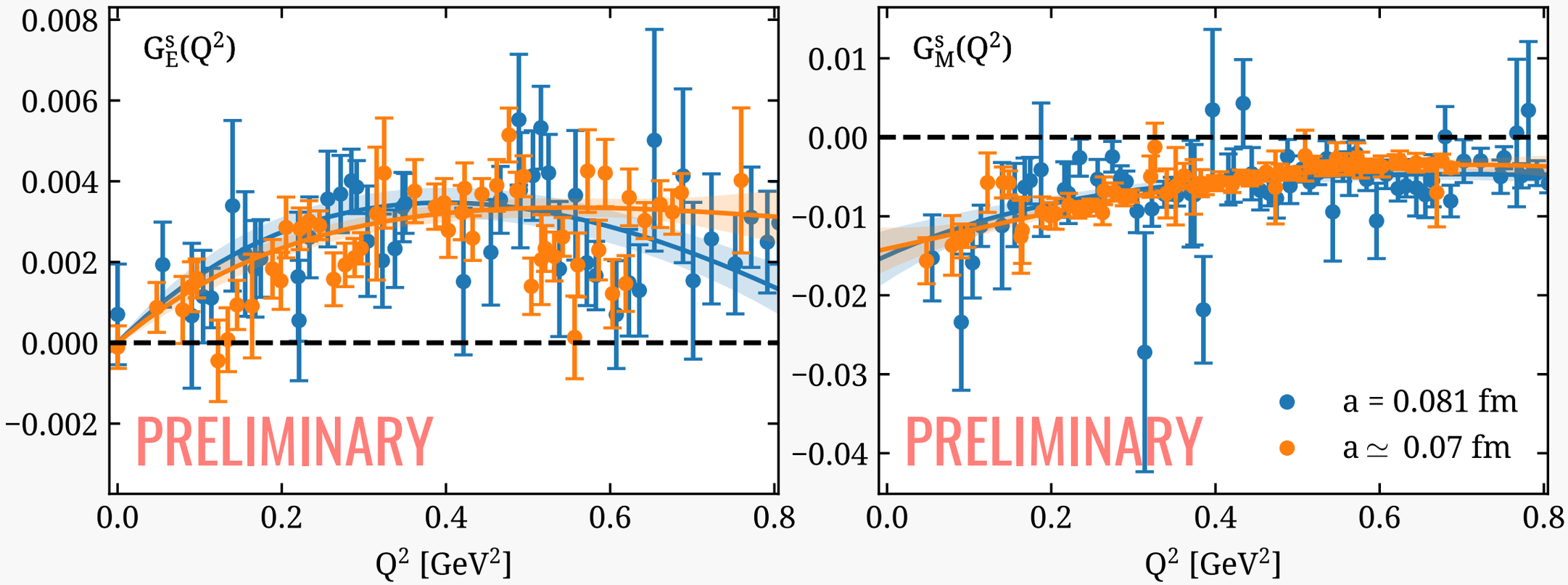


Strange EM form factors (N_f=2+1+1, a=0.081 fm)

- High accuracy determination at physical point:
  - Color-spin dilution
  - Hierarchical probing (2^3-distance) →512 Hadamard vectors
  - High-statistics two-point functions
- No deflation necessary for strange

Phys. Rev. **D101** (2020) 031501, arXiv:1909.10744 [hep-lat]

Strange EM form factors



Strange EM form factors: not well known experimentally

- Preliminary finer lattice spacing $a{=}0.07\ \mathrm{fm}$
- Compare to best experimental results so far:
  - HAPPEX: $G_M^s(Q^2 \sim 0.62\ GeV^2) = -0.070(67)$
  - A4: $G_E^s(Q^2 \sim 0.22\ GeV^2) = 0.050(38)(19)$
    $G_M^s(Q^2 \sim 0.22\ GeV^2) = 0.14(11)(11)$

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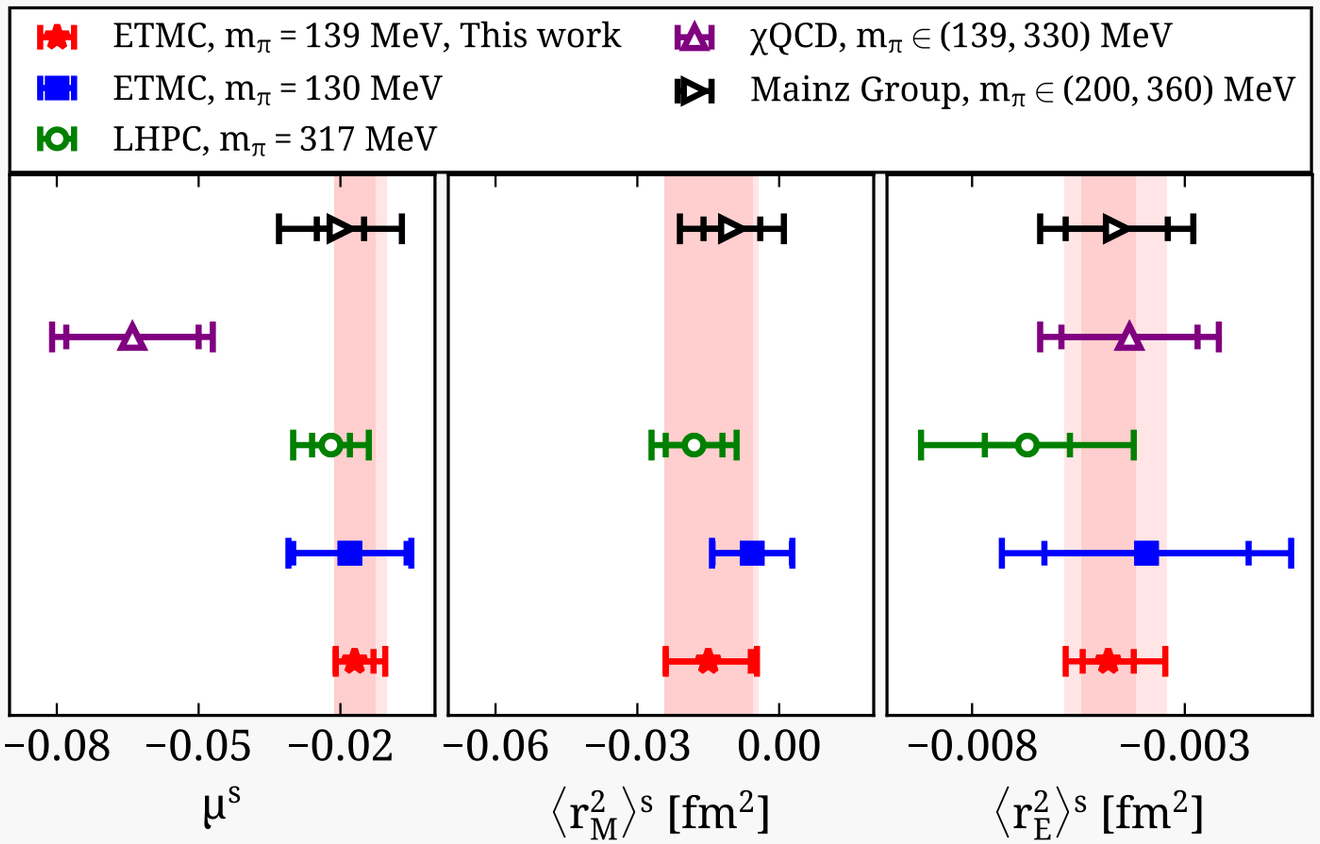
Strange EM form factors



Strange EM form factor: radii and moments

- High accuracy also reflected in radius determination
- Note consistency between various lattice groups

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Axial Form Factors

Matrix element:

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p',s')\mathcal{O}_\mu u(p,s)$$

$$\mathcal{O}_\mu = \gamma_\mu\gamma_5 G_A(Q^2) - \frac{Q_\mu}{2M_N}\gamma_5 G_P(Q^2)$$

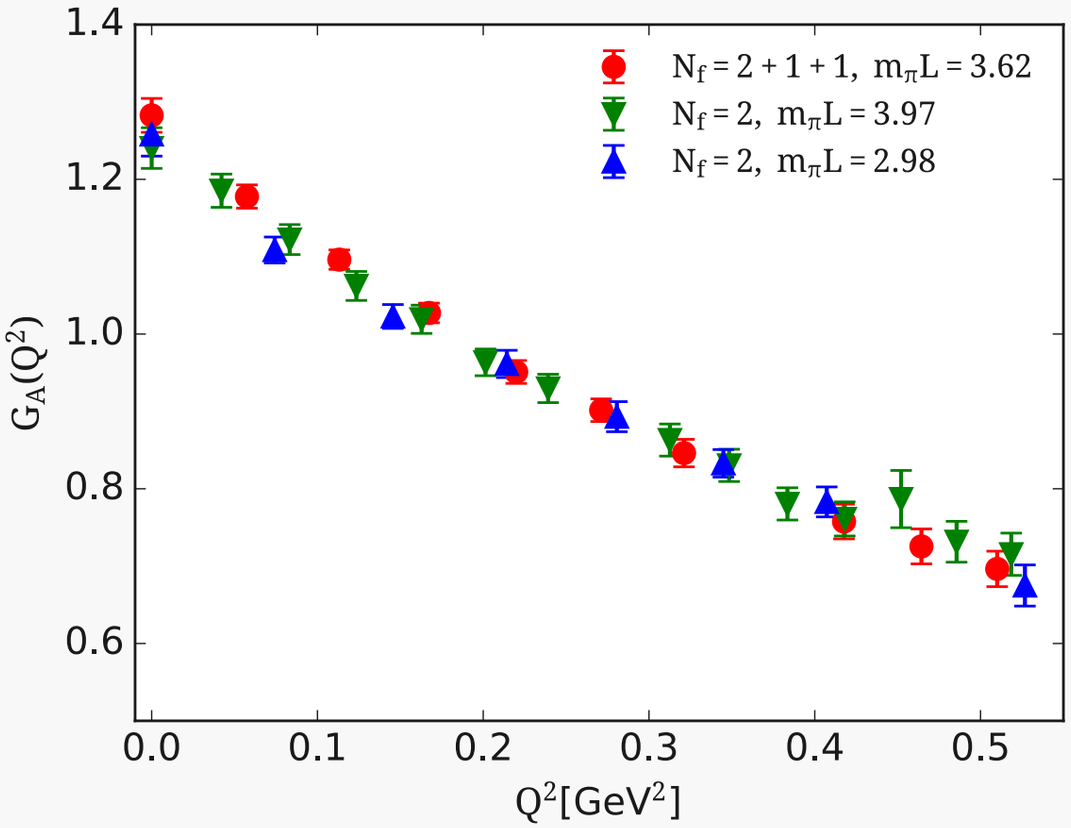
Sachs form-factors:

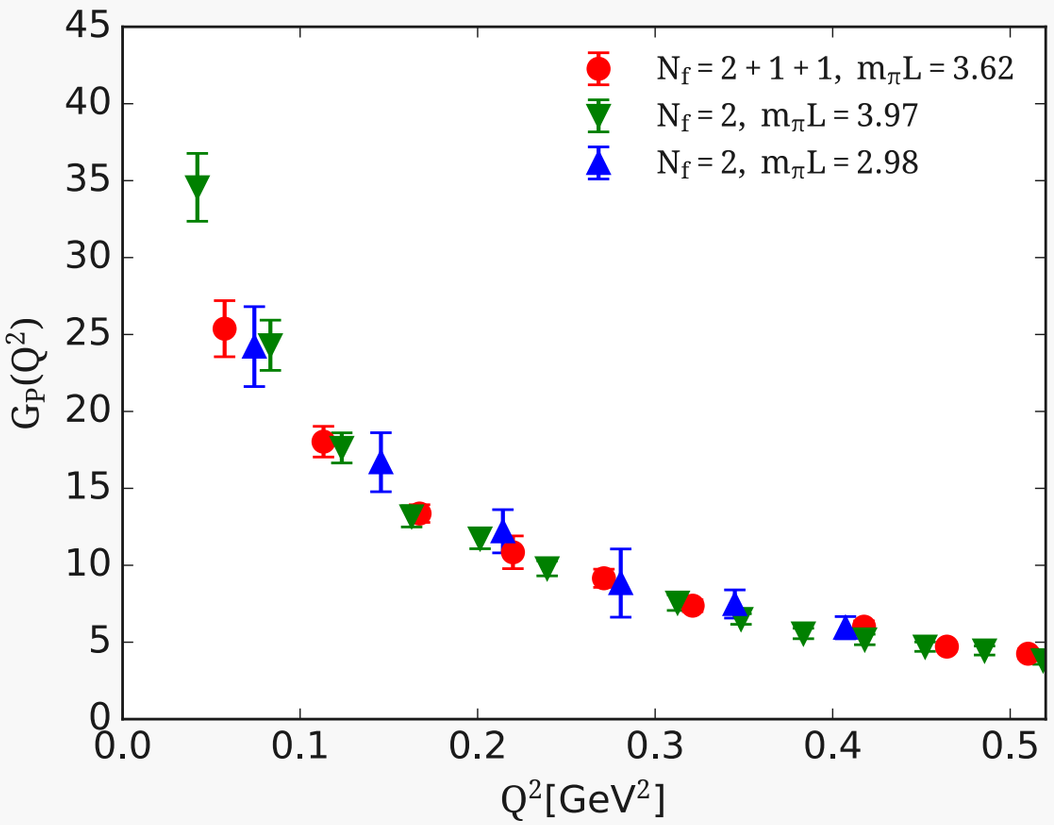
- G_A: Axial form factor
- G_P: Induced pseudoscalar form factor

Flavor combinations:

- Isovector: $A^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$
- Isoscalar: $A^{u+d} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$
- Octet: $A^{u+d-2s} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s$
- Singlet: $A^{u+d+s} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$

Isovector Axial FFs



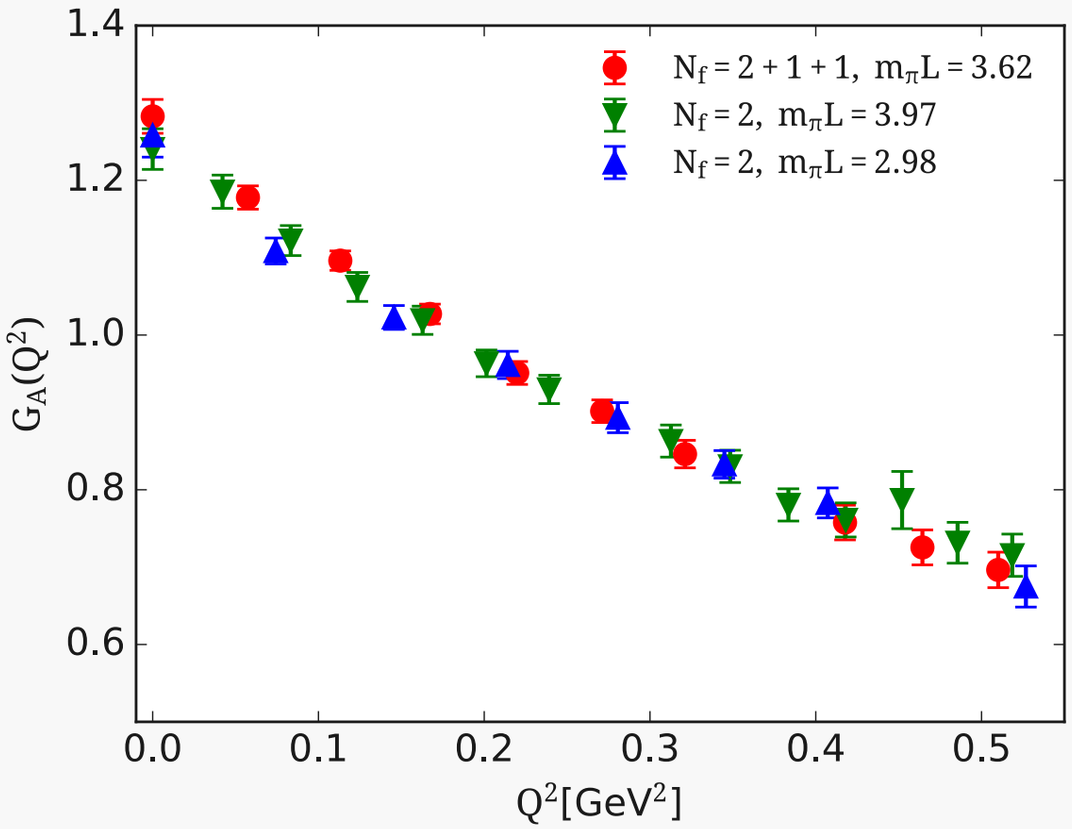


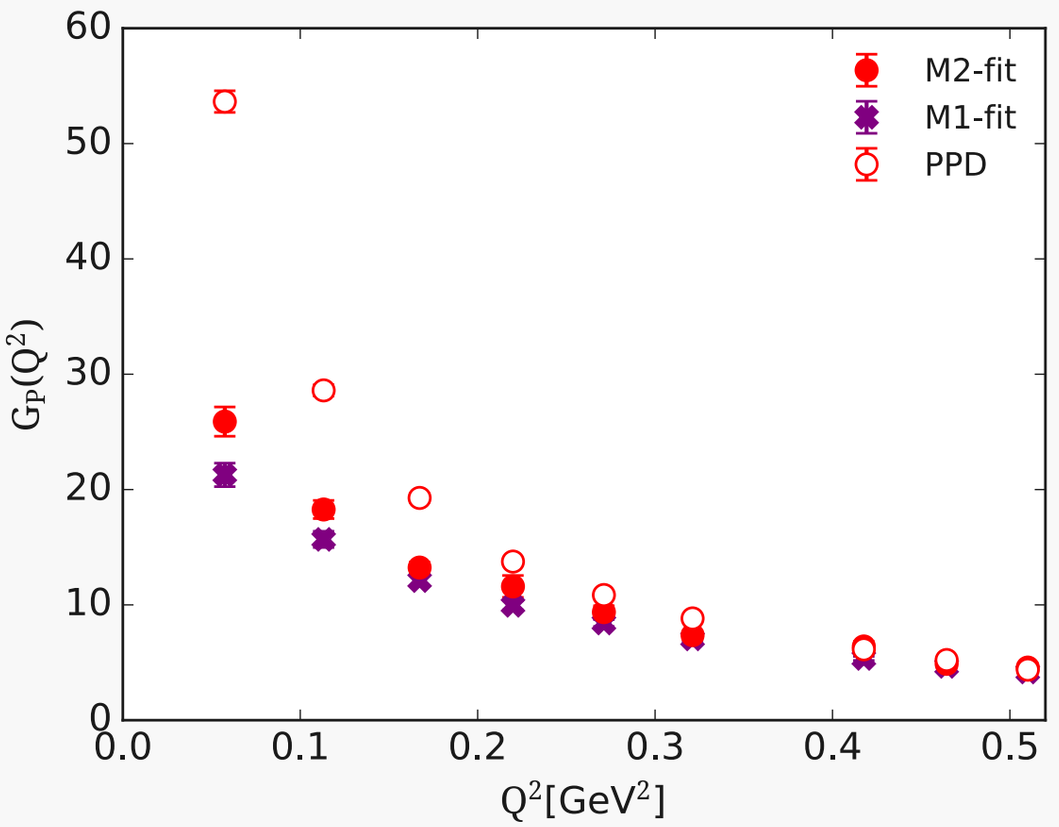
Isovector axial $G_A(Q^2)$ and induced pseudoscalar $G_P(Q^2)$

- Compare volume effects (two N_f=2) and cut-off effects (N_f=2 vs N_f=2+1+1)

Phys. Rev. **D103** (2021) 034509, arXiv:2011.13342 [hep-lat]

Isovector Axial FFs



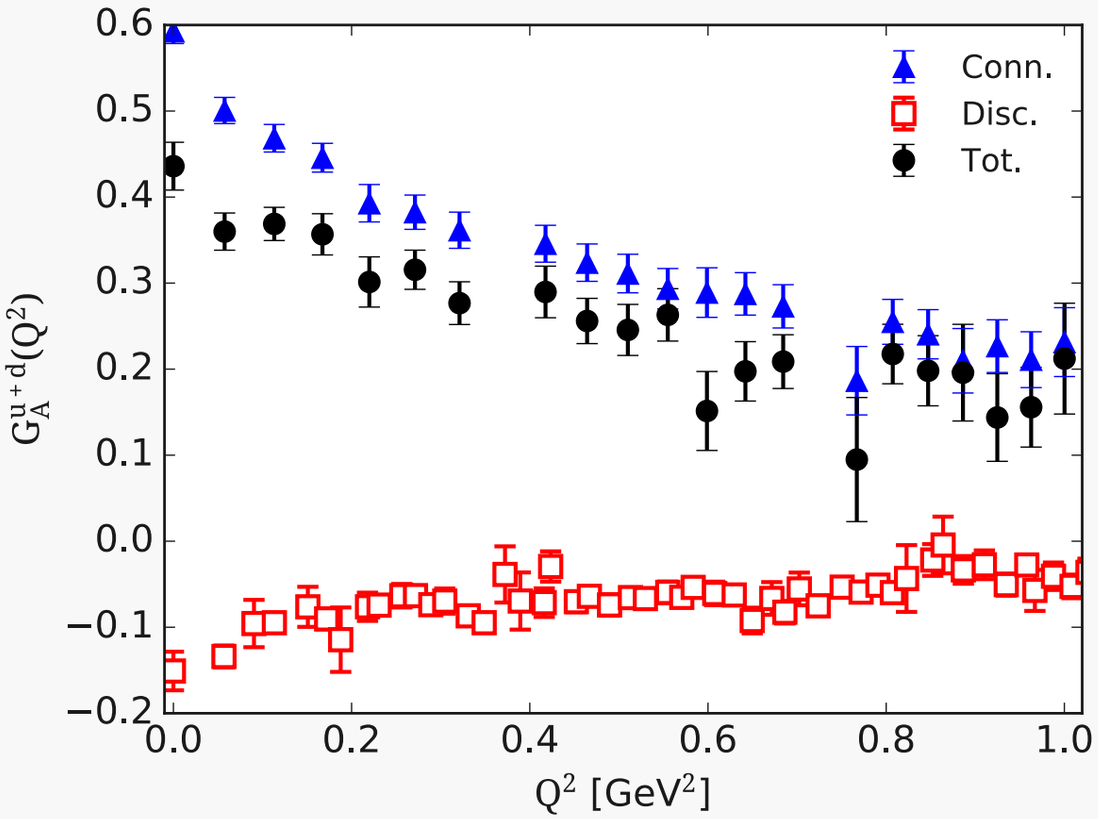


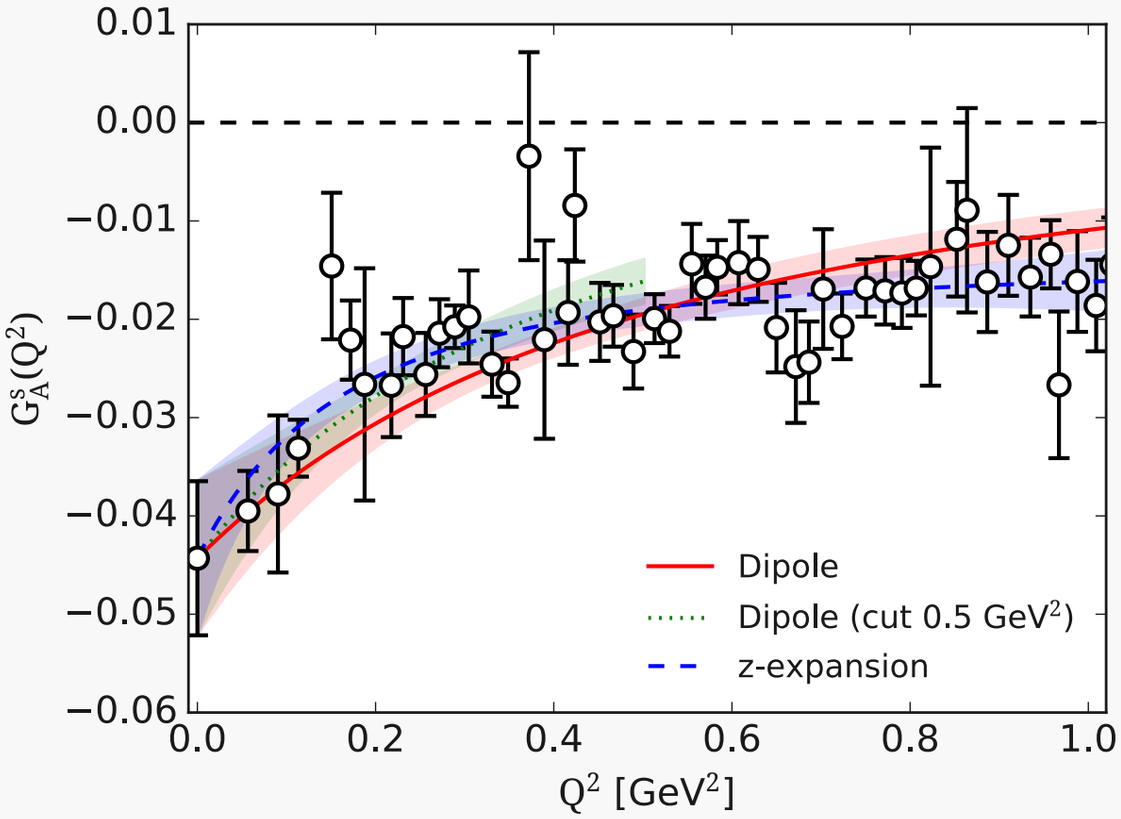
Isovector axial $G_A(Q^2)$ and induced pseudoscalar $G_P(Q^2)$

- Compare volume effects (two $N_f{=}2$) and cut-off effects ($N_f{=}2$ vs $N_f{=}2{+}1{+}1$)
- $G_P(Q^2)$ from PCAC and pion pole dominance (PPD)
  - $M1$ fit: same parameter for excited state between 3pt and 2pt
  - $M2$ fit: different parameters in 3pt and 2pt

Phys. Rev. **D103** (2021) 034509, arXiv:2011.13342 [hep-lat]

Isoscalar and strange FFs





Contribution of disconnected diagrams (N_f=2+1+1, a=0.081 fm)

- Significant contribution of disconnected to u+d combination
- Negative disconnected contribution: subtracts from connected
- Good signal for strange contribution: clearly non-zero and negative

$\Delta\Sigma^{u,s,d}$

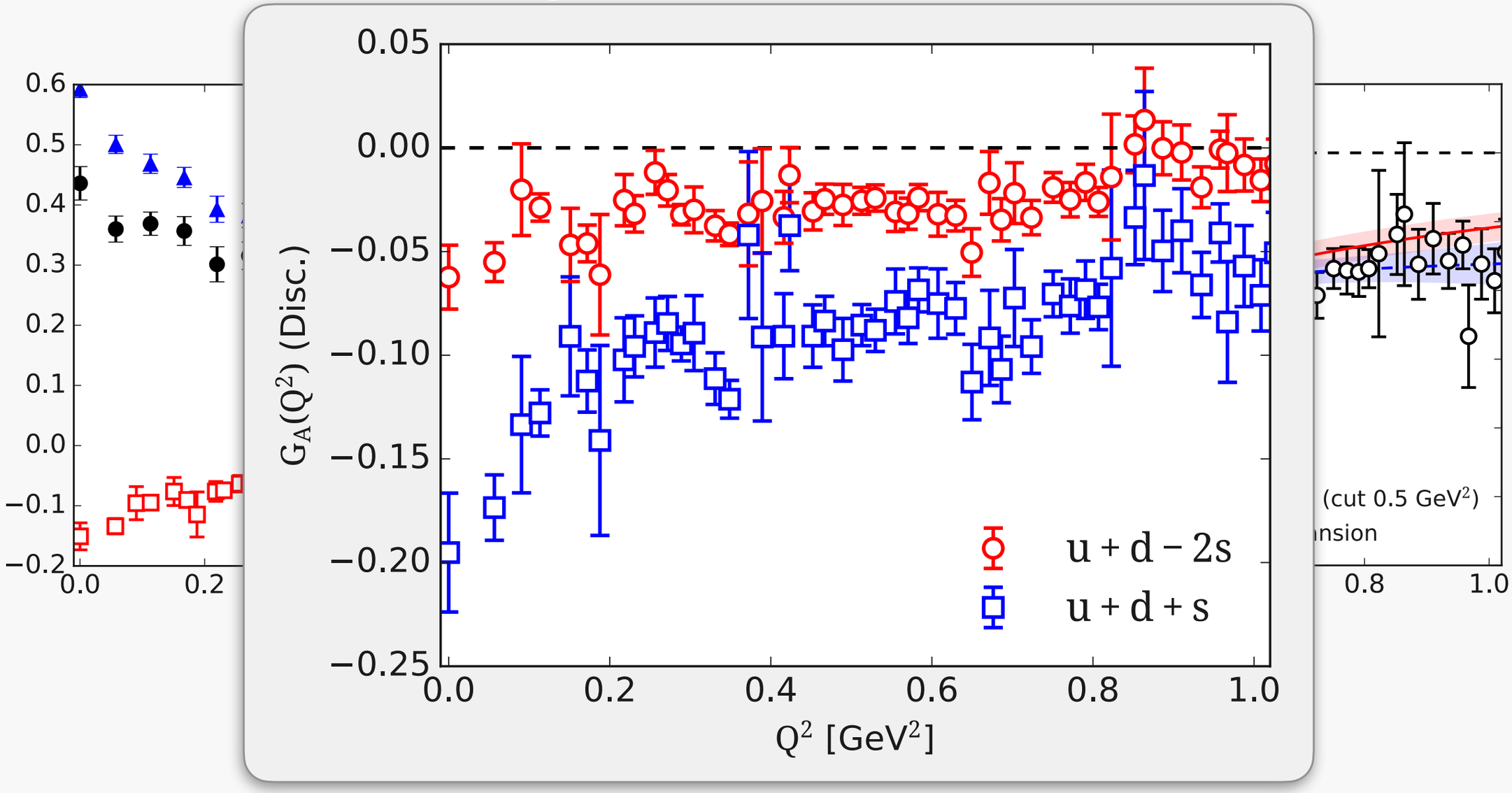
See talk by

K. Hadjiyiannakou

Thu. @ 13:00 EDT

C. Alexandrou *et al.* "*Quark flavor decomposition of the nucleon axial form factors*", arXiv:2106.13468

Isoscalar and strange FFs

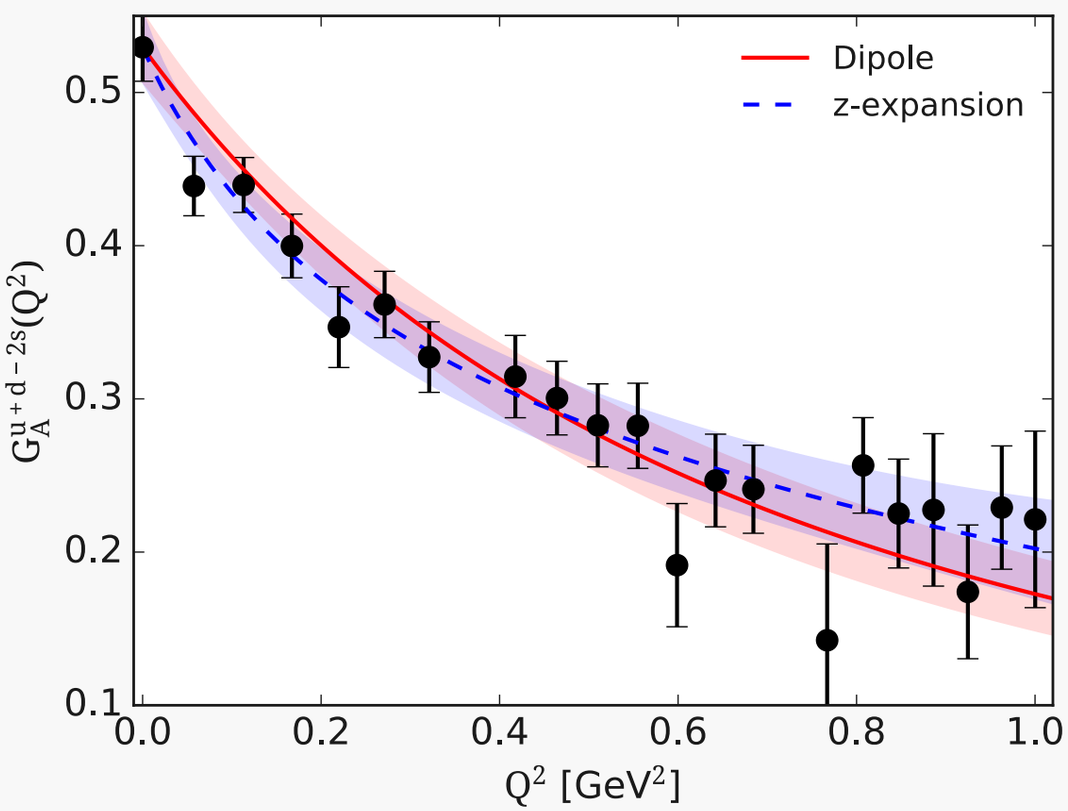


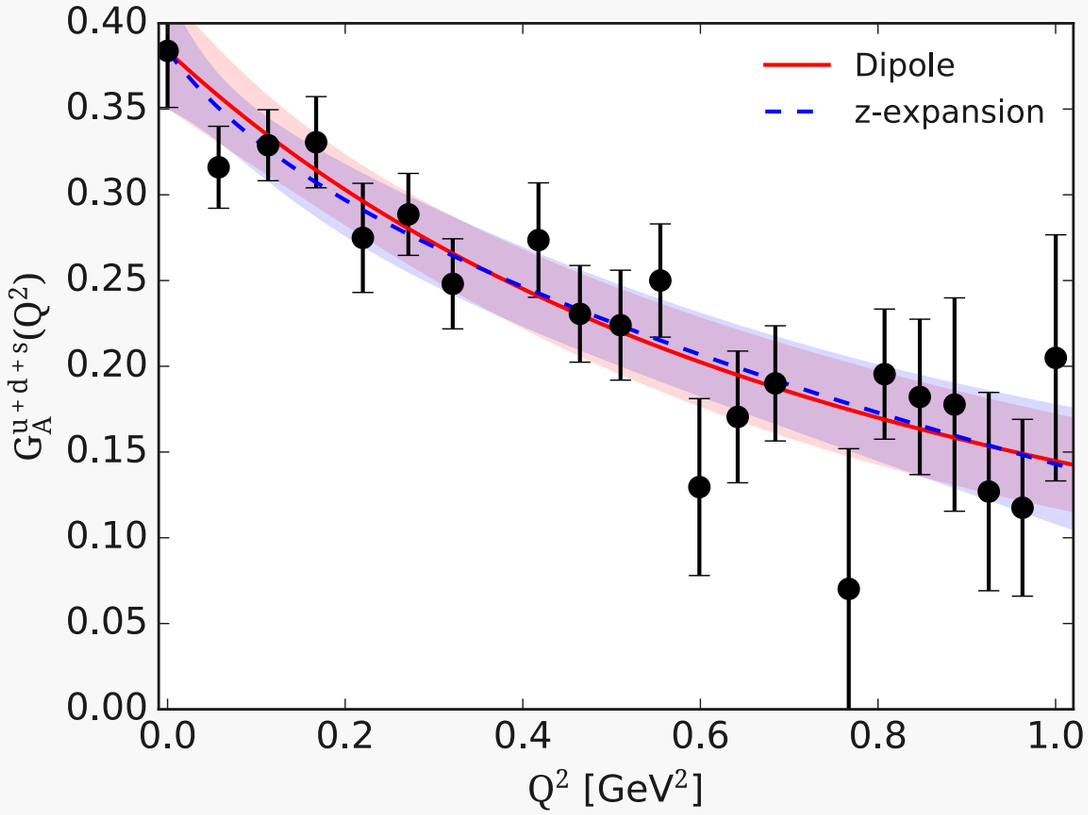
Contribution of disconnected diagrams (N_f=2+1+1, a=0.081 fm)

- At SU(3) flavour limit disconnected contribution to octet is zero ($u+d-2s$)

C. Alexandrou *et al.* "*Quark flavor decomposition of the nucleon axial form factors*", arXiv:2106.13468

Octet and singlet





First look at octet and singlet axial form factor at physical point

- Radii via dipole and z-expansion fits

Fit Type	Q^2_{max} [GeV^2]	$\sqrt{\langle (r_A^{u+d-2s})^2 \rangle}$ [fm]	χ^2/d.o.f	$\sqrt{\langle (r_A^{u+d+s})^2 \rangle}$ [fm]	χ^2/d.o.f
Dipole	$\simeq 0.5$	0.623(59)	1.07	0.545(104)	0.68
	$\simeq 1$	0.592(52)	1.04	0.542(81)	0.65
z-expansion	$\simeq 0.5$	0.780(108)	0.45	0.673(221)	0.50
	$\simeq 1$	0.761(113)	0.57	0.650(221)	0.59

C. Alexandrou *et al.* "*Quark flavor decomposition of the nucleon axial form factors*", `arXiv:2106.13468`

Summary & Outlook

Nucleon EM and Axial form factors at the physical point

- Multiple sink-source separation with increasing statistics
- Very good signal for disconnected contributions
  - Confirmation of few % level of contribution of light disconnected contributions to EM ffs at physical point
  - High-accuracy calculation of strange EM ffs at physical point
  - First look at SU(3) flavour combinations for Axial ffs directly at physical point
- Ongoing
  - Analysis of smaller lattice spacing and larger volume ensembles
- Outlook
  - Complete $V \to \infty$ and $a \to 0$ extrapolation using $N_f{=}2{+}1{+}1$ ensembles all at the physical point

Acknowledgements

RESEARCH & INNOVATION FOUNDATION

ΕΥΡΩΠΑΪΚΗ ΕΝΩΣΗ

2016-2020 ReSTART SEARCH

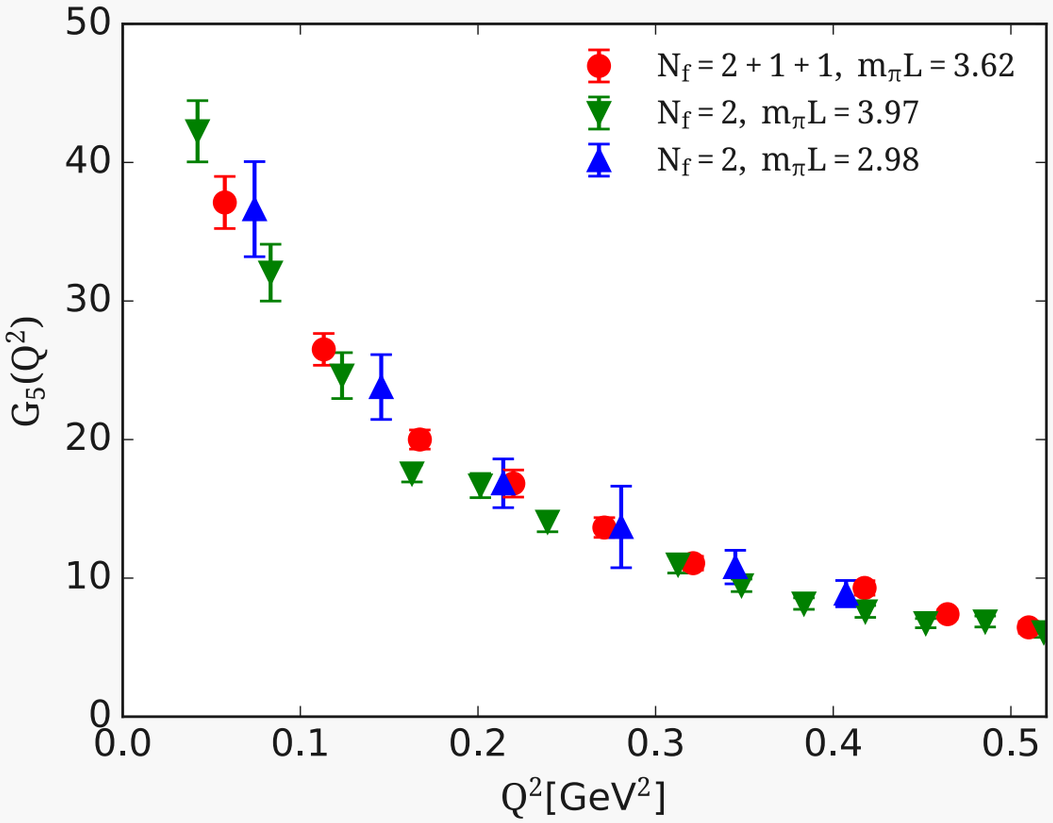
ΠΡΟΓΡΑΜΜΑΤΑ ΕΡΕΥΝΑΣ, ΤΕΧΝΟΛΟΓΙΚΗΣ ΑΝΑΠΤΥΞΗΣ ΚΑΙ ΚΑΙΝΟΤΟΜΙΑΣ

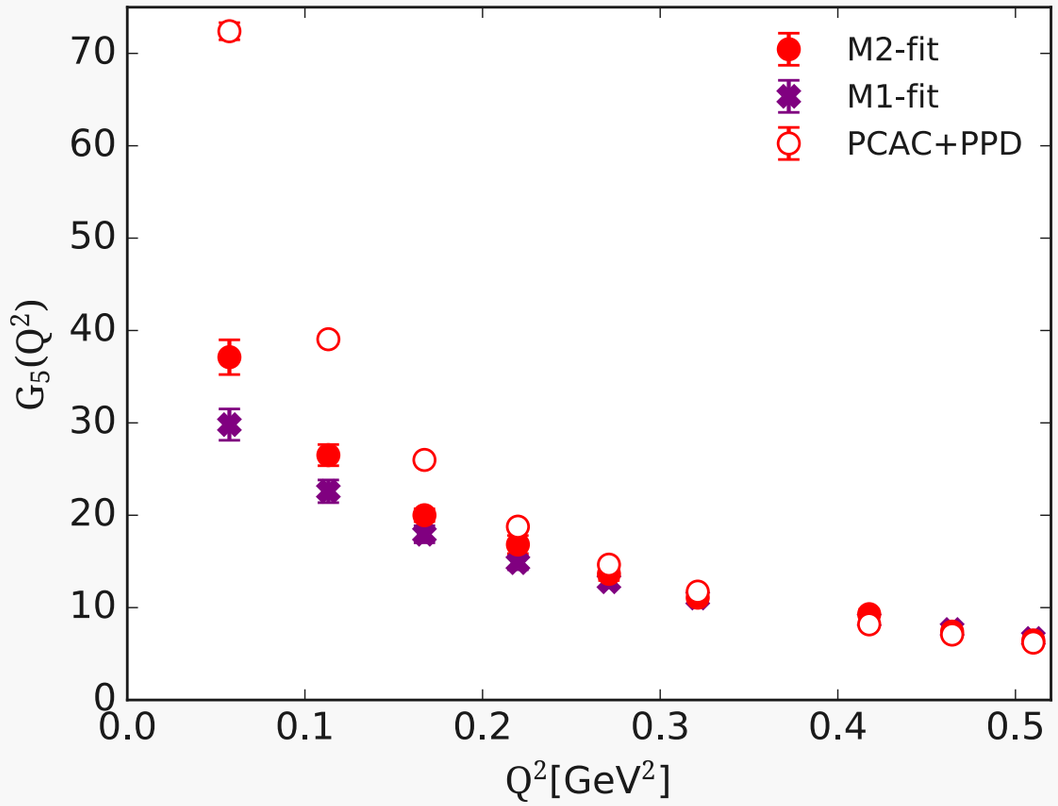
ΚΥΠΡΙΑΚΗ ΔΗΜΟΚΡΑΤΙΑ

ΔΙΑΡΘΡΩΤΙΚΑ ΤΑΜΕΙΑ
οι ιδέες μας, πρόξη και ανάπτυξη

EXCELLENCE/0918/0129

BACKUP

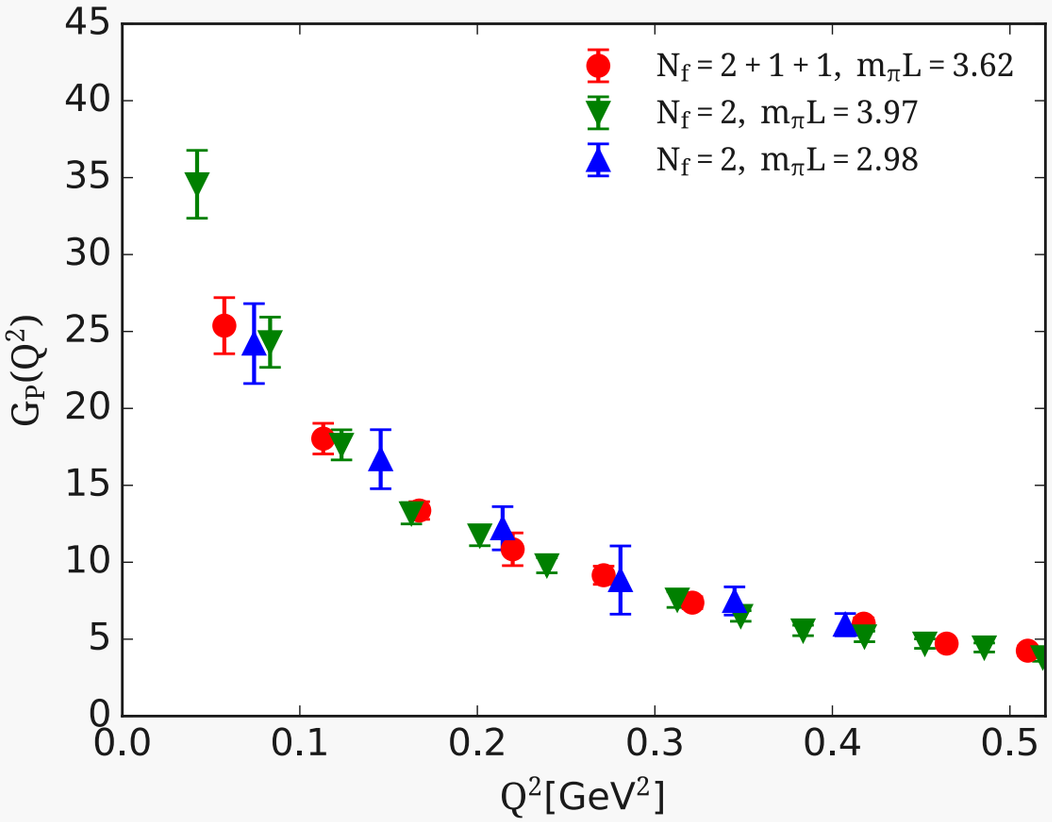


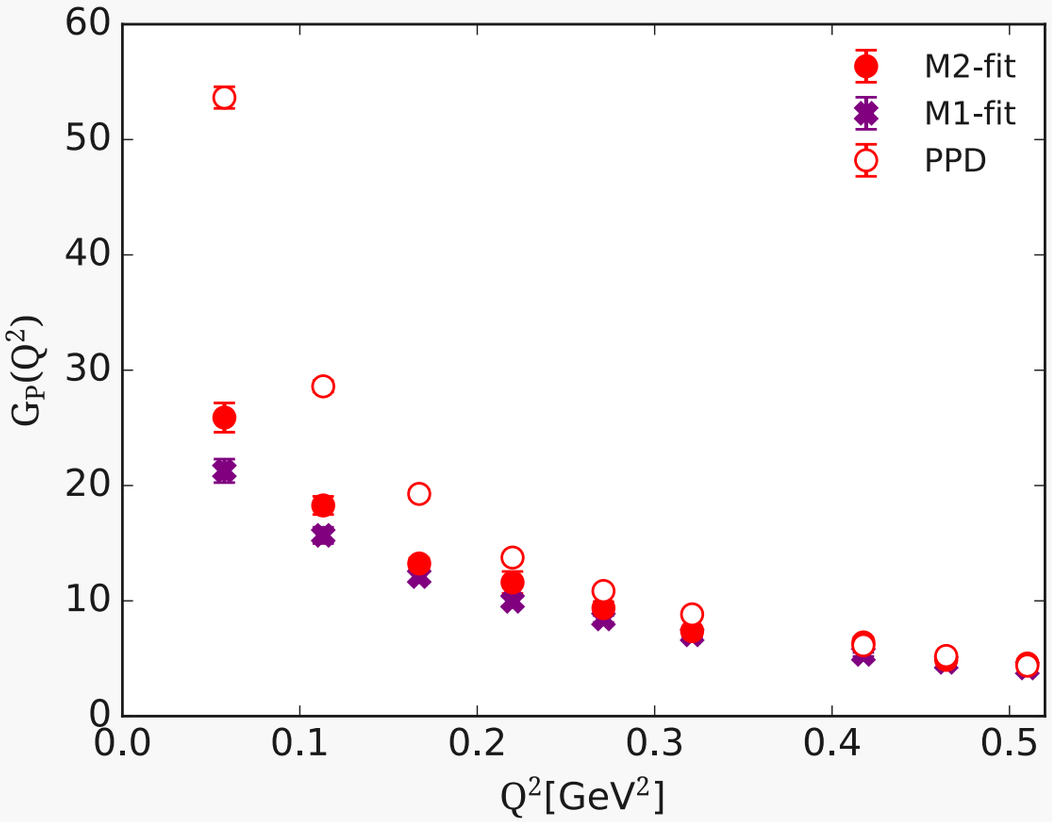


Isovector pseudo-scalar form factor

- Three ensembles from twisted mass at physical point
- Compare to expectation from PCAC and pion pole dominance
- $M1$ fit: same parameter for excited state between 3pt and 2pt
- $M2$ fit: different parameters in 3pt and 2pt

BACKUP

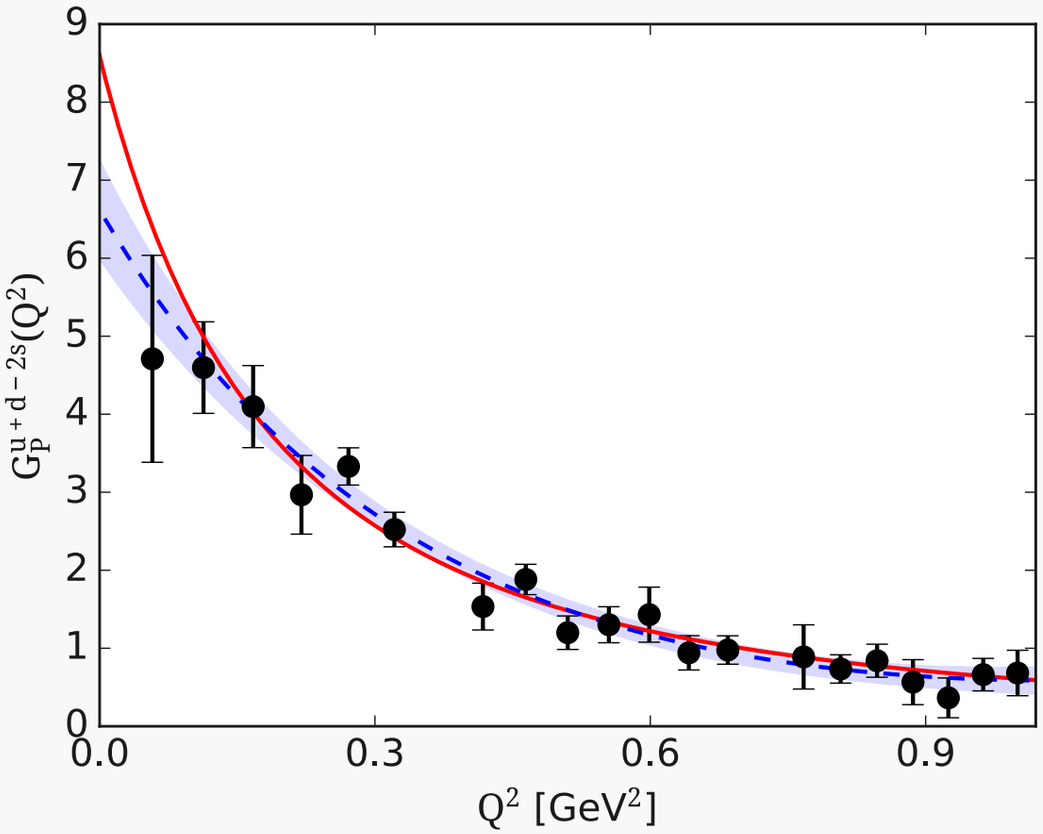


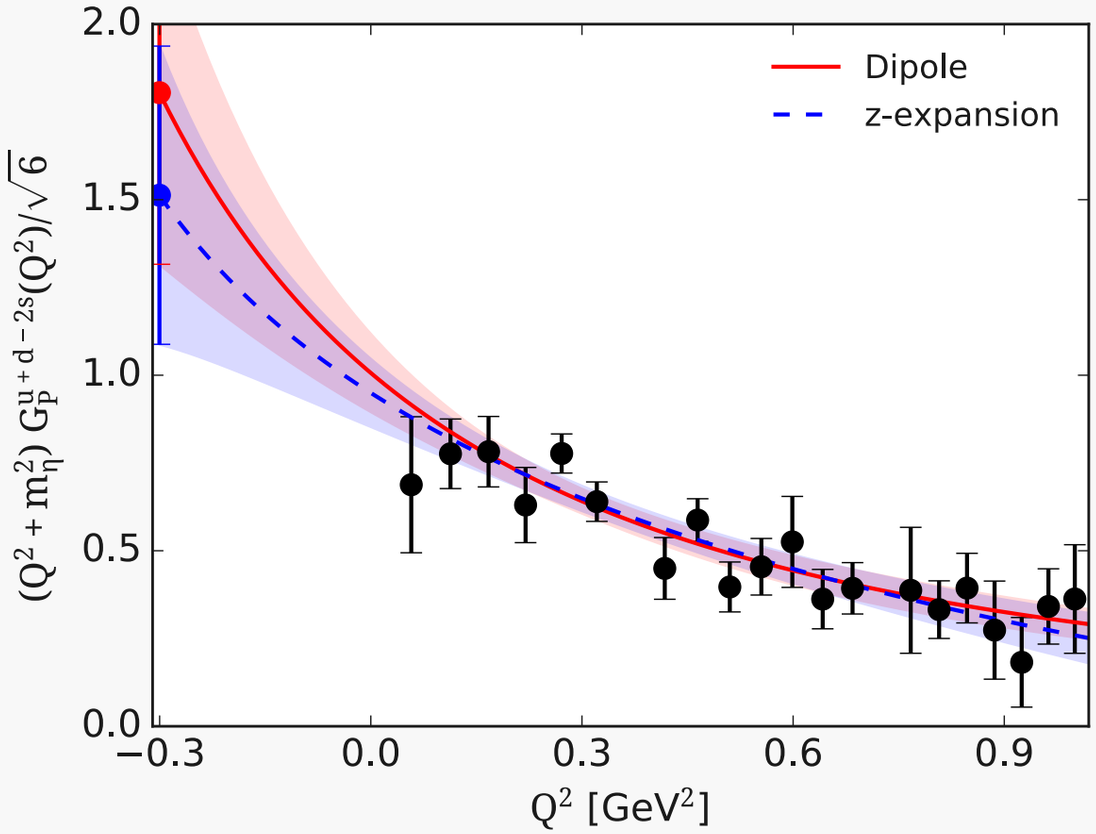


Isovector induced pseudo-scalar form factor

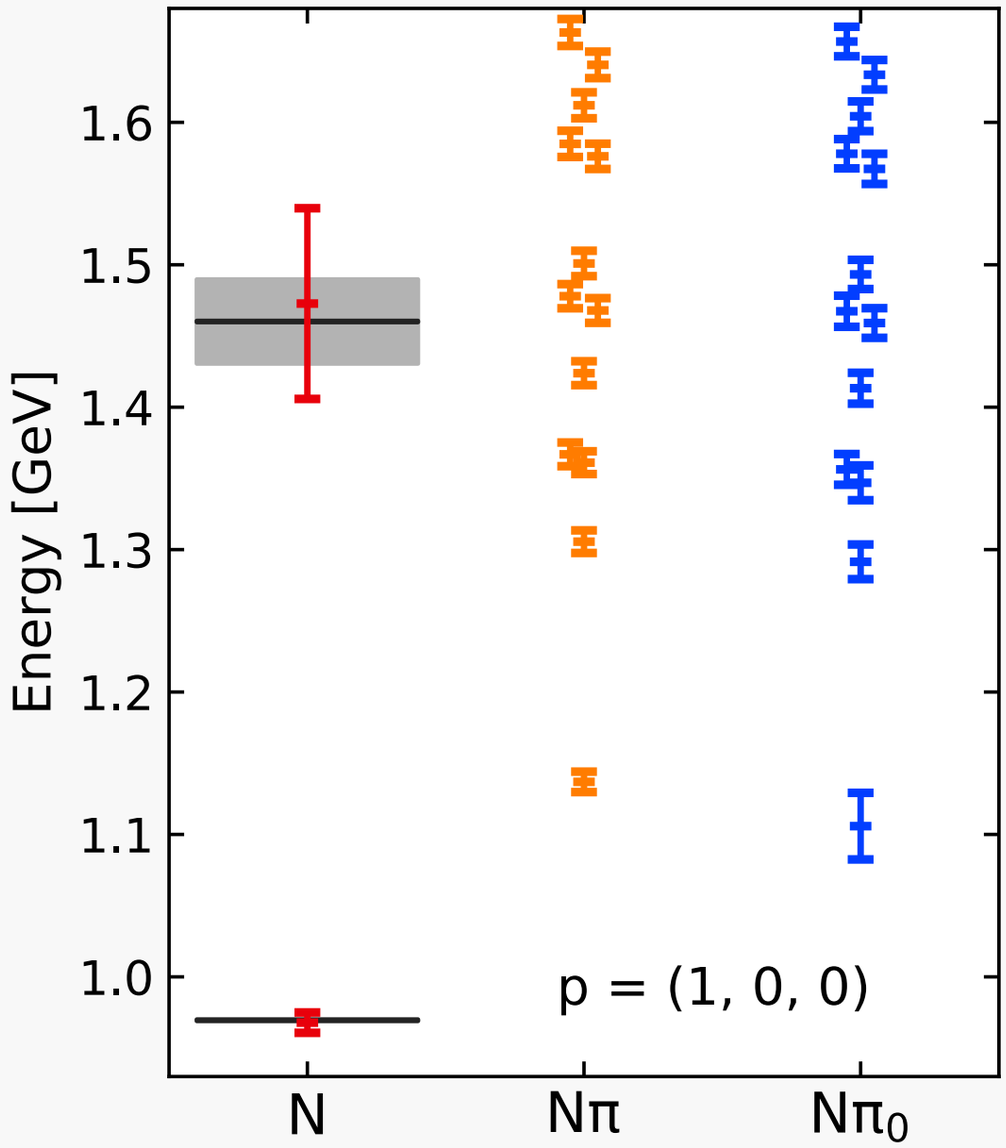
- Three ensembles from twisted mass at physical point
- Compare to expectation from PCAC and pion pole dominance
- $M1$ fit: same parameter for excited state between 3pt and 2pt
- $M2$ fit: different parameters in 3pt and 2pt

BACKUP


9
8
7
6
5
4
3
2
1
0
0.0
0.3
0.6
0.9
$G_P^{u+d-2s}(Q^2)$
Q^2 [GeV^2]


2.0
1.5
1.0
0.5
0.0
−0.3
0.0
0.3
0.6
0.9
Dipole
z-expansion
$(Q^2+m_\eta^2)\,G_P^{u+d-2s}(Q^2)/\sqrt{6}$
Q^2 [GeV^2]

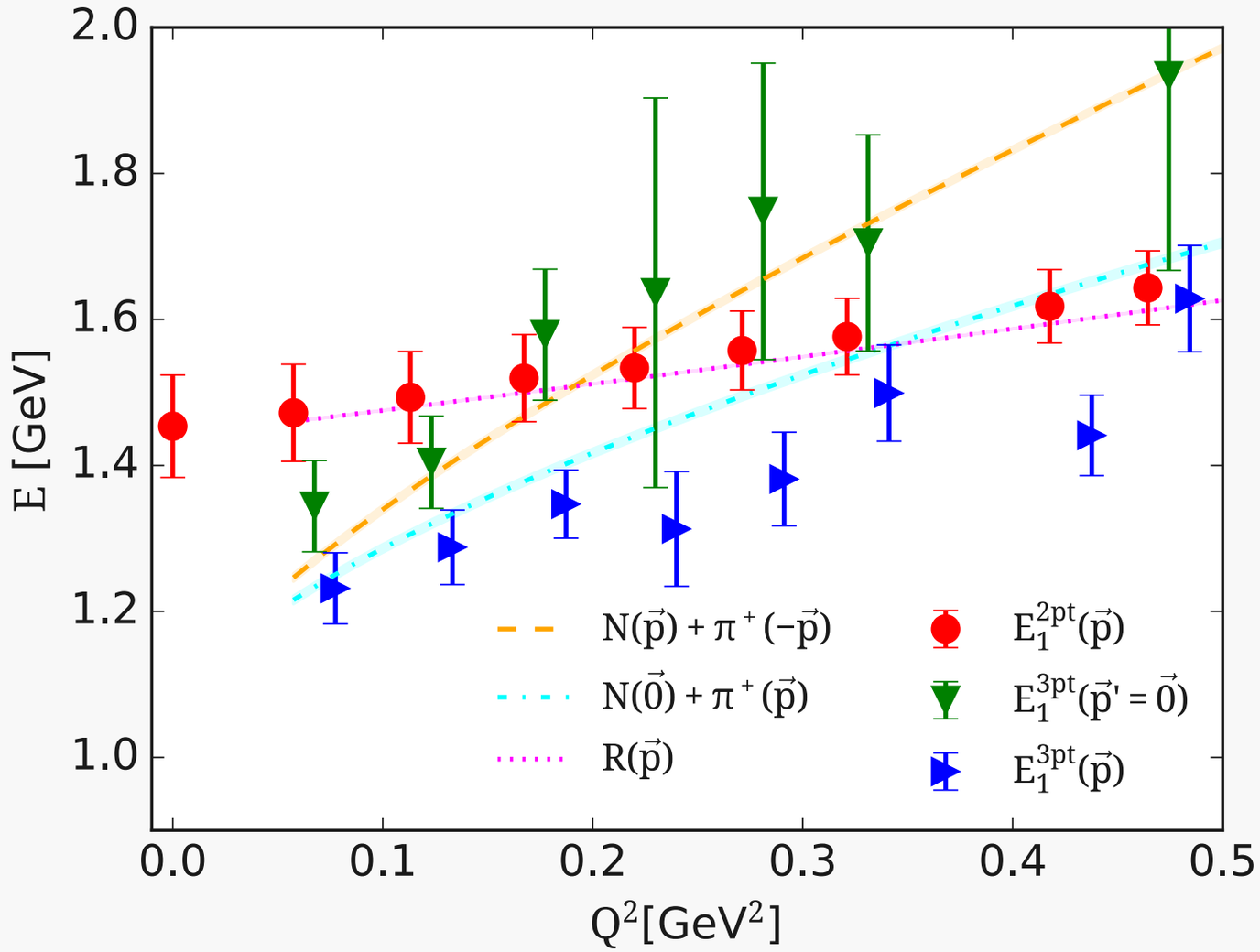
BACKUP



Nucleon states obtain from two-point function

- $N\pi$: energies of non-interacting states with m_N and m_π from two-point function
- Nucleon excited state from two-point function consistent with Roper

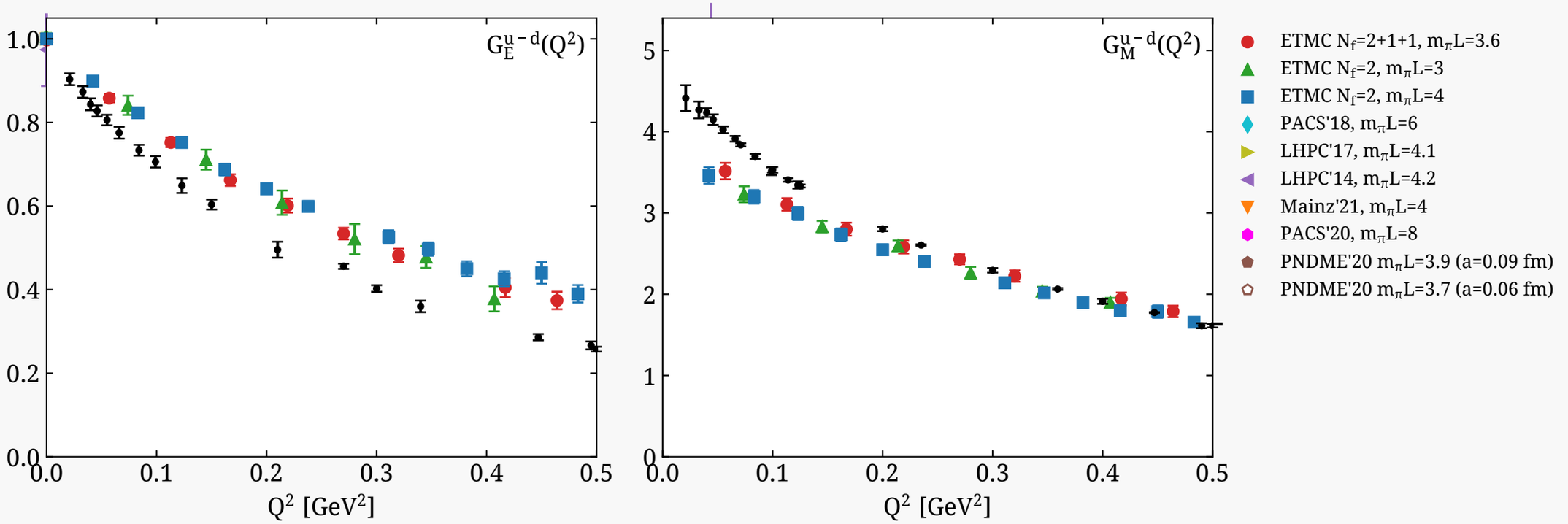
BACKUP



Excited state analysis using "$A4$" three-point function (as in `arXiv:905.06470`)

- Excited state from two-point function follows Roper dispersion relation
- Excited state from "$A4$" three-point approximately follow non-interacting πN

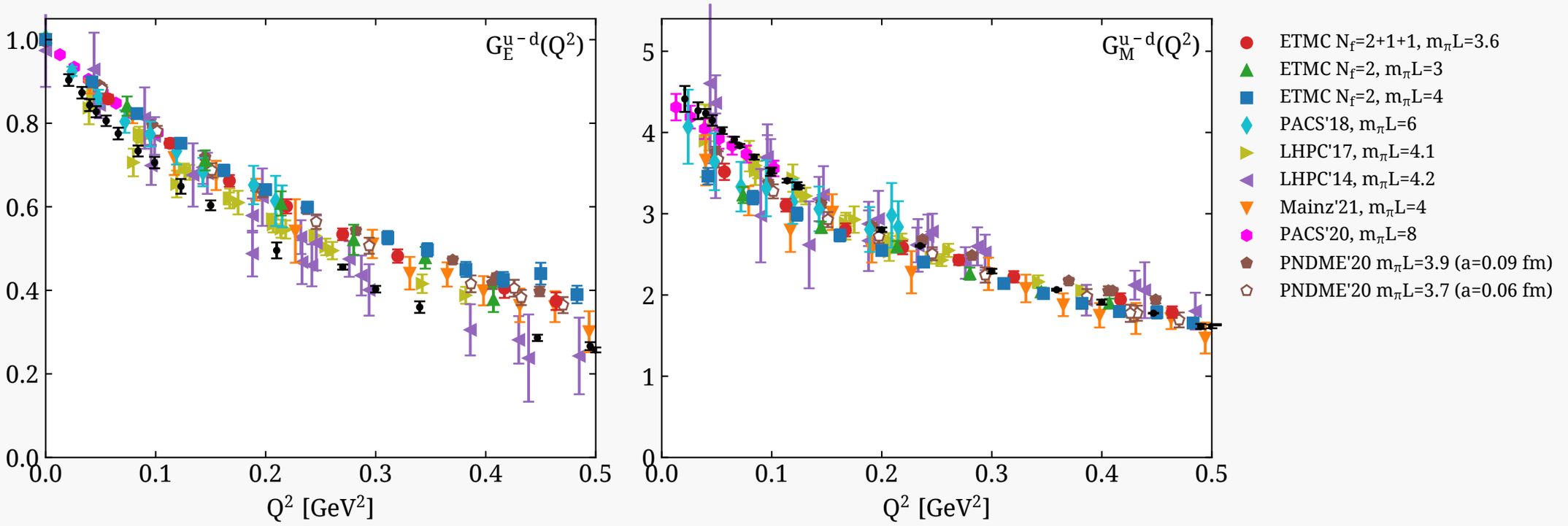
BACKUP



- *Showing*: physical point ensembles by ETMC (N_f=2 and N_f=2+1+1)
  - Three volumes $m_\pi L = \{3, 3.6, 4\}$
  - Two lattice spacings $a = \{0.093, 0.081\}$ fm

Phys. Rev. **D100** (2019) 014509, arXiv:1812.10311 [hep-lat]

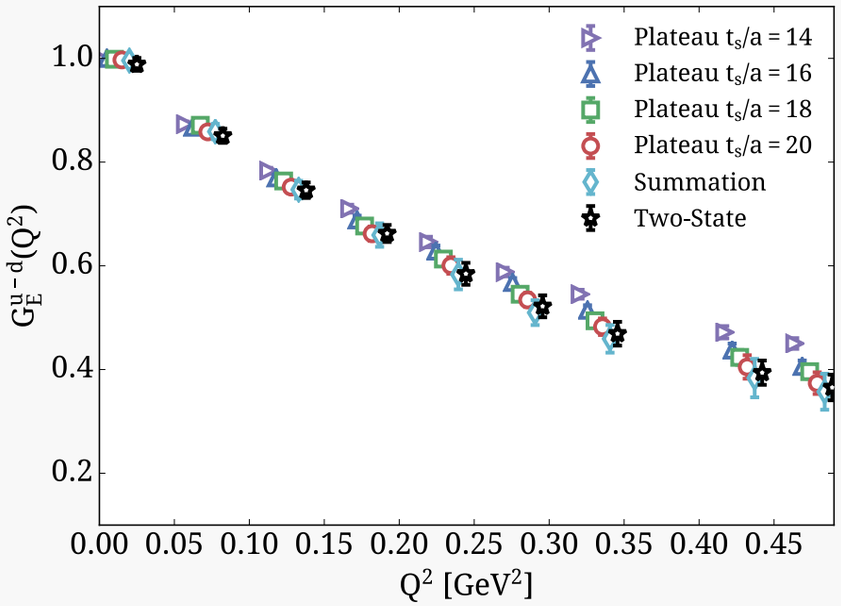
BACKUP

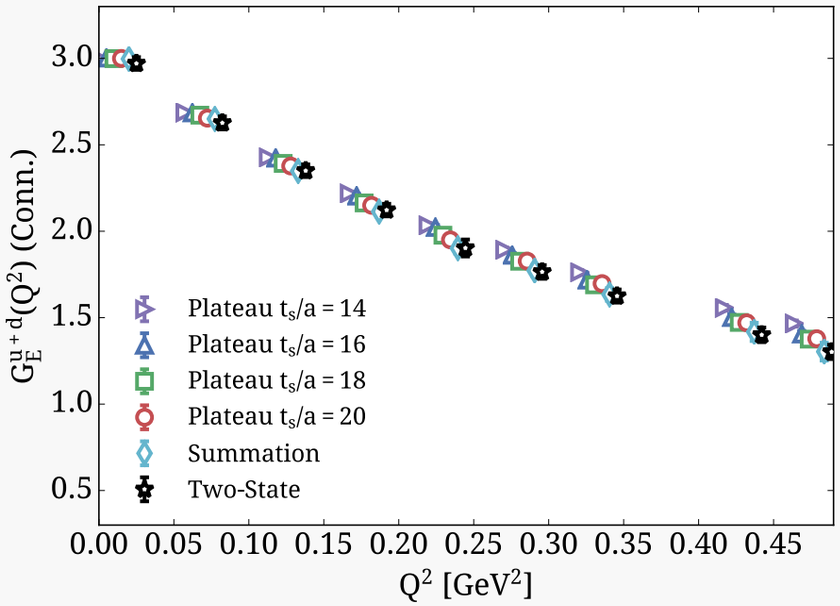


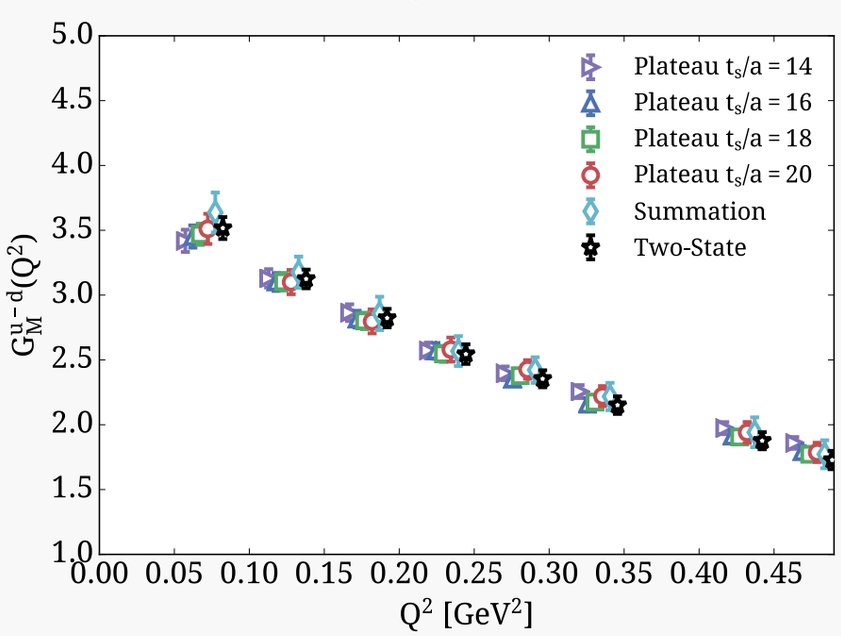
- *Showing:* physical point ensembles by ETMC (N_f=2 and N_f=2+1+1)
  - Three volumes $m_\pi L$ = {3, 3.6, 4}
  - Two lattice spacings a = {0.093, 0.081} fm
- *Added:* Recent physical point ensembles by other collaborations
  - $m_\pi L$ up to 8 (PACS)
  - Finest lattice spacing $a \simeq$ 0.06 fm (PNDME; clover on HISQ)

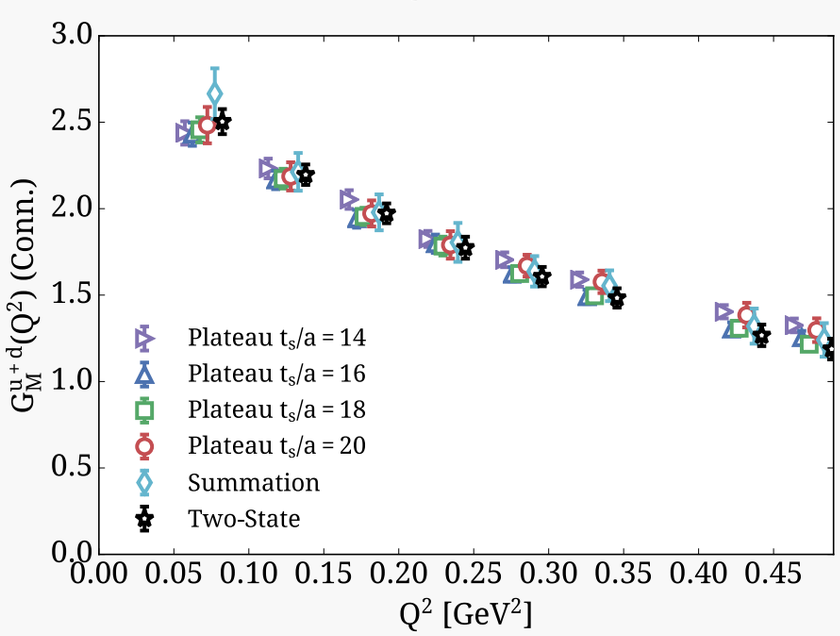
Phys. Rev. **D100** (2019) 014509, arXiv:1812.10311 [hep-lat]

BACKUP









- Convergence with plateau at t_s/a=20 (~ 1.6 fm)

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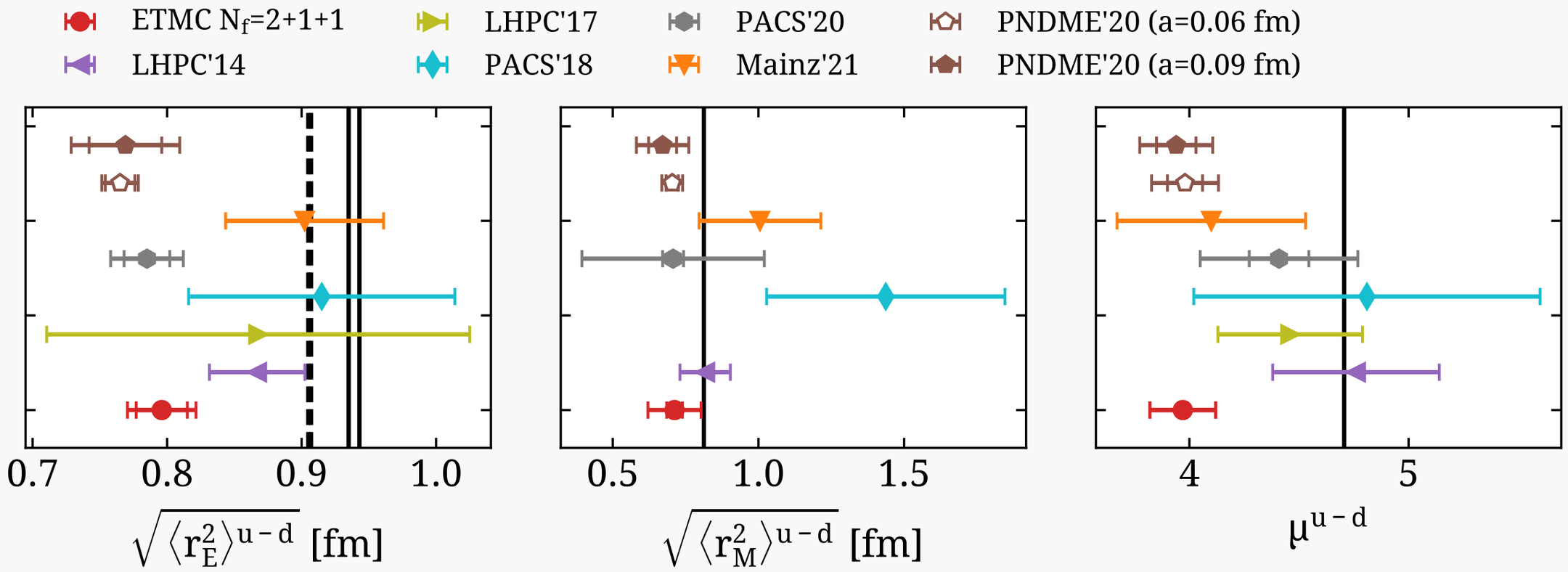
BACKUP



- *Showing:* Recent physical point ensembles by other collaborations
  - $m_\pi L$ up to 8 (PACS)
  - Finest lattice spacing $a \simeq 0.06$ fm (PNDME; clover on HISQ)

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