Gauge-invariant renormalization of fermion bilinears and energy-momentum tensor on the lattice

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Outline

A. Introduction: Gauge-invariant renormalization scheme (GIRS)

B. Study of multiplicative renormalization with GIRS:
Application to the fermion bilinear operators

$$\mathcal{O}_\Gamma(x) = \overline{\psi}(x)\Gamma\psi(x), \qquad \Gamma = \mathbf{1}, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$$

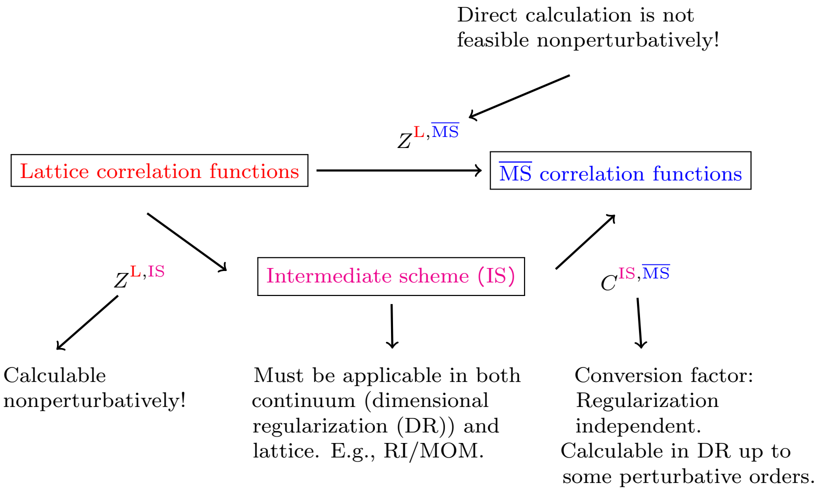
C. Study of operator mixing with GIRS:
Application to the QCD traceless energy-momentum tensor (EMT)

$$\begin{aligned} \overline{T}^G_{\mu\nu} &= F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{d}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}, \\ \overline{T}^F_{\mu\nu} &= \sum_f \left[\frac{1}{2}\left(\bar{\psi}_f\gamma_\mu \overleftrightarrow{D}_\nu\psi_f + \bar{\psi}_f\gamma_\nu \overleftrightarrow{D}_\mu\psi_f\right) - \frac{1}{d}\delta_{\mu\nu}\left(\bar{\psi}_f\gamma_\rho \overleftrightarrow{D}_\rho\psi_f\right)\right] \end{aligned}$$

D. Conclusions and future prospects

Present work: Phys. Rev. D 103, 094509 (2021), arXiv:2102.00858

Renormalization



Gauge-invariant renormalization scheme (GIRS)

- **Older studies of coordinate space renormalization prescriptions:**
  1. V. Gimenez et al., PLB598 (2004) 227
  2. K. G. Chetyrkin, A. Maier, NPB844 (2011) 266
  3. K. Cichy, K. Jansen, P. Korcyl, NPB865 (2012) 268, NPB913 (2016) 278
  4. M. Tomii, N. H. Christ, PRD99 (2019) 014515

- Consider on-shell Green's functions of gauge-invariant operators at different spacetime points in coordinate space, e.g.,

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle, \quad (x \neq y)$$

- Impose renormalization conditions of the following form (similar to RI/MOM):

$$\langle \mathcal{O}_1^R(x)\mathcal{O}_2^R(y)\rangle = |_{|x-y|=\frac{1}{\mu}} = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle^{\text{tree}}|_{|x-y|=\frac{1}{\mu}} \Rightarrow$$

$$Z_{\mathcal{O}_1}(a\mu, g(a))\, Z_{\mathcal{O}_2}(a\mu, g(a))\, \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle|_{|x-y|=\frac{1}{\mu}} = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle^{\text{tree}}|_{|x-y|=\frac{1}{\mu}},$$

where $\mathcal{O}_i^{\text{R}} = Z_{\mathcal{O}_i}\mathcal{O}_i$.

Gauge-invariant renormalization scheme (GIRS)

- **Good features:**
  1. No need for gauge-fixing $\Rightarrow$ No problems with Gribov copies.
  2. When mixing occurs, gauge-variant(GV) operators can be excluded from the renormalization procedure.
  3. Contact terms are automatically excluded ($1/\mu \neq 0$).
  4. Perturbative matching of GIRS and $\overline{\text{MS}}$ scheme is possible at high perturbative orders (in most cases).

- **Practical issues/Challenges:**
  1. Green's functions in coordinate space require calculation of diagrams with one extra loop.
  2. When mixing occurs, $n > 2$-point Green's functions are often needed (typically more noisy).
  3. Narrow renormalization window:
     $\Lambda_{\text{QCD}} \ll \mu \ll a^{-1}, \qquad \mu = |x-y|^{-1} = (|n|a)^{-1}, n_\mu \in \mathbb{Z}.$

- **Extensions of GIRS:**
  In our work: Integration (sum, on the lattice) over time slices (t-GIRS):

$$\int d^3\vec{x}\ \langle \mathcal{O}_1(\vec{x},t)\mathcal{O}_2(\vec{0},0)\rangle, \qquad t \neq 0$$

Application of GIRS to fermion bilinears

- **Fermion bilinear operators:**

$$\mathcal{O}_\Gamma(x) = \overline{\psi}(x)\Gamma\psi(x), \qquad \Gamma = \mathbf{1}, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$$

- **Renormalization factors:**

$$\mathcal{O}_\Gamma^R = Z_\Gamma^{B,R}\mathcal{O}_\Gamma^B, \quad (\text{B} = \text{regularization scheme, R} = \text{renormalization scheme})$$

- **Renormalization condition:**

$$\langle \mathcal{O}_\Gamma^{\text{GIRS}}(x)\mathcal{O}_\Gamma^{\text{GIRS}}(0)\rangle|_{x=\bar{x}} = \langle \mathcal{O}_\Gamma(x)\mathcal{O}_\Gamma(0)\rangle^{\text{tree}}|_{x=\bar{x}}$$

$$\Rightarrow \boxed{Z_\Gamma^{\text{L,GIRS}} = \sqrt{\frac{\langle \mathcal{O}_\Gamma(\bar{x})\mathcal{O}_\Gamma(0)\rangle^{\text{tree}}}{\langle \mathcal{O}_\Gamma^{\text{L}}(\bar{x})\mathcal{O}_\Gamma^{\text{L}}(0)\rangle}}}$$

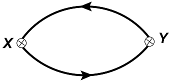
Alternatively,

$$\int d^3\vec{x}\, \langle \mathcal{O}_\Gamma^{\text{t-GIRS}}(\vec{x},t)\mathcal{O}_\Gamma^{\text{t-GIRS}}(\vec{0},0)\rangle\Big|_{t=\bar{t}} = \int d^3\vec{x}\, \langle \mathcal{O}_\Gamma^{\text{t-GIRS}}(\vec{x},t)\mathcal{O}_\Gamma^{\text{t-GIRS}}(\vec{0},0)\rangle^{\text{tree}}\Big|_{t=\bar{t}}$$

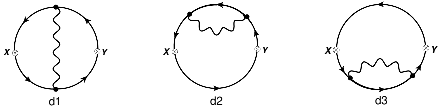
$$\Rightarrow \boxed{Z_\Gamma^{\text{L,t-GIRS}} = \sqrt{\frac{\int d^3\vec{x}\, \langle \mathcal{O}_\Gamma(\vec{x},\bar{t})\mathcal{O}_\Gamma(\vec{0},0)\rangle^{\text{tree}}}{\int d^3\vec{x}\, \langle \mathcal{O}_\Gamma^{\text{L}}(\vec{x},\bar{t})\mathcal{O}_\Gamma^{\text{L}}(\vec{0},0)\rangle}}}$$

Application of GIRS to fermion bilinears

- **Continuum calculation (DR):** $\otimes = \mathcal{O}_\Gamma$



“tree-level” $\mathcal{O}(g^0)$ contribution



“one-loop” $\mathcal{O}(g^2)$ contributions

$$\langle \mathcal{O}_{\mathbf{1}}^{\overline{\rm MS}}(x)\ \mathcal{O}_{\mathbf{1}}^{\overline{\rm MS}}(0)\rangle = \frac{N_c}{\pi^4\ (x^2)^3}\left[1+\frac{g_{\overline{\rm MS}}^2\ C_F}{16\pi^2}\left(2+6\ln(\bar\mu^2x^2)+12\gamma_E-12\ln(2)\right)+\ \mathcal{O}(g_{\overline{\rm MS}}^4)\right],$$

$$\langle \mathcal{O}_{\gamma_5}^{\overline{\rm MS}}(x)\ \mathcal{O}_{\gamma_5}^{\overline{\rm MS}}(0)\rangle = -\langle \mathcal{O}_{\mathbf{1}}^{\overline{\rm MS}}(x)\mathcal{O}_{\mathbf{1}}^{\overline{\rm MS}}(0)\rangle - c_{\rm HV}\ \frac{N_c}{\pi^4\ (x^2)^3}\left[16\ \frac{g_{\overline{\rm MS}}^2\ C_F}{16\pi^2}\ +\ \mathcal{O}(g_{\overline{\rm MS}}^4)\right],$$

$$\langle \mathcal{O}_{\gamma_\mu}^{\overline{\rm MS}}(x)\ \mathcal{O}_{\gamma_\nu}^{\overline{\rm MS}}(0)\rangle = -\frac{N_c}{\pi^4\ (x^2)^3}\left[\left(\delta_{\mu\nu}-2\frac{x_\mu\ x_\nu}{x^2}\right)\left(1+3\frac{g_{\overline{\rm MS}}^2\ C_F}{16\pi^2}\right)\right]+\ \mathcal{O}(g_{\overline{\rm MS}}^4),$$

$$\langle \mathcal{O}_{\gamma_5\gamma_\mu}^{\overline{\rm MS}}(x)\ \mathcal{O}_{\gamma_5\gamma_\nu}^{\overline{\rm MS}}(0)\rangle = \langle \mathcal{O}_{\gamma_\mu}^{\overline{\rm MS}}(x)\mathcal{O}_{\gamma_\nu}^{\overline{\rm MS}}(0)\rangle + c_{\rm HV}\ \frac{N_c}{\pi^4\ (x^2)^3}\left[\left(\delta_{\mu\nu}-2\frac{x_\mu\ x_\nu}{x^2}\right)8\frac{g_{\overline{\rm MS}}^2\ C_F}{16\pi^2}\ +\ \mathcal{O}(g_{\overline{\rm MS}}^4)\right],$$

$$\begin{aligned}\langle \mathcal{O}_{\sigma_{\mu\nu}}^{\overline{\rm MS}}(x)\ \mathcal{O}_{\sigma_{\rho\sigma}}^{\overline{\rm MS}}(0)\rangle \quad = \quad & -\frac{N_c}{\pi^4\ (x^2)^3}\Big[(\delta_{\mu\rho}\ \delta_{\nu\sigma}-\delta_{\mu\sigma}\ \delta_{\nu\rho})- \\ & 2\ (\delta_{\mu\rho}\frac{x_\nu\ x_\sigma}{x^2}-\delta_{\mu\sigma}\frac{x_\nu\ x_\rho}{x^2}-\delta_{\nu\rho}\ \frac{x_\mu\ x_\sigma}{x^2}+\delta_{\nu\sigma}\frac{x_\mu\ x_\rho}{x^2})\Big]\times \\ & \left[1+\frac{g_{\overline{\rm MS}}^2\ C_F}{16\pi^2}\left(6-2\ln(\bar\mu^2x^2)-4\gamma_E+4\ln(2)\right)+\ \mathcal{O}(g_{\overline{\rm MS}}^4)\right],\end{aligned}$$

where $c_{\rm HV}=0$ (1) for NDR (HV) prescription of γ_5.

Application of GIRS to fermion bilinears

- **Conversion to $\overline{\text{MS}}$:**

$$C_\Gamma^{\text{GIRS},\overline{\text{MS}}} \equiv \frac{Z_\Gamma^{\text{DR},\overline{\text{MS}}}}{Z_\Gamma^{\text{DR,GIRS}}} = \frac{Z_\Gamma^{\text{L},\overline{\text{MS}}}}{Z_\Gamma^{\text{L,GIRS}}}.$$

- **Conversion factors from t-GIRS to $\overline{\text{MS}}$:**

Nonzero contributions for $\Gamma = \{1,\ \gamma_5,\ \gamma_i,\ \gamma_5\gamma_i,\ \sigma_{4i},\ \sigma_{ij}\}$

$$\begin{aligned}
C_{\mathcal{O}_{\mathbf{1}}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2\ C_F}{16\pi^2}\left(-\frac{1}{2} + 6\ln(\bar\mu\bar t) + 6\gamma_E\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4),\\
C_{\mathcal{O}_{\gamma 5}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2\ C_F}{16\pi^2}\left(-\frac{1}{2} + 6\ln(\bar\mu\bar t) + 6\gamma_E + 8\,c_{\text{HV}}\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4),\\
C_{\mathcal{O}_{\gamma i}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2\ C_F}{16\pi^2}\left(\frac{3}{2}\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4),\\
C_{\mathcal{O}_{\gamma 5\,\gamma i}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2\ C_F}{16\pi^2}\left(\frac{3}{2} + 4\,c_{\text{HV}}\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4),\\
C_{\mathcal{O}_{\sigma 4i}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2 C_F}{16\pi^2}\left(\frac{25}{6} - 2\ln(\bar\mu\bar t) - 2\gamma_E\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4),\\
C_{\mathcal{O}_{\sigma ij}}^{\text{t-GIRS},\overline{\text{MS}}} &= 1 + \frac{g_{\overline{\text{MS}}}^2\ C_F}{16\pi^2}\left(\frac{25}{6} - 2\ln(\bar\mu\bar t) - 2\gamma_E\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4).
\end{aligned}$$

Energy-Momentum Tensor (EMT)

- **Gauge-invariant symmetric energy-momentum tensor (EMT) of QCD:** Decomposition into traceless and trace parts [X. Ji, PRD 52 (1995) 271]

$$T_{\mu\nu}(x) = \overline{T}^G_{\mu\nu}(x) + \overline{T}^F_{\mu\nu}(x) + \hat{T}^G_{\mu\nu}(x) + \hat{T}^F_{\mu\nu}(x),$$

where $\overline{T}^G_{\mu\nu}(x) = F_{\{\mu\rho}F_{\nu\}\rho}$, $\quad \overline{T}^F_{\mu\nu}(x) = \sum_f \bar{\psi}_f \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} \psi_f$,

$$\hat{T}^G_{\mu\nu}(x) = -\frac{2\beta(g)}{g}\frac{\delta_{\mu\nu}}{4} F_{\rho\sigma}F_{\rho\sigma}, \quad \hat{T}^F_{\mu\nu}(x) = (1+\gamma_m)\frac{\delta_{\mu\nu}}{4} m \sum_f \bar{\psi}_f \psi_f,$$

$\{\ldots\}$ = symmetrization over Lorentz indices μ, ν and subtraction of the trace.

- **Nucleon matrix elements of traceless EMT:** [X. Ji, J. Phys.G24, (1998) 1181]

$$\langle N(\vec{p}', s')|\overline{T}^{G(F)}_{\mu\nu}|N(\vec{p}, s)\rangle =$$

$$\overline{u}_N(\vec{p}', s')\left[A^{G(F)}_{20}(q^2)\; i\gamma_{\{\mu}P_{\nu\}} + B^{G(F)}_{20}(q^2)\frac{iP_{\{\mu}\sigma_{\nu\}\rho}q_\rho}{2m_N} + C^{G(F)}_{20}(q^2)\frac{q_{\{\mu}q_{\nu\}}}{m_N}\right]u_N(\vec{p}, s),$$

where $P \equiv (p' + p)/2$, $q \equiv p' - p$.

$\langle x\rangle^{G(F)} = A^{G(F)}_{20}(0)$, **average momentum fraction**

$J = \frac{1}{2}\left[A^G_{20}(0) + A^F_{20}(0) + B^G_{20}(0) + B^F_{20}(0)\right]$ **nucleon spin**

Renormalization of EMT

- **Set of mixing operators in the case of traceless EMT:** [S. Caracciolo, P. Menotti, A. Pelissetto, NPB 375 (1992) 195]:

$$
\begin{aligned}
\mathcal{O}_{1\,\mu\nu} &= \overline{T}^{G}_{\mu\nu} = F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{d}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}, \\
\mathcal{O}_{2\,\mu\nu} &= \overline{T}^{F}_{\mu\nu} = \sum_f \left[\frac{1}{2}\left(\bar{\psi}_f \gamma_\mu \overleftrightarrow{D}_\nu \psi_f + \bar{\psi}_f \gamma_\nu \overleftrightarrow{D}_\mu \psi_f \right) - \frac{1}{d}\delta_{\mu\nu}\left(\bar{\psi}_f \gamma_\rho \overleftrightarrow{D}_\rho \psi_f \right) \right], \\
\mathcal{O}_{3\,\mu\nu} &= \frac{1}{\alpha}\left[\left(\partial_\mu A^a_\nu + \partial_\nu A^a_\mu\right)\partial_\rho A^a_\rho - \frac{2}{d}\delta_{\mu\nu}\partial_\rho A^a_\rho \partial_\sigma A^a_\sigma \right] \\
&\quad - \left[\bar{c}^a \partial_\mu (D_\nu c)^a + \bar{c}^a \partial_\nu (D_\mu c)^a - \frac{2}{d}\delta_{\mu\nu}\bar{c}^a \partial_\rho (D_\rho c)^a \right], \\
\mathcal{O}_{4\,\mu\nu} &= -\frac{1}{\alpha}\left[\left(A^a_\mu \partial_\nu + A^a_\nu \partial_\mu\right)\left(\partial_\rho A^a_\rho\right) - \frac{2}{d}\delta_{\mu\nu}A^a_\rho \partial_\rho \partial_\sigma A^a_\sigma \right] \\
&\quad + \left[\partial_\mu \bar{c}^a D_\nu c^a + \partial_\nu \bar{c}^a D_\mu c^a - \frac{2}{d}\delta_{\mu\nu}\partial_\rho \bar{c}^a D_\rho c^a \right], \\
\mathcal{O}_{5\,\mu\nu} &= A^a_\mu \frac{\delta S}{\delta A^a_\nu} + A^a_\nu \frac{\delta S}{\delta A^a_\mu} - \frac{2}{d}\delta_{\mu\nu}A^a_\rho \frac{\delta S}{\delta A^a_\rho}.
\end{aligned}
$$

- **Previous calculations based on RI/MOM schemes:**
  Partial solution: Gauge-variant operators ($\mathcal{O}_{3\,\mu\nu}, \mathcal{O}_{4\,\mu\nu}, \mathcal{O}_{5\,\mu\nu}$) are neglected or included using lattice perturbation theory

Application of GIRS to EMT

- **Green's functions in GIRS:**
  We consider on-shell Green's functions of gauge-invariant operators in coordinate space (similar to the application of GIRS to the fermion bilinears):

$$\text{E.g.,} \quad \langle \overline{T}^G_{\mu\nu}(x) \overline{T}^F_{\rho\sigma}(y) \rangle, \quad (x \neq y)$$

- **Mixing matrix:** (off-diagonal elements $\mu \neq \nu$)
  Gauge-variant operators do not contribute in such Green's functions.

$$\begin{pmatrix} \bar{T}^{G,R}_{\mu\nu} \\ \bar{T}^{F,R}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{GG} & Z_{GF} \\ Z_{FG} & Z_{FF} \end{pmatrix} \begin{pmatrix} \bar{T}^{G}_{\mu\nu} \\ \bar{T}^{F}_{\mu\nu} \end{pmatrix}$$

- **Renormalization conditions:**
  - 3 conditions from two-point functions: $(\mu \neq \nu,\ \rho \neq \sigma)$

$$\begin{aligned}
\langle \overline{T}^{G\ \mathrm{GIRS}}_{\mu\nu}(x) \overline{T}^{G\ \mathrm{GIRS}}_{\rho\sigma}(0) \rangle|_{x=\bar{x}} &= \langle \overline{T}^{G}_{\mu\nu}(x) \overline{T}^{G}_{\rho\sigma}(0) \rangle^{\mathrm{tree}}|_{x=\bar{x}}, \\
\langle \overline{T}^{F\ \mathrm{GIRS}}_{\mu\nu}(x) \overline{T}^{F\ \mathrm{GIRS}}_{\rho\sigma}(0) \rangle|_{x=\bar{x}} &= \langle \overline{T}^{F}_{\mu\nu}(x) \overline{T}^{F}_{\rho\sigma}(0) \rangle^{\mathrm{tree}}|_{x=\bar{x}}, \\
\langle \overline{T}^{G\ \mathrm{GIRS}}_{\mu\nu}(x) \overline{T}^{F\ \mathrm{GIRS}}_{\rho\sigma}(0) \rangle|_{x=\bar{x}} &= \langle \overline{T}^{G}_{\mu\nu}(x) \overline{T}^{F}_{\rho\sigma}(0) \rangle^{\mathrm{tree}}|_{x=\bar{x}} = 0.
\end{aligned}$$

  - A 4th condition necessarily from three-point functions: $(\mu \neq \nu)$
    E.g.,

$$\langle \mathcal{O}^{\mathrm{GIRS}}_{\Gamma}(x) \overline{T}^{G\ \mathrm{GIRS}}_{\mu\nu}(0) \mathcal{O}^{\mathrm{GIRS}}_{\Gamma'}(-x) \rangle|_{x=\bar{x}} = \langle \mathcal{O}_{\Gamma}(x) \overline{T}^{G}_{\mu\nu}(0) \mathcal{O}_{\Gamma'}(-x) \rangle^{\mathrm{tree}}|_{x=\bar{x}} = 0.$$

Application of GIRS to EMT

- **Solution to the system of renormalization conditions:**

$$Z^{\rm L,GIRS}_{GG} = \sqrt{\frac{\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^G_{\rho\sigma}(0)\rangle^{\rm tree}}{\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^G_{\rho\sigma}(0)\rangle - 2\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle R_1 + \langle \overline{T}^F_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle R_1^2}},$$

$$Z^{\rm L,GIRS}_{GF} = -Z^{\rm L,GIRS}_{GG} R_1,$$

$$Z^{\rm L,GIRS}_{FG} = -Z^{\rm L,GIRS}_{FF} R_2,$$

$$Z^{\rm L,GIRS}_{FF} = \sqrt{\frac{\langle \overline{T}^F_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle^{\rm tree}}{\langle \overline{T}^F_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle - 2\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle R_2 + \langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^G_{\rho\sigma}(0)\rangle R_2^2}},$$

where $R_1 = \frac{\langle \mathcal{O}_\Gamma(\bar{x})\overline{T}^G_{\mu\nu}(0)\mathcal{O}_{\Gamma'}(-\bar{x})\rangle}{\langle \mathcal{O}_\Gamma(\bar{x})\overline{T}^F_{\mu\nu}(0)\mathcal{O}_{\Gamma'}(-\bar{x})\rangle}$, $R_2 = \frac{\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle - R_1\langle \overline{T}^F_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle}{\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^G_{\rho\sigma}(0)\rangle - R_1\langle \overline{T}^G_{\mu\nu}(\bar{x})\overline{T}^F_{\rho\sigma}(0)\rangle}$.

- **Conditions in t-GIRS:**
  Nonzero contributions for $\mu = \rho = i$, $\nu = \sigma = j$, $i \neq j$ and e.g., $\Gamma = \gamma_i$, $\Gamma' = \gamma_j$

$$\int d^3\vec{x}\ \langle \overline{T}^{G\,\text{t-GIRS}}_{ij}(\vec{x},t)\overline{T}^{G\,\text{t-GIRS}}_{ij}(\vec{0},0)\rangle|_{t=\bar{t}} = \text{tree},$$

$$\int d^3\vec{x}\ \langle \overline{T}^{F\,\text{t-GIRS}}_{ij}(\vec{x},t)\overline{T}^{F\,\text{t-GIRS}}_{ij}(\vec{0},0)\rangle|_{t=\bar{t}} = \text{tree},$$

$$\int d^3\vec{x}\ \langle \overline{T}^{G\,\text{t-GIRS}}_{ij}(\vec{x},t)\overline{T}^{F\,\text{t-GIRS}}_{ij}(\vec{0},0)\rangle|_{t=\bar{t}} = \text{tree},$$

$$\int d^3\vec{x}\ \langle \mathcal{O}^{\text{t-GIRS}}_{\gamma_i}(\vec{x},t)\overline{T}^{G\,\text{t-GIRS}}_{ij}(\vec{0},0)\mathcal{O}^{\text{t-GIRS}}_{\gamma_j}(-\vec{x},-t)\rangle|_{t=\bar{t}} = \text{tree}.$$

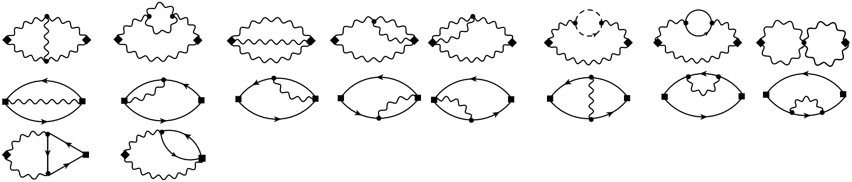
Application of GIRS to EMT

- **Continuum calculation (DR):** Two-point functions

"Tree-level" $\mathcal{O}(g^0)$ contributions:

◆ = $\overline{T}^G_{\mu\nu}$, ■ = $\overline{T}^F_{\mu\nu}$

"One-loop" $\mathcal{O}(g^2)$ contributions:



$$\langle \overline{T}^{G\,\overline{\rm MS}}_{\mu\nu}(x)\overline{T}^{G\,\overline{\rm MS}}_{\rho\sigma}(0)\rangle = \frac{2(N_c^2-1)}{\pi^4(x^2)^4}\, s_{\mu\nu\rho\sigma}(x)\left[1-\frac{g^2_{\overline{\rm MS}}}{16\pi^2}\left(\frac{4N_f}{3}\left(\frac{1}{6}+\ln(\bar\mu^2x^2)+2\gamma_E-2\ln(2)\right)+\frac{20N_c}{9}\right)+\mathcal{O}(g^4_{\overline{\rm MS}})\right],$$

$$\langle \overline{T}^{F\,\overline{\rm MS}}_{\mu\nu}(x)\overline{T}^{F\,\overline{\rm MS}}_{\rho\sigma}(0)\rangle = \frac{N_cN_f}{\pi^4(x^2)^4}\, s_{\mu\nu\rho\sigma}(x)\left[1-\frac{g^2_{\overline{\rm MS}}}{16\pi^2}\,\frac{16C_F}{3}\left(-\frac{59}{48}+\ln(\bar\mu^2x^2)+2\gamma_E-2\ln(2)\right)+\mathcal{O}(g^4_{\overline{\rm MS}})\right],$$

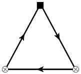
$$\langle \overline{T}^{G\,\overline{\rm MS}}_{\mu\nu}(x)\overline{T}^{F\,\overline{\rm MS}}_{\rho\sigma}(0)\rangle = \frac{N_cN_f}{\pi^4(x^2)^4}\, s_{\mu\nu\rho\sigma}(x)\left[\frac{g^2_{\overline{\rm MS}}}{16\pi^2}\,\frac{16C_F}{3}\left(-\frac{1}{6}+\ln(\bar\mu^2x^2)+2\gamma_E-2\ln(2)\right)+\mathcal{O}(g^4_{\overline{\rm MS}})\right],$$

where $s_{\mu\nu\rho\sigma}(x)=(\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\rho})+8\,\frac{x_\mu x_\nu x_\rho x_\sigma}{(x^2)^2}-2\left(\delta_{\mu\rho}\frac{x_\nu\, x_\sigma}{x^2}+\delta_{\mu\sigma}\frac{x_\nu\, x_\rho}{x^2}+\delta_{\nu\rho}\frac{x_\mu\, x_\sigma}{x^2}+\delta_{\nu\sigma}\frac{x_\mu\, x_\rho}{x^2}\right).$

Application of GIRS to EMT

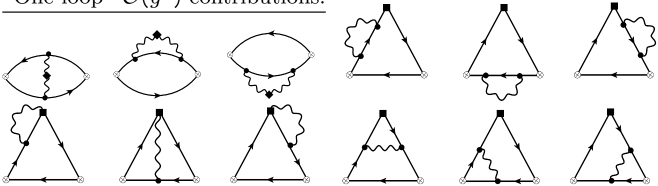
- **Continuum calculation (DR):** Three-point functions

"Tree-level" $\mathcal{O}(g^0)$ contribution:



◆ = $\overline{T}^G_{\mu\nu}$, ■ = $\overline{T}^F_{\mu\nu}$, ⊗ = $\mathcal{O}_\Gamma$

"One-loop" $\mathcal{O}(g^2)$ contributions:



Diagrams having the arrows of the fermion lines in counterclockwise direction must also be considered.

$$\langle \mathcal{O}^{\overline{\rm MS}}_{\gamma\rho}(x)\overline{T}^{G\ \overline{\rm MS}}_{\mu\nu}(0)\mathcal{O}_{\gamma\sigma}^{\ \overline{\rm MS}}(-x)\rangle = \frac{N_c N_f}{4\pi^6 (x^2)^5}\frac{g^2_{\overline{\rm MS}}}{16\pi^2}\frac{8C_F}{3}\Big\{\Big(2s^{[1]}_{\mu\nu\rho\sigma}-8s^{[2]}_{\mu\nu\rho\sigma}+s^{[3]}_{\mu\nu\rho\sigma}\Big)\Big(-1.701491+\ln(\bar\mu^2x^2)\Big) + \frac{1}{2}s^{[1]}_{\mu\nu\rho\sigma} + \frac{3}{4}s^{[4]}_{\mu\nu\rho\sigma} - s^{[3]}_{\mu\nu\rho\sigma} + \mathcal{O}(g^4_{\overline{\rm MS}})\Big\},$$

$$\langle \mathcal{O}^{\overline{\rm MS}}_{\gamma\rho}(x)\overline{T}^{F\ \overline{\rm MS}}_{\mu\nu}(0)\mathcal{O}_{\gamma\sigma}^{\ \overline{\rm MS}}(-x)\rangle = \frac{N_c N_f}{4\pi^6 (x^2)^5}\Big\{\Big(2s^{[1]}_{\mu\nu\rho\sigma}-8s^{[2]}_{\mu\nu\rho\sigma}+s^{[3]}_{\mu\nu\rho\sigma}\Big)\Big[1-\frac{g^2_{\overline{\rm MS}}}{16\pi^2}\frac{8C_F}{3}\Big(-3.201491+\ln(\bar\mu^2x^2)\Big)\Big] - \frac{g^2_{\overline{\rm MS}}}{16\pi^2}\frac{8C_F}{3}\Big[\frac{11}{4}s^{[1]}_{\mu\nu\rho\sigma} + \frac{9}{8}s^{[4]}_{\mu\nu\rho\sigma} - s^{[3]}_{\mu\nu\rho\sigma}\Big] + \mathcal{O}(g^4_{\overline{\rm MS}})\Big\},$$

where $s^{[1]}_{\mu\nu\rho\sigma} = \frac{x_\mu x_\nu}{x^2}\delta_{\rho\sigma}$, $s^{[2]}_{\mu\nu\rho\sigma} = \frac{x_\mu x_\nu x_\rho x_\sigma}{(x^2)^2}$, $s^{[3]}_{\mu\nu\rho\sigma} = \delta_{\mu\rho}\frac{x_\nu\ x_\sigma}{x^2}+\delta_{\mu\sigma}\frac{x_\nu\ x_\rho}{x^2}+\delta_{\nu\rho}\frac{x_\mu\ x_\sigma}{x^2}+\delta_{\nu\sigma}\frac{x_\mu\ x_\rho}{x^2}$,
$s^{[4]}_{\mu\nu\rho\sigma} = \delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\rho}$. (see our paper for other choices of bilinear operators)

Application of GIRS to EMT

- **Conversion to $\overline{\mathrm{MS}}$:**

$$\begin{pmatrix} C_{GG}^{\mathrm{GIRS},\overline{\mathrm{MS}}} & C_{GF}^{\mathrm{GIRS},\overline{\mathrm{MS}}} \\ C_{FG}^{\mathrm{GIRS},\overline{\mathrm{MS}}} & C_{FF}^{\mathrm{GIRS},\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{GG}^{\mathrm{DR},\overline{\mathrm{MS}}} & Z_{GF}^{\mathrm{DR},\overline{\mathrm{MS}}} \\ Z_{FG}^{\mathrm{DR},\overline{\mathrm{MS}}} & Z_{FF}^{\mathrm{DR},\overline{\mathrm{MS}}} \end{pmatrix} \cdot \begin{pmatrix} Z_{GG}^{\mathrm{DR,GIRS}} & Z_{GF}^{\mathrm{DR,GIRS}} \\ Z_{FG}^{\mathrm{DR,GIRS}} & Z_{FF}^{\mathrm{DR,GIRS}} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} Z_{GG}^{\mathrm{L},\overline{\mathrm{MS}}} & Z_{GF}^{\mathrm{L},\overline{\mathrm{MS}}} \\ Z_{FG}^{\mathrm{L},\overline{\mathrm{MS}}} & Z_{FF}^{\mathrm{L},\overline{\mathrm{MS}}} \end{pmatrix} \cdot \begin{pmatrix} Z_{GG}^{\mathrm{L,GIRS}} & Z_{GF}^{\mathrm{L,GIRS}} \\ Z_{FG}^{\mathrm{L,GIRS}} & Z_{FF}^{\mathrm{L,GIRS}} \end{pmatrix}^{-1}$$

- **Conversion factors from t-GIRS to $\overline{\mathrm{MS}}$:**

$$C_{GG}^{\mathrm{t-GIRS},\overline{\mathrm{MS}}} = 1 - \frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}\Big[\frac{10}{9}N_c + 0.236288 N_f + \frac{2}{3}N_f \ln(\bar{\mu}^2\bar{t}^2)\Big] + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

$$C_{GF}^{\mathrm{t-GIRS},\overline{\mathrm{MS}}} = -\frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}C_F\Big[-7.848365 - \frac{8}{3}\ln(\bar{\mu}^2\bar{t}^2)\Big] + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

$$C_{FG}^{\mathrm{t-GIRS},\overline{\mathrm{MS}}} = -\frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}N_f\Big[1.933961 - \frac{2}{3}\ln(\bar{\mu}^2\bar{t}^2)\Big] + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

$$C_{FF}^{\mathrm{t-GIRS},\overline{\mathrm{MS}}} = 1 - \frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}C_F\Big[-2.777072 + \frac{8}{3}\ln(\bar{\mu}^2\bar{t}^2)\Big] + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4).$$

Conclusions and future prospects

- The application of GIRS in the renormalization of fermion bilinears and of traceless EMT is being studied.

- Extensions of GIRS (integration over timeslices) are being explored.

- One-loop conversion factors between different variants of GIRS and $\overline{\text{MS}}$ are extracted, via two-loop computations.

- Studies of GIRS by lattice simulations are in progress.

- **Further applications of GIRS:**
  - **Gluino Glue** in Supersymmetric Yang-Mills Theory on the lattice: **see talk by M. Costa**
  - **Supercurrent** in Supersymmetric Yang-Mills Theory on the lattice: **see poster by A. Skouroupathis, I. Soler Calero**
  - **Trace part of EMT**: More complicated renormalization!

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THANK YOU!