Direct Measurement and Renormalisation of Quark and Gluon Momentum Fractions in the Quenched Approximation

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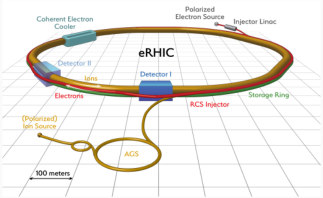
Direct Measurement of Glue

With upcoming experiments, such as eRHIC,
gluonic hadron structure is becoming more accessible.

Experiments are
best complimented by a strong theoretical foundation.

Lattice calculations of
gluonic quantities are typically impacted by statistical noise, due to entirely disconnected contributions.

Having many techniques for calculating gluon operator matrix elements may help.



https://www.bnl.gov/cad/eRhic/

Feynman–Hellmann Method

An indirect method for computing 3–point correlation functions.

Compute 2–point functions in the presence of a modification to the action:

$$S \to S_\lambda = S + \lambda \mathcal{O} \tag{1}$$

for some relevant operator $\mathcal{O}$.

Matrix elements are extracted from the λ dependence of the energy of hadrons.

Feynman–Hellmann Method on the Glue

When applying the method to extract a gluon operator, we modify the *ensemble weight factor*;

$$\exp(-S_g[U]) \rightarrow \exp(-S_g[U] - \lambda\mathcal{O}) \quad (2)$$

i.e. modification at the point of gauge ensemble generation.

Feynman–Hellmann Method: Measurements in This Work

Will discuss two distinct but related uses of the Feynman–Hellmann method

- Measurement of gluonic momentum fractions in hadrons,
- Determination of renormalisation factors of such momentum fractions.

Momentum Fractions

$\langle x \rangle_f$: total fraction of hadron momentum carried by parton f

Take the operator

$$\mathcal{O}_g^{(b)} = \frac{1}{3}\beta \sum_{x,i} \mathrm{Re}\,\mathrm{Tr}(1 - P_{i4}(x)) - \frac{1}{3}\beta \sum_{x,i<j} \mathrm{Re}\,\mathrm{Tr}(1 - P_{ij}(x)), \tag{3}$$

has matrix element:

$$\frac{\langle N(p)|\, \mathcal{O}_g^{(b)}\, |N(p)\rangle}{\langle N(p)|N(p)\rangle} = \frac{1}{2E}\left(2p_4^2 - \frac{2}{3}|\vec{p}|^2\right)\langle x \rangle_g . \tag{4}$$

Choose $\mathcal{O}^{(b)}$ so we may set $\vec{p} = 0$.

Momentum Fraction from Feynman–Hellmann

After some algebra from Feynman–Hellmann expression,

$$\langle x \rangle_g = \frac{4}{3aE_0} \frac{\partial(aE_0)}{\partial \lambda}\bigg|_{\lambda=0}, \tag{5}$$

for hadron ground state energy aE_0 in lattice units.

See [QCDSF-UKQCD, Phys.Lett.B(2012)] for details.

Shift in ground state energy encodes the matrix element.

Feynman–Hellmann Modification

Compare $\mathcal{O}_g^{(b)}$ and Wilson gauge action S_g:

$$\mathcal{O}_g^{(b)} = \frac{1}{3}\beta \sum_{x,i} \mathrm{Re}\,\mathrm{Tr}(1 - P_{i4}(x)) - \frac{1}{3}\beta \sum_{x,i<j} \mathrm{Re}\,\mathrm{Tr}(1 - P_{ij}(x)),$$

$$S_g = \frac{1}{3}\beta \sum_{x,i} \mathrm{Re}\,\mathrm{Tr}(1 - P_{i4}(x)) + \frac{1}{3}\beta \sum_{x,i<j} \mathrm{Re}\,\mathrm{Tr}(1 - P_{ij}(x)),$$

FH modification, $S_g + \lambda\mathcal{O}_g^{(b)}$, equivalent to anisotropy.

Details given in [QCDSF-UKQCD, Phys.Lett.B(2012)].

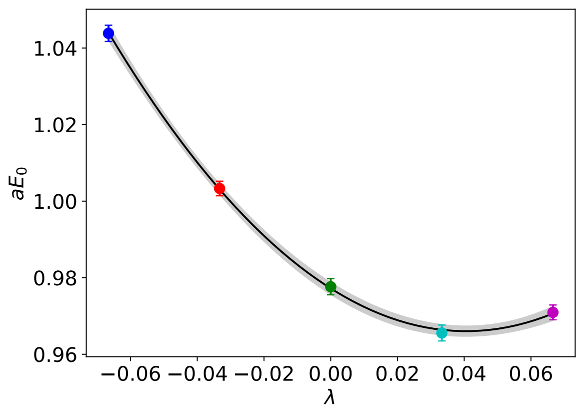
Gauge Field Configurations

- Lattice volume of $24^3 \times 48$,
- $\beta = 6.00$,
- standard Wilson action in the quenched approximation.

- 5 values of λ, at $-0.0666, -0.0333, 0, +0.0333, +0.0666$.
- 1000 configurations on each λ, $\implies$ 5000 configurations in total.

These gauge configurations will also be utilised in the calculation of the renormalisation factor in the next section.

Momentum Fraction - Results



Nucleon, aE_0 vs λ, at $am_\pi = 0.555$.

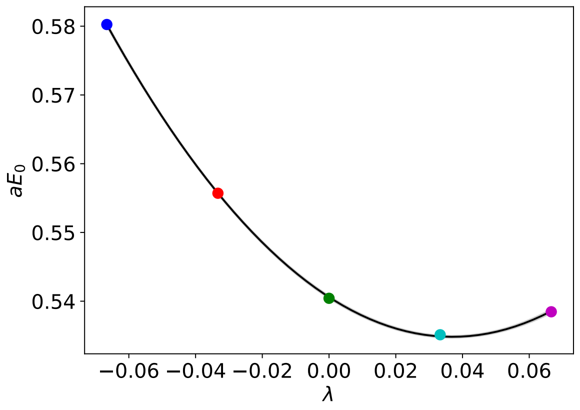
Nucleon:
$\langle x \rangle_g^{\mathrm{Lat}} = 0.751(26)$.

Data points are entirely uncorrelated, from separate ensembles.

Good agreement with quadratic fit, no significant cubic term.

Momentum Fraction - Results

Pion: $\langle x \rangle_g^{\mathrm{Lat}} = 0.7718(72)$.



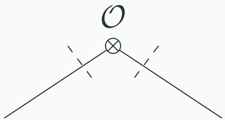
Pion, aE_0 vs λ, at $am_\pi = 0.555$.

Renormalisation Factor

Find a renormalisation factor for $\mathcal{O}_g^{(b)}$ from the same gauge configurations.

The amputated vertex function

$$\Gamma_{\mathcal{O}} = D^{-1} \ \langle\phi|\, \mathcal{O}\, |\phi\rangle\, D^{-1} = \tag{6}$$



Gauge dependent quantities, so we work in Landau gauge.

$RI - MOM$ scheme:

$$\Gamma_{\mathcal{O}}^{RI-MOM} = Z_{\{\text{field}\}}^{-1} Z_{\mathcal{O}} \Gamma_{\mathcal{O}}^{\text{Lat}}, \tag{7}$$

$$\text{Tr}\{\Gamma_{\mathcal{O}}^{RI-MOM}(p)[\Gamma_{\mathcal{O}}^{0}(p)]^{-1}\}\Big|_{p^2=\mu^2} = 1, \tag{8}$$

Feynman–Hellmann method

Extract 3–point function from gluon propagator:

$$\left.\frac{\partial}{\partial\lambda}D(p,\lambda)\right|_{\lambda=0} = \langle A(p)|\,\mathcal{O}^{(b)}\,|A(p)\rangle = \tag{9}$$

gg,

$$\implies \Gamma_{gg} = D^{-1}(p,0)\left.\left(\frac{\partial}{\partial\lambda}D(p,\lambda)\right)\right|_{\lambda=0} D^{-1}(p,0) \tag{10}$$

Avoiding Problematic Vertex Functions

Gluon Propagator and Vertex not invertible in Landau gauge

Problematic for $RI - MOM$ schemes, addressed in [ETM, Phys.Rev.D(2020)].

Find an alternative $RI - MOM$ condition.

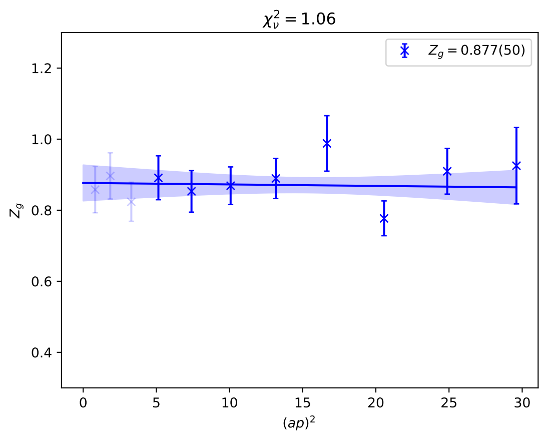
Look for something invertible, or at least doesn't vanish.

- [QCDSF-UKQCD-CSSM, PoS(2019)] gives some suggestions.
- Proposal here; restrict to space components,

$$Z_{gg} = \sum_{i,j=1}^{3} (\Gamma_{ij})_{g}^{\mathrm{Born}}(p)[(\Gamma_{ji})_{gg}^{\mathrm{Lat}}(p)]^{-1}.$$

Gluon Renormalisation Factor

Well behaved in p^2, good agreement with linear fit.



Z_g vs $(ap)^2$, along $p = k(1,1,1,0)$ diagonal.

Mixing

Under renormalisation, the quark and gluon sectors mix.

So

$$\mathcal{O}^{\mathrm{R}} = Z\mathcal{O}^{\mathrm{Lat}} \tag{11}$$

becomes

$$\begin{pmatrix} \mathcal{O}_q \\ \mathcal{O}_g \end{pmatrix}^{\mathrm{R}} = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} \mathcal{O}_q \\ \mathcal{O}_g \end{pmatrix}^{\mathrm{Lat}}, \tag{12}$$

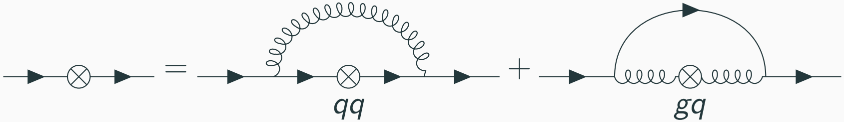
$$Z_q = Z_{qq} + Z_{gq}, \quad \text{and} \quad Z_g = Z_{qg} + Z_{gg}. \tag{13}$$

Even when quenched, glue is present in quark contributions,

so $Z_{qg} = 0$, but $Z_{gq} \neq 0$.

Quark Renormalisation Factor

In the quark sector, account for mixing from gluon operator.



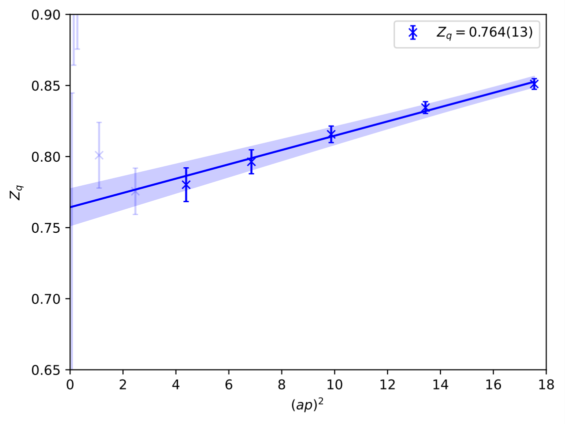
(14)

$$Z_q^{-1} = \mathrm{Tr}\{\Gamma_{qq}^{\mathrm{Lat}}(p)[Z_2^{-1}\Gamma_q^{\mathrm{Born}}(p) - Z_{gg}\Gamma_{gq}^{\mathrm{Lat}}(p)]^{-1}\}, \tag{15}$$

with

$$\Gamma_q^{\mathrm{Born}} = p_4\gamma_4 - \frac{1}{3}\sum_{i=1}^{3} p_i\gamma_i. \tag{16}$$

Z_q



Z_q vs $(ap)^2$

Renormalisation Matrix Components

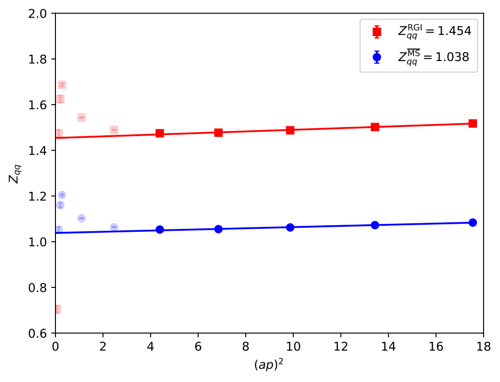
Use conventional renormalisation scheme to define diagonal matrix components:

$$Z_{qq}^{-1} = \frac{1}{Z_2} \mathrm{Tr}\{\Gamma_{qq}^{\mathrm{Lat}}(p)[\Gamma_{qq}^{\mathrm{Born}}(p)]^{-1}\} \tag{17}$$

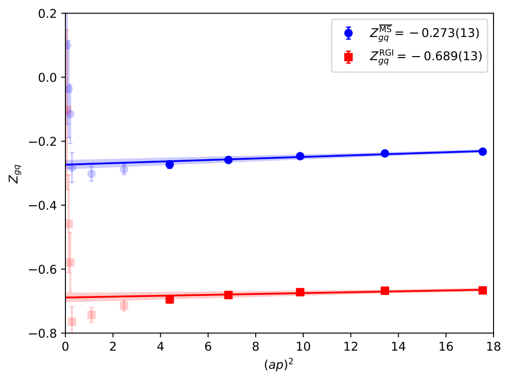
and

$$Z_{gq} = Z_q - Z_{qq}. \tag{18}$$

Renormalisation Matrix Components: Z_{qq} & Z_{gq}



Z_{qq} vs $(ap)^2$



Z_{gq} vs $(ap)^2$

Sum Rule

Expect renormalised fractions to sum to unity.

Using prev results for $\langle x \rangle_g^{\mathrm{Lat}}$,

and $\langle x \rangle_q^{\mathrm{Lat}}$ from established 3–point correlator method (as outlined in [QCDSF, Phys.Rev.D(2005)]),

	$\langle x \rangle_q^{\overline{\mathrm{MS}}}$	$\langle x \rangle_g^{\overline{\mathrm{MS}}}$	Sum
Nucleon	0.603(45)	0.500(39)	1.103(53)
Pion	0.477(10)	0.551(35)	1.028(36)

Preliminary results are sensible.

Summary

Having multiple methods of investigation for statistically troublesome quantities makes for more versatile and informative analysis.

The Feynman–Hellmann method provides a viable option for direct calculation of gluonic operator matrix elements.

With this renormalisation factor, further investigations into this operator are available.

Applications of this method into more physically relevant simulations (e.g. dynamical quarks) are of interest.

C. Alexandrou et al. "Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass". In: *Physical Review D* 101.9 (May 2020), p. 094513. ISSN: 2470-0010. DOI: 10.1103/PhysRevD.101.094513. URL: https://link.aps.org/doi/10.1103/PhysRevD.101.094513.

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R. Horsley et al. "A lattice study of the glue in the nucleon". In: *Physics Letters B* 714.2-5 (Aug. 2012), pp. 312–316. ISSN: 03702693. DOI: 10.1016/j.physletb.2012.07.004. arXiv: 1205.6410. URL: https://linkinghub.elsevier.com/retrieve/pii/S0370269312007447.

R. Horsley et al. "Determining the glue component of the nucleon". In: *Proceedings of Science* 363.June 2019 (2019), pp. 0–6. ISSN: 18248039. DOI: 10.22323/1.363.0220. arXiv: 2001.07639.