Lattice QCD calculation of the proton electromagnetic polarizability

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Outline

- Background
- Polarizabilities and Compton Scattering
- Lattice Calculation of Proton 4-point Function
- To Do List

Backgroud

Motivation

- Electromagneitc polarizabilities are fundamental parameters to describe hadron inner structure.
- Important input parameters for nuclear physics.
- Characterize the second-order response of a proton to an intense electromagnetic field.

Our Method

- Calculating a proton 4-point correlation function on lattice, can be used in multiple applications.

See Yang Fu's talk at 5:00 am today.

Background

- Thomson Scattering: photon scattering with charged particles, cross section independent of wave length.
- Rayleigh Scattering: photon scattering with polarizable particles, cross section $\sigma \propto \lambda^{-4}$.

Non-relativistic quantum machanics discription of Compton scattering,[1]

$$t_{CS} = \vec{\varepsilon}'^{*} \cdot \vec{\varepsilon}(-\frac{Q^2}{M} + \alpha_E E_\gamma E'_\gamma) + \beta_M(\vec{q}' \times \vec{\varepsilon}'^{*}) \cdot (\vec{q} \times \vec{\varepsilon}) \tag{1}$$

In this work, we will extract α_E and β_M from calculating a lattice 4-point correlation function.

[1] Ericson T , J Hüfner. Low-frequency photon scattering by nuclei[J]. Nuclear Physics, 2016, 57(2):604-616.

Parameterization of Compton Scattering

In quatum field theory, the Compton scattering amplitude,

$$T_{VCS} = \epsilon_\mu \epsilon_\nu^{'*} T^{\mu\nu}$$
$$T^{\mu\nu} = i \int d^4 x e^{-iq\cdot x} \langle P(p_f) | \hat{T}[J^\mu(x) J^\nu(0)] | P(p_i) \rangle \tag{2}$$

Separating Born and non-Born terms of Compton tensor, we can parameterize the non-Born part with scalor functions B_i,

$$\begin{aligned} T^{\mu\nu} = & \, T^{\mu\nu}_{Born} + (q^2 - q^\mu q^\nu) B_1(q^2) \\ & + [(pq)^2 g^{\mu\nu} - (pq)(q^\mu p^\nu + p^\mu q^\nu) + q^2 p^\mu p^\nu] B_2(q^2) + \mathcal{O}(q^2) \end{aligned} \tag{3}$$

Comparing with Eq.(1), relations to α_E and β_M are,

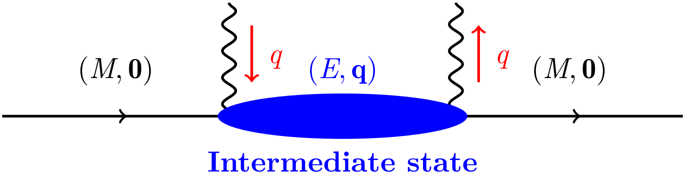
$$\begin{aligned} 2M\alpha_E &= -B_1(0) - M^2 B_2(0) \\ 2M\beta_M &= B_1(0) \end{aligned} \tag{4}$$

Separating Born and Non-Born Terms

Compton tensor calculated on lattice,

$$T^L_{\mu\nu} = \sum_{T,V} e^{-iQ\cdot x} \underbrace{\left\langle P(p_f) \right| \hat{T}[J_\mu(x) J_\nu(0)] \left| P(p_i) \right\rangle}_{H^L_{\mu\nu}(x)} \\ = \underbrace{T^{GS}_{\mu\nu} + \Delta T^{Born}_{\mu\nu}}_{Born\ term(T^{Born}_{\mu\nu})} + T^{NB}_{\mu\nu} \tag{5}$$

- Born term($T^{Born}_{\mu\nu}$): Proton state
- Ground state($T^{GS}_{\mu\nu}$): On-shell proton state.
- Non-Born term($T^{NB}_{\mu\nu}$): Excited states



Off-shell Contribution in Born Term

To extract polarizabilities, we need only the non-Born Compton tensor $T^{NB}_{\mu\nu}$.

- T^{GS}: Long distance contribution, estimate with lattice data at large t_s.
- $\Delta T^{Born} = T^{Born} - T^{GS}$: Short distance contribution, dependence on q^2 can be parameterized with constants.

Let $q = (0, \vec{\xi})$, electronic polarizability can be extracted with,

$$
\begin{aligned}
2M\alpha_E &= \frac{\partial(T^L_{00} - T^{GS}_{00})}{\partial|\vec{\xi}|^2} - \frac{\partial(\Delta T^{Born}_{00})}{\partial|\vec{\xi}|^2}\Bigg|_{\xi\to 0} \\
&= \alpha_{em}\frac{2+\kappa^2}{2M^2} + \int d^4x(-\frac{1}{6}\vec{x}^2)(H^L_{00}(x) - H^{GS}_{00}(x))
\end{aligned}
\tag{6}
$$

Ensemble for Calculation

Ensemble	m_π[MeV]	m_p[MeV]	L/a	T/a	a[fm]	N_{conf}
24D	141.7(2)	935(5)	24	64	0.1944	124

- The calculation is based on Domain Wall Fermion ensemble generated by RBC/UKQCD[1].
- 1024 propagators per configuration: 16 sources per time slice.
- Random field selection method[2] is used, 864 sinks per time slice.

[1] Blum T et al. Domain wall QCD with physical quark masses[J]. Physical Review D, 2016, 93(7):074505.

[2] Li Y , Xia S C , Feng X , et al. Field sparsening for the construction of the correlation functions in lattice QCD[J]. 2021.

Lattice Calculation of Proton 4-point Function

Proton Compton tensor is obtained by calculating a proton 4-point function on lattice,

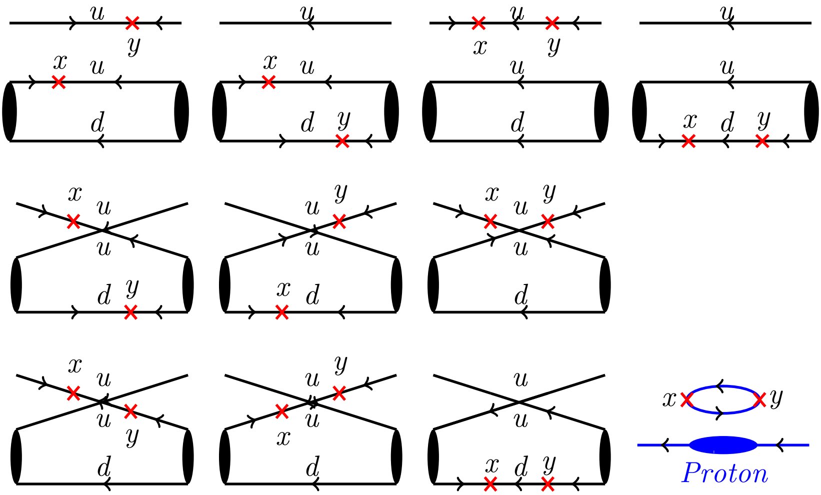
$$H^L_{\mu\nu}(t,\vec{x}) = L^3 \frac{\left\langle N(t+\Delta t) J_\mu(t,\vec{x}) J_\nu(0) \bar{N}(-\Delta t) \right\rangle_L}{\left\langle N(t+\Delta t) \bar{N}(-\Delta t) \right\rangle_L} \tag{7}$$

With proton interpolator,

$$N(x) = \varepsilon_{abc} P_+ u_a(x) \left(u_b^T(x) C\gamma_5 d_c(x) \right) \tag{8}$$

- Currently, we choose $\Delta t = 2a$.
- Connected diagrams only.
- 10 diagrams are involved.

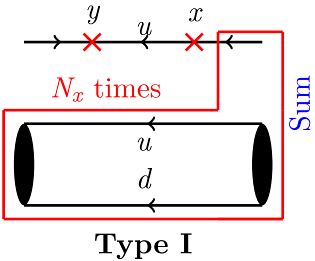
Feymann Diagrams in Proton 4-point Correlator

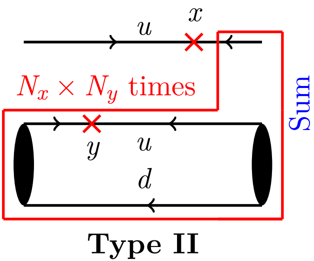


*disconnected diagrams excluded for our preliminary results

Calculating Procedure of Proton 4-point Function

- The calculation of the diagrams requires a contraction of 5 propagators.





$$N_x = 16, N_y = 864$$

- Computating time for a Type II diagrams is about 16 times longer than Type I.
- Currently, 1/16 statistics is calculated for Type II diagrams.

Fitting the Elastic Term

Recall the eqution for extracting α_E

$$2M\alpha_E = \alpha_{em}\frac{2+\kappa^2}{2M^2} + \int d^4x(-\frac{1}{6}\vec{x}^2)(H_{00}^L(x) - H_{00}^{GS}(x)) \qquad (9)$$

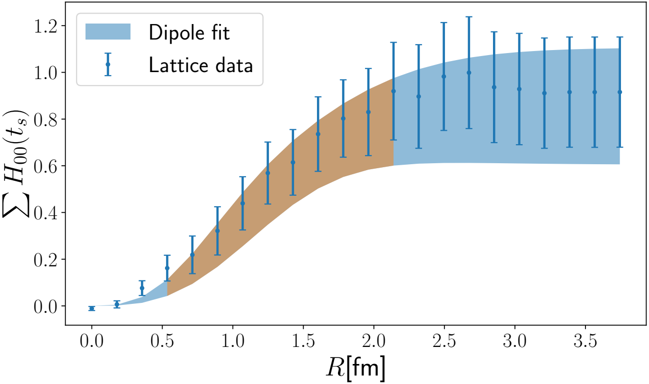
H_{00}^{GS} can be estimated with a dipole model of form factor,

$$G_E(Q^2) = \frac{G_E(0)}{(1+\frac{r_E^2 Q^2}{12})^2} \qquad (10)$$

At a large enough time slice t_s, we fit $G_E(0)$ and r_E in G_E with

$$\sum_{|\mathbf{x}|<R} H_{00}^{GS}(t_s,\mathbf{x}) = \frac{1}{\pi}\int_{-\infty}^{+\infty} dQ \frac{M}{E}(E+M) G_E^2(Q^2) e^{-(E-M)t_s} \times \left(-R\cos QR + \frac{\sin QR}{Q}\right) \qquad (11)$$

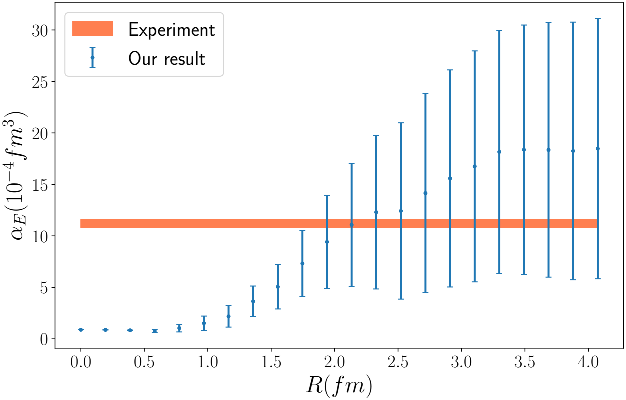
Fitting Result for Ground State



$$G_E(Q^2) = \frac{G_E(0)}{(1 + r_E^2 Q^2/12)^2}$$

	$G_E(0)$	r_E[fm]
$t_s = 4a$	0.92(14)	0.84(27)
Physical	1	0.8409(4)

Preliminary Results on Extracting α_E



- Ground state contribution fitted at $t_s = 4a$.
- Should check with $t_s = 5a$, but is too noisy.
- Our result: $\alpha_E \approx 18(13) \times 10^{-4} fm^3$
- Experiment: $\alpha_E \approx 11.2(0.4) \times 10^{-4} fm^3$.

To Do List

- Improve our fitting method, reduce error induced in the ground state contribution.
- β_M requires more efforts on estimating the ground state contribution, will be performed afterwards.
- Put our calculation on a larger 64×32^3 lattice. Improve the accuracy.

Thanks!