Variations to the z-Expansion of the Form Factor Describing the Decay of B Mesons

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Introduction

- Using differential decay data collected from the Belle Collaboration to study the parameterizations of the form factor describing the decay: $B^0 \to D^- \ell^+ \nu_\ell$
(R. Glattauer et al. (Belle Collaboration), PRD 93 (2016))

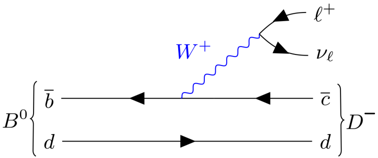


Figure: Edited copy from E. Waheed et al., PRD 100 (2019)

- Form factor is a function of hadronic recoil w

$$w = \frac{P_B \cdot P_D}{m_B m_D} = \frac{m_B^2 + m_D^2 - q^2}{2 m_B m_D}$$

Goals

- Looking at the fits in the lattice regime
  - Hadronic recoil: $w \lesssim 1.24$
  - Useful for Monte Carlo Simulations (Lattice QCD)
- Which parameterization of the form factor produces the best results?
- How many data points are needed to include in the fit to accurately predict the remaining data?
- How well do lattice-QCD calculation results fit this data?
- Looking at the χ^2 values of my fits
  - χ^2_{fitted} and $\chi^2_{predicted}$

Theory: Decay Rate

- The differential decay rate describing this decay (R. Glattauer et. al, PRD 93, 032006 (2016))

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2(w^2 - 1)^{3/2}\eta_{EW}^2|V_{cb}|^2|G(w)|^2$$

- We considered 3 parameterizations of the form factor $G(w)$
  1. BGL (uses z-expansion)
  2. BCL (uses z-expansion)
  3. CLN

Theory: BGL

- Boyd, Grinstein and Lebed (BGL) Parameterization (R. Glattauer et. al, PRD 93, 032006 (2016)):

$$G(w)^2 = \frac{4r}{(1+r)^2} f_+(w)^2$$

$$f_+(z) = \frac{1}{\phi_+(z)} \sum_{n=0}^{N} a_{+,n} z^n$$

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \; ; \; r = \frac{m_D}{m_B}$$

$$\phi_+(z) = 1.1213(1+z)^2(1-z)^{1/2}[(1+r)(1-z) + 2\sqrt{r}(1+z)]^{-5}$$

Theory: BCL

- Bourrely, Caprini, and Lellouch (BCL) Parameterization (C. Bourrely, I. Caprini, and L. Lellouch, PRD 79, 013008 (2010)):

$$G(w)^2 = \frac{4r}{(1+r)^2} f_+(w)^2$$

$$f_+(z) = \frac{1}{P_+(z)} \sum_{n=o}^{N-1} b_{+,n} \left(z^n - (-1)^{n-N} \frac{n}{N} z^N \right)$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \; ; \; r = \frac{m_D}{m_B}$$

$$P_+(z) = 1 - \frac{q^2(z)}{m_{B_c^*}^2}$$

Theory: BCL Continued

- BCL Parameterization:

$$t_{+} = (m_B + m_D)^2$$

$$t_0 = (m_B + m_D)\left(\sqrt{m_B} - \sqrt{m_D}\right)^2$$

- Using an estimated value for $m_{B_c^*}$ from (Q. Li et. al, PRD 99, 096020 (2019))

$$m_{B_c^*} = 6.326 \text{ GeV}$$

Theory: CLN

- Caprini, Lellouch, and Neubert (CLN) Parameterization (R. Glattauer et. al, PRD 93, 032006 (2016)):

$$G(z) = G(1)\left(1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3\right)$$

- Fixed number of fit parameters: $G(1)$ and ρ^2

Results

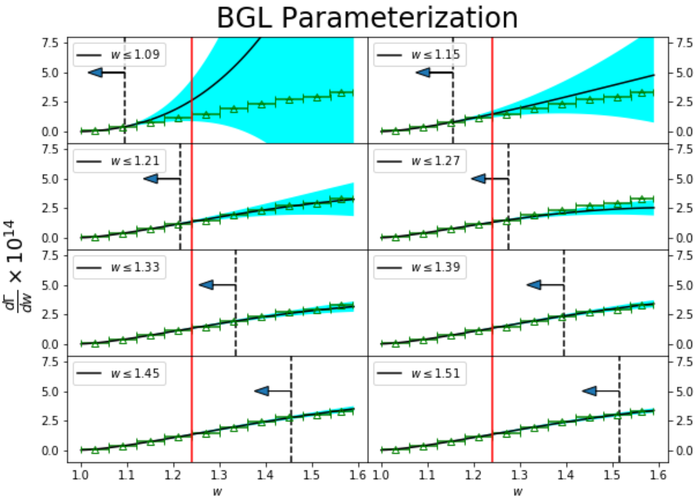
- Using Least-Square-Fitting methods, we fit the free parameters of the different parameterizations:
  1. BGL: $a_{+,0}$ and $a_{+,1}$
  2. BCL: $b_{+,0}$ and $b_{+,1}$
  3. CLN: $G(1)$ and ρ^2
- Using the $\chi^2_{reduced}$ values from our fits to determine the accuracy of the fit and its predictiveness
  - $\chi^2_{reduced,fitted}$ and $\chi^2_{reduced,predicted}$

Results: BGL

$$f_{+,BGL}(z) = \frac{1}{\phi_+(z)} \sum_{n=0}^{N} a_{+,n} z^n$$

BGL $\chi^2_{reduced}$ Values			
w-range for fit	# of points used in fit	$\chi^2_{reduced,fitted}$	$\chi^2_{reduced,predicted}$
$w \leq 1.09$	2	—	2844.9767
$w \leq 1.15$	3	0.5923	20.3533
$w \leq 1.21$	4	0.4018	0.4906
$w \leq 1.27$	5	0.3761	10.3272
$w \leq 1.33$	6	0.3944	1.2176
$w \leq 1.39$	7	0.3793	0.1660
$w \leq 1.45$	8	0.3434	0.3218
$w \leq 1.51$	9	0.3243	0.3300

Results: BGL Continued

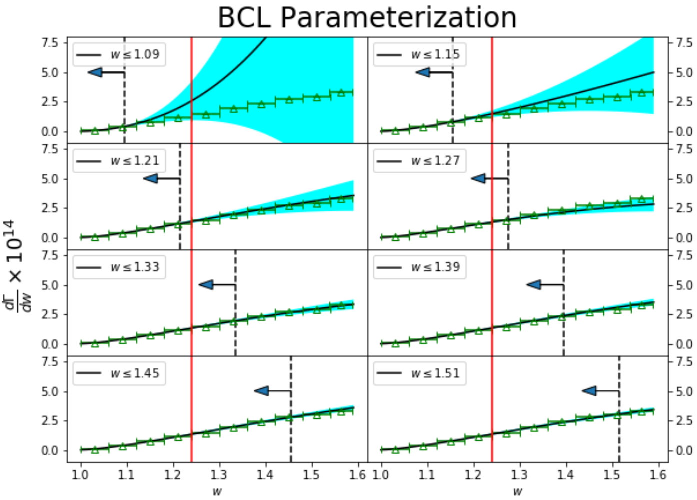


Results: BCL

$$f_{+,BCL}(z) = \frac{1}{P_+(z)} \sum_{n=o}^{N-1} b_{+,n} \left(z^n - (-1)^{n-N} \frac{n}{N} z^N \right)$$

BCL $\chi^2_{reduced}$ Values			
w-range for fit	# of points used in fit	$\chi^2_{reduced,fitted}$	$\chi^2_{reduced,predicted}$
$w \leq 1.09$	2	—	2137.4863
$w \leq 1.15$	3	0.5965	25.9358
$w \leq 1.21$	4	0.4167	0.4507
$w \leq 1.27$	5	0.4116	5.6754
$w \leq 1.33$	6	0.3908	0.4200
$w \leq 1.39$	7	0.3543	0.2445
$w \leq 1.45$	8	0.3071	0.6166
$w \leq 1.51$	9	0.3264	0.1194

Results: BCL Continued

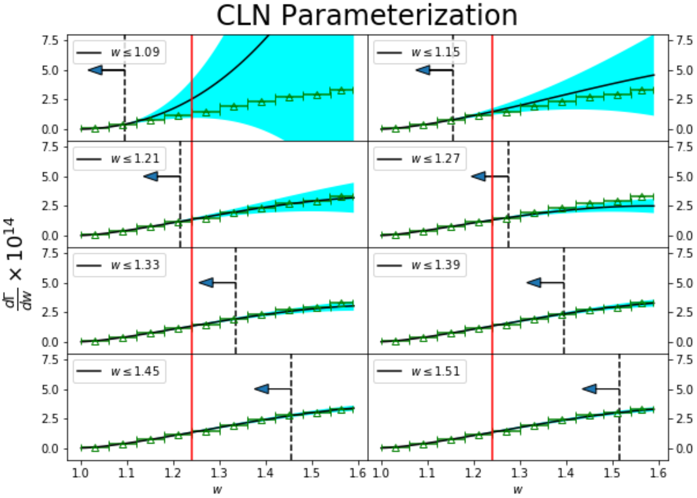


Results: CLN

$$G_{CLN}(z) = G(1)\left(1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3\right)$$

CLN $\chi^2_{reduced}$ Values			
w-range for fit	# of points used in fit	$\chi^2_{reduced,fitted}$	$\chi^2_{reduced,predicted}$
$w \leq 1.09$	2	—	1934.0241
$w \leq 1.15$	3	0.5954	17.8212
$w \leq 1.21$	4	0.4161	0.5545
$w \leq 1.27$	5	0.4060	10.1728
$w \leq 1.33$	6	0.4020	1.8833
$w \leq 1.39$	7	0.3874	0.3951
$w \leq 1.45$	8	0.3627	0.2266
$w \leq 1.51$	9	0.3210	0.7370

Results: CLN Continued



Using Results from Lattice Calculations

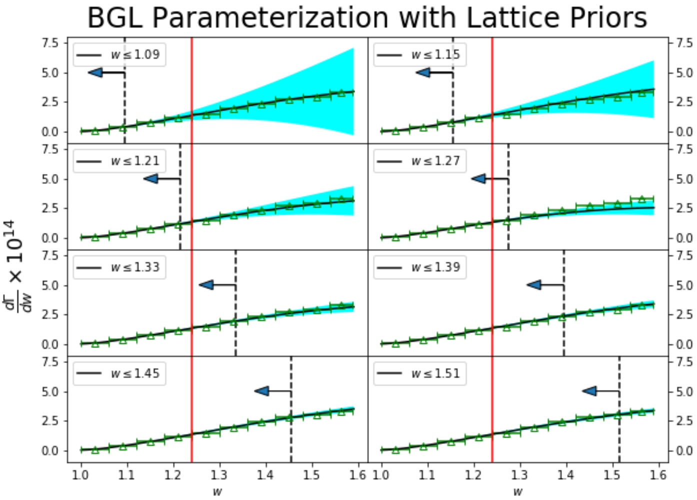
- Using the results of Lattice-QCD calculations for the free parameters of the BGL parameterization given from (J. A. Bailey and et. al, PRD 92, 014024 (2015))
  - $a_{+,0}$ and $a_{+,1}$
- Using these free parameters as priors for python least square fitting function
- Uncertainties in their parameters are so low, the fit is over-constrained
- Scaled the uncertainties up by a factor of 30 (still very small)

Results: BGL with Lattice Data

$$f_{+,BGL}(z) = \frac{1}{\phi_+(z)} \sum_{n=0}^{N} a_{+,n} z^n$$

BGL $\chi^2_{reduced}$ Values with Lattice Priors			
w-range for fit	# of points used in fit	$\chi^2_{reduced,fitted}$	$\chi^2_{reduced,predicted}$
$w \leq 1.09$	2	—	3.2982
$w \leq 1.15$	3	0.8136	0.6562
$w \leq 1.21$	4	0.4254	1.6925
$w \leq 1.27$	5	0.3734	10.1269
$w \leq 1.33$	6	0.3921	1.5060
$w \leq 1.39$	7	0.3720	0.1885
$w \leq 1.45$	8	0.3353	0.3046
$w \leq 1.51$	9	0.3186	0.3312

Results: BGL with Lattice Data Continued



Conclusion

w-range for fit	BGL $\chi^2_{reduced,predicted}$	BGL $\chi^2_{reduced,predicted}$ with Lattice Data	BCL $\chi^2_{reduced,predicted}$
$w \leq 1.15$	20.3533	0.6562	25.9358
$w \leq 1.21$	0.4906	1.6925	0.4507
$w \leq 1.27$	10.3272	10.1269	5.6754
$w \leq 1.33$	1.2176	1.5060	0.4200

- BCL has primarily not been used to study the decay $B^0 \to D^- \ell^+ \nu_\ell$, but it appears to slightly outperform BGL in the lattice regime
- The $\chi^2_{reduced,predicted}$ spike for $w \leq 1.27$ warrants further investigation to determine minimum number of points for accurate fit
- The results of lattice-QCD calculations appear to also fit this data well

The End

Thank you!

Questions?