ν N inclusive scattering cross sections on the lattice

Jun-Sik Yoo[1] Hiroshi Ohki[2] Shoji Hashimoto[1,3]

[1]Institute of Particle and Nuclear Studies, KEK junsik@post.kek.jp

[2]Dept.of Physics, Nara Women's University

[3]The Graduate University for Advanced Studies (SOKENDAI)

July 29, 2021

Introduction

lepton-Nucleon scattering

- quasielastic, resonance, deep inelastic scatterings
- Various final states $X = N, N\pi, \Delta$, ...

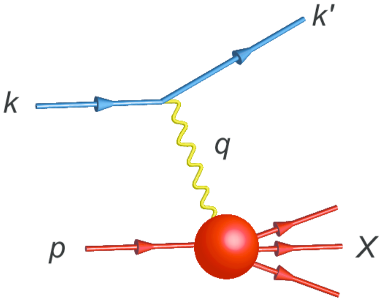


Figure 1: lepton-Nucleon scattering

Introduction

Obstacles to compute the inclusive scattering

- High-energy: Factorization not obvious
- Intermediate energy: Multiparticle states
- Low energy: inverse Laplace problem

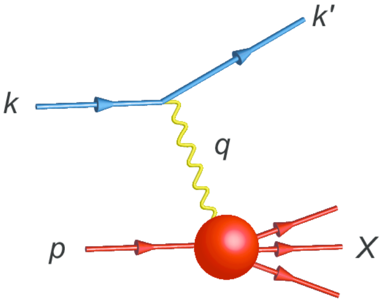
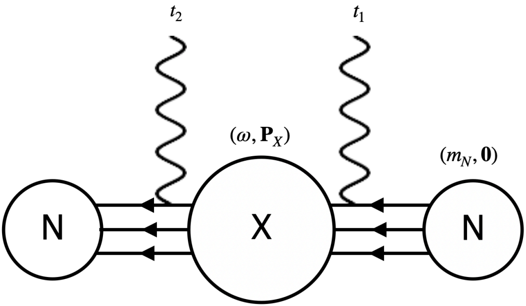


Figure 2: lepton-Nucleon scattering

Introduction

A new method to compute the inclusive scattering amplitude[Fuk+20; Lia+20].

- Consider the energy integrals, which come from the kinematical factor of the ℓN scattering – circumvent the inverse Laplace problem



$$\rho(\omega, \mathbf{P}_X) = \sum_{X(\mathbf{P}_X)} \delta(\omega - E_{X(\mathbf{P}_X)}) |\langle X(\mathbf{P}_X) | J | N \rangle|^2$$

Figure 3: Spectral function

Chebyshev Approximation

Shfted Chebyshev Polynomial $T_j^*(x), j = 1, ..., N$

- recurrent relation

$$T_0^*(x) = 1,\ T_1^*(x) = 2x - 1,\ T_{j+1}^* = 2(2x-1)\,T_j^*(x) - T_{j-1}^*(x) \tag{1}$$

- Approximation of the kinematical factor writes:

$$K(\omega, \mathbf{q}) \simeq \frac{c_0^*(\mathbf{q})}{2} + \sum_{j=1}^{N} c_j^*(\mathbf{q})\,T_j^*(z), \tag{2}$$

where $z = e^{-\omega}$ and $c_j^*(\mathbf{q})$

Then the total cross section reduces to:

$$\sigma(E) = \int_0^{E^2} d\mathbf{q}^2 \int d\omega K(\omega; \mathbf{q})\rho(\omega, \mathbf{q}) \tag{3}$$

Contractions

4pt-function with two current insertion is computed.

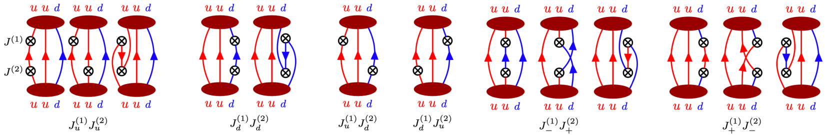


Figure 4: 4pt-functio contractions for (a) neutral currents (b) charged current

The contraction of 4pt-function is computed

- with momenta configs [0,0,0],[0,0,1],[0,1,1]

Lattice Setups

2+1 flavor Domain wall fermion generated by RBC/UKQCD [All+07]

Item	Value
Size	$16^3 \times 32$
gauge	Iwasaki
fermion	Domain wall fermion
lattice spacing	$a = 0.12$fm ($a^{-1} = 1.62(5)\,GeV$)
m_π	377 MeV
$m_\pi L$	3.7
Z_A	0.7162(2)
AMA	1 exact per config.
CG residual	exact(10^{-9})
# of configs.	100

Table 1: Lattice ensemble

The decomposition of nucleon form factors for axial and vector current is as follows.

$$\langle N(p_f)|J^V_\mu|N(p_i)\rangle = \bar{u}(p_f)\left(\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right)u(p_i) \qquad (4)$$

$$\langle N(p_f)|J^A_\mu|N(p_i)\rangle = \bar{u}(p_f)\left(\gamma_\mu\gamma_5 G_A(q^2) + \frac{q_\mu}{m_N}G_P(q^2)\right)\gamma_5 u(p_i) \qquad (5)$$

We use the model calculation of nucleon form factor, and compute it to our reference. [Bra+06].

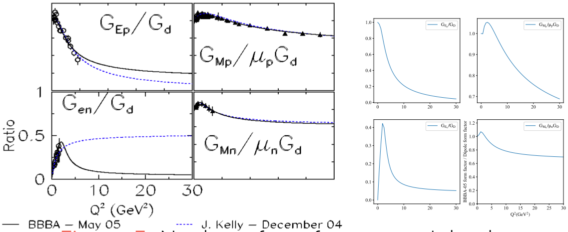


Figure 5: Nucleon form factor model values.

Hadronic tensors

$$H_{\mu\nu} = \langle N | J_\mu(-\mathbf{q}, t_2) J_\nu(\mathbf{q}; t_1) | N \rangle \qquad (6)$$

We project this Hadronic tensors to the certain direction and see how it decays by time separation.
$\texttt{ppH} = \hat{\mathbb{p}}^\mu \hat{\mathbb{p}}^\nu H_{\mu\nu}$
$\texttt{nnH} = \hat{\mathbb{n}}^\mu \hat{\mathbb{n}}^\nu H_{\mu\nu}$,
where $\hat{\mathbb{p}} = [1, 0, 0, 0], \hat{\mathbb{n}}$ a unit vector s.t. $\hat{\mathbb{n}} \perp \hat{\mathbb{p}}, \hat{\mathbb{n}} \perp \mathbb{q}$.

Put a nucleon source at $t_{src} = 0$, put sink at $t_{snk} = 8$, and the first current at the $t_1 = 2$.

ppH results

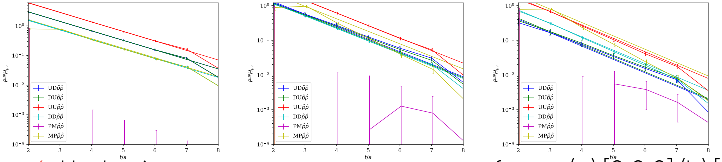


Figure 6: Hadronic tensors vector-vector current for p = (a) [0 0 0] (b) [0 0 1] (c) [0 1 1] .

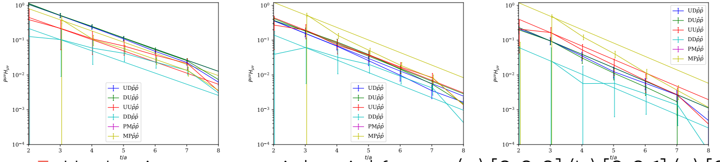


Figure 7: Hadronic tensors axial-axial for p = (a) [0 0 0] (b) [0 0 1] (c) [0 1 1] .

Lattice computations
Hadronic Tensors

`nnH` results

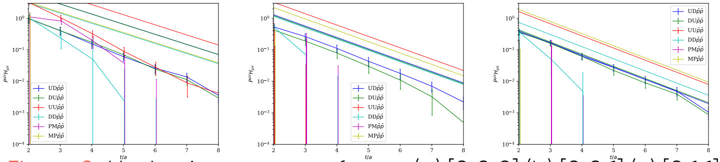


Figure 8: Hadronic tensors pp for p = (a) [0 0 0] (b) [0 0 1] (c) [0 1 1] .

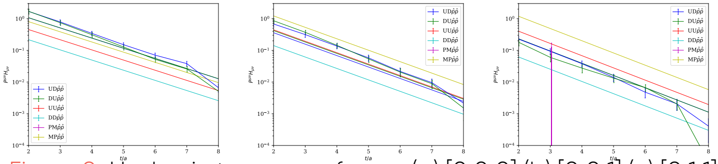


Figure 9: Hadronic tensors nn for p = (a) [0 0 0] (b) [0 0 1] (c) [0 1 1] .

Lattice computations
Inclusive total cross sections

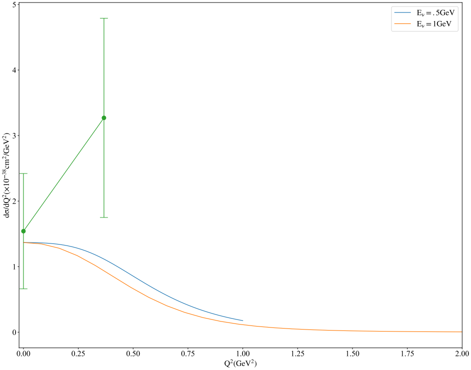


Figure 10: The total cross section of

Conclusion

- The first attempt to compute the ℓN inclusive scattering cross section.
  Four point function contractions are worked out, obtained a numerical result on a small lattice, for νN.
- So far, νN[NC] has been considered. Should look at the charged and EM currents.
- Careful test of the systematic errors to be performed

Appendix

Consider the perturbed Dirac operator due to the background field

$$\not{D} \longrightarrow \not{D} + \epsilon_1 \Gamma^{(1)} + \epsilon_2 \Gamma^{(2)} \tag{7}$$

then the two-point function, say,

$$C^{J_u^{(1)} J_u^{(2)}}_{2pt_c onn.} = N[\mathcal{F}^{\epsilon_1 \epsilon_2}, \mathcal{F}] \tag{8}$$

The numerical derivative of 2pt function converges to the 4pt-functions.

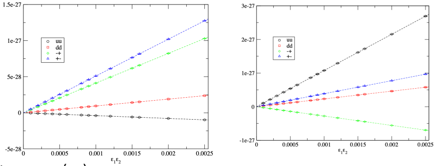


Figure 11: (a) Real and (b) Imaginary part for the perturbed c2pt result for different current insertions

References I

[All+07] C. Allton et al. "2+1 flavor domain wall QCD on a (2 fm)*83 lattice: Light meson spectroscopy with L(s) = 16". In: *Phys. Rev. D* 76 (2007), p. 014504. DOI: 10.1103/PhysRevD.76.014504. arXiv: hep-lat/0701013.

[Bra+06] R. Bradford, A. Bodek, Howard Scott Budd, and J. Arrington. "A New parameterization of the nucleon elastic form-factors". In: *Nucl. Phys. B Proc. Suppl.* 159 (2006). Ed. by F. Cavanna, J. G. Morfin, and T. Nakaya, pp. 127–132. DOI: 10.1016/j.nuclphysbps.2006.08.028. arXiv: hep-ex/0602017.

[Fuk+20] Hidenori Fukaya, Shoji Hashimoto, Takashi Kaneko, and Hiroshi Ohki. "Towards fully nonperturbative computations of inelastic ℓN scattering cross sections from lattice QCD". In: *Phys. Rev. D* 102.11 (2020), p. 114516. DOI: 10.1103/PhysRevD.102.114516. arXiv: 2010.01253 [hep-lat].

[Lia+20] Jian Liang, Terrence Draper, Keh-Fei Liu, Alexander Rothkopf, and Yi-Bo Yang. "Towards the nucleon hadronic tensor from lattice QCD". In: *Phys. Rev. D* 101.11 (2020), p. 114503. DOI: 10.1103/PhysRevD.101.114503. arXiv: 1906.05312 [hep-ph].