Charged and neutral pion magnetic polarisabilities using the background field method

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Energy & Two-point Correlation Functions

- Directly calculate hadron energies in an external magnetic field
- The energy of a pion in an external magnetic field is

$$E^2(B) = m_\pi + (2\,n + 1)\ |qe\,B| - 4\,\pi\,m_\pi\,\beta_\pi\ |B|^2 + \mathcal{O}\left(B^3\right)$$

- Periodic spatial boundary conditions impose a quantisation for a uniform field

$$a^2\,qe\,B^2 = \frac{2\,\pi\,k}{N_x\,N_y}$$

- $k_d = 0, 1, 2, \ldots$ for the field strength experienced by the d quark

Wilson Term Mass Renormalisation

- Wilson term causes unphysical quark mass renormalisation in background magnetic field
- In free-field limit this change is

$$m_{[w]}(B) = m(0) + \frac{a}{2}|qe\,B|$$

- First discussed by Bali *et al.* 1510.03899, 1707.05600

Background field corrected clover action

- Allow QCD and electromagnetic field strengths to have different clover coefficients

$$c_{cl} \to C_{SW}^{NP}\, F_{\mu\nu}^{QCD} + C_{EM}\, F_{\mu\nu}^{EM}$$

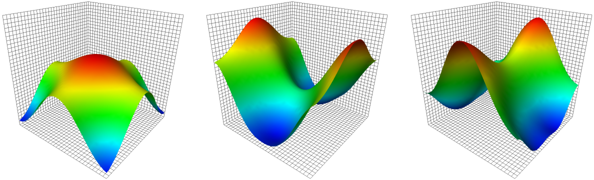
- and set C_{EM} such that Wilson Landau shift is cancelled

$$C_{EM} = C_{EM}^{Tree}$$

- Details may be found in our paper, 1910.14244

Quark Operators

- Standard lattice QCD interpolators are inefficient at isolating energy eigenstates in a background magnetic field
- Quarks are charged!
  - Quarks experience Landau type effects
  - QCD causes quarks to hadronise for composite Landau energy
- Competing effects, introduce a quark projection operator that includes QCD and QED



$SU(3) \times U(1)$ Projection Operator

- Two-dimensional lattice Laplacian operator

$$\Delta_{\vec{x},\vec{x}'} = 4\,\delta_{\vec{x},\vec{x}'} - \sum_{\mu=1,2} U_\mu\left(\vec{x}\right)\,\delta_{\vec{x}+\hat{\mu},\vec{x}'} + U_\mu^\dagger\left(\vec{x}-\hat{\mu}\right)\,\delta_{\vec{x}-\hat{\mu},\vec{x}'},$$

- Use low-lying eigenmodes of the 2D Laplacian $\left(\psi_{i\,\vec{B}}\right)$ to project the propagator

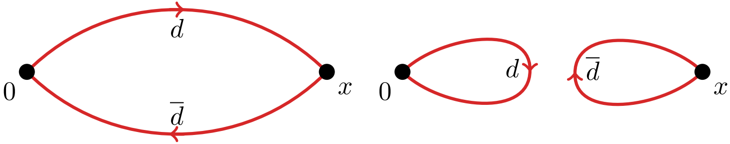
$$P_n\left(\vec{x},t;\vec{x}',t'\right) = \sum_{i=1}^{n} \left\langle \vec{x},t \,\middle|\, \psi_{i,\vec{B}} \right\rangle \left\langle \psi_{i\,\vec{B}} \,\middle|\, \vec{x}',t' \right\rangle \delta_{zz'}\,\delta_{tt'}$$

- Projected propagator is

$$S_n(\vec{x},t;\vec{0},0) = \sum_{\vec{x}'} P_n(\vec{x},t;\vec{x}',t)\,S(\vec{x}',t;\vec{0},0)$$

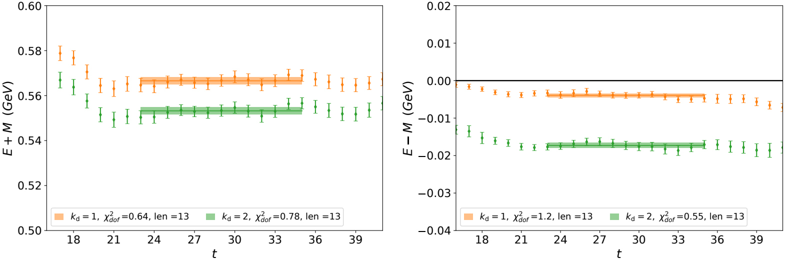
Neutral Pion

- Consider the neutral (connected pion) $\overline{q}\,\gamma_5\,q$
  - for $\overline{q}\,q = \overline{u}u = \pi^0_u$ and $\overline{d}d = \pi^0_d$



- This approach is common in the literature due to the expense of including disconnected contributions of the full neutral pion

π^0 Energy Shifts

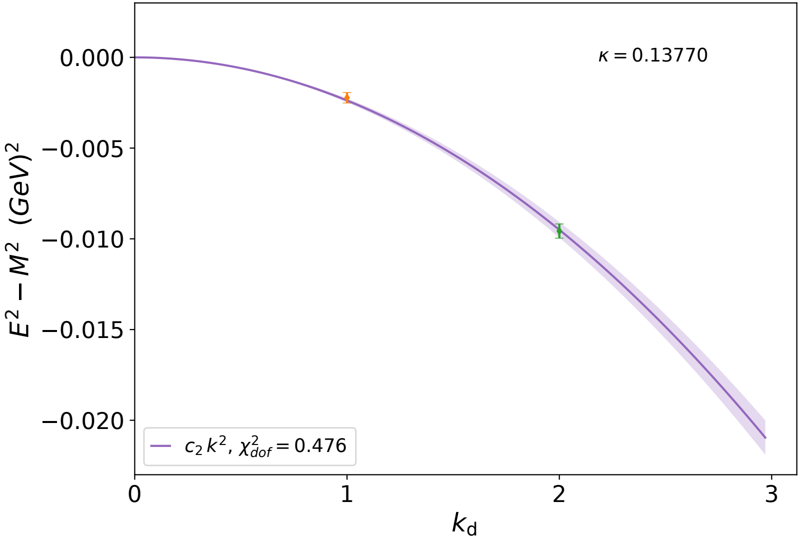


- Fit to these energy shifts $E^2(B) - m_\pi^2$

$$(E_{\pi^0}(B) - m_{\pi^0})(E_{\pi^0}(B) + m_{\pi^0}) = -4\pi\, m_\pi\, \beta_{\pi^0}\, B^2 = c_2\, k_d^2$$

- where k is the field quanta from background magnetic field quantisation condition

π^0 Polarisability Fitting

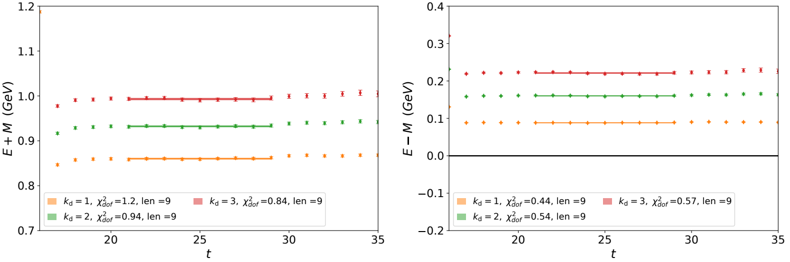


Hadronic Landau Projection

- Charged particles such as π^+ experience hadronic level Landau effects
- In external magnetic field along $\hat{z}$-axis
  - Energy eigenstates of π^+ are no longer eigenstates of p_x, p_y momentum components
- Hence project (x, y) dependence of correlator onto lowest Landau level

$$G\left(p_z, \vec{B}, t\right) = \sum_{\vec{r}} \psi_{\vec{B}}(x,\, y)\; \mathrm{e}^{-i\, p_z\, z}\; \langle\Omega|\, T\left\{\overline{\chi}(r, t)\; \chi(0)\right\} |\Omega\rangle$$

π^+ Energy Shift

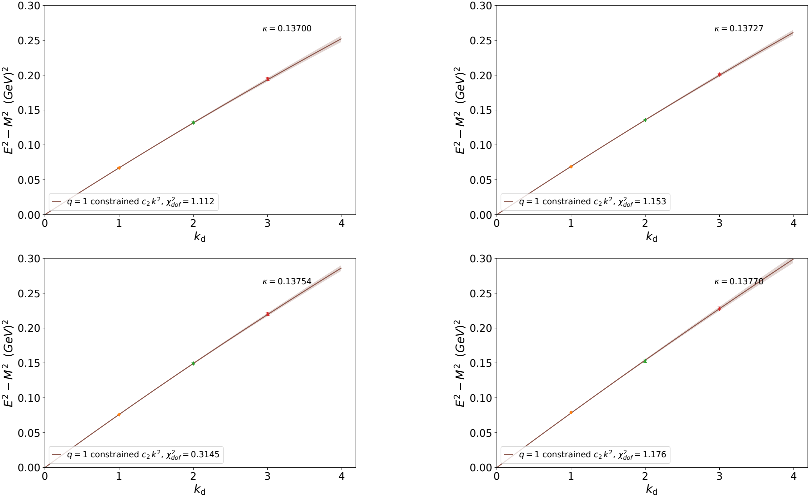


- Fit to these energy shifts $E^2(B) - m_\pi^2$

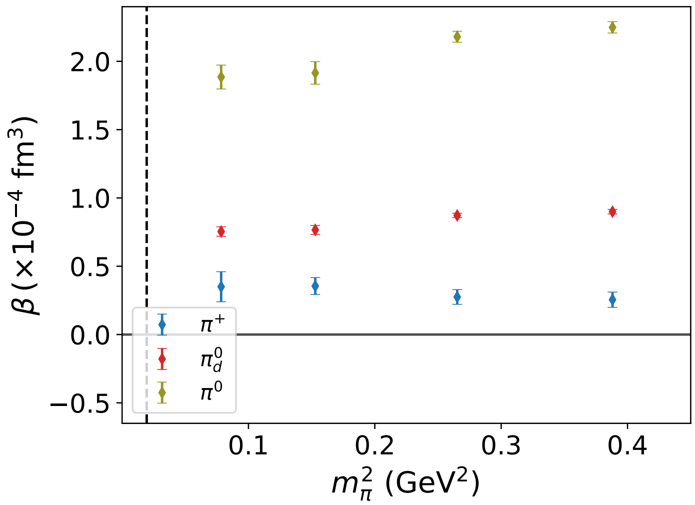
$$E^2(B) - m_\pi^2 - |qe\,B| = -4\,\pi\,m_\pi\,\beta_{\pi^+}\,B^2 = c_2\,k^2$$

- where k is the field quanta from background magnetic field quantisation condition

π^+ Polarisability Fits for four masses



π^0 and π^+ Polarisability



Connection to Physical

Chiral extrapolation of the charged-pion magnetic polarizability with Padé approximant

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The background magnetic-field formalism of Lattice QCD has been used recently to calculate the magnetic polarizability of the charged pion. These $n_f = 2 + 1$ numerical simulations are electro-quenched, such that the virtual sea-quarks of the QCD vacuum do not interact with the background field. To understand the impact of this, we draw on partially quenched chiral perturbation theory. In this case, the leading term proportional to $1/M_\pi$ arises at tree level from $\mathcal{L}_4$. To describe the results from lattice QCD, while maintaining the exact leading terms of chiral perturbation theory, we introduce a Padé approximant designed to reproduce the slow variation observed in the lattice QCD results. Two-loop contributions are introduced to assess the systematic uncertainty associated with higher-order terms of the expansion. Upon extrapolation, the magnetic polarizability of the charged pion at the physical pion mass is found to be $\beta_{\pi^\pm} = -1.70\,(14)_{\text{stat}}(25)_{\text{syst}} \times 10^{-4}$ fm^3, in good agreement with the recent experimental measurement.

Chiral extrapolation of the magnetic polarizability of the neutral pion

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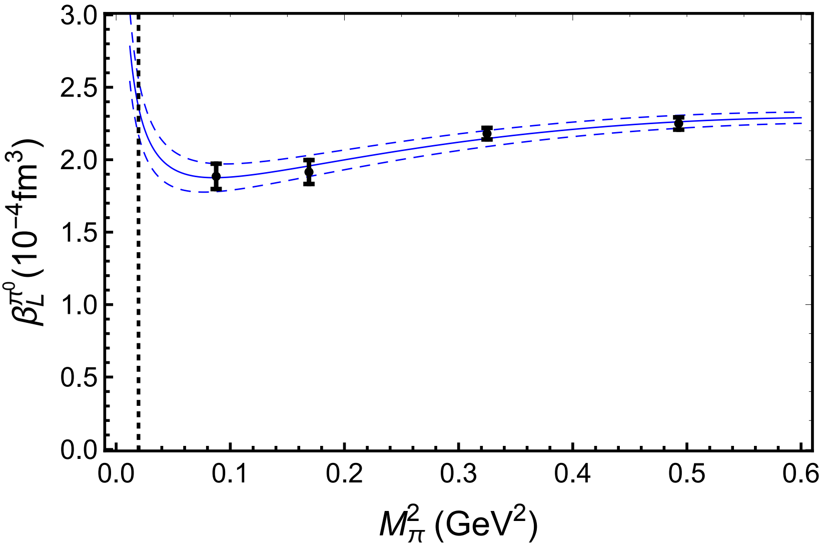
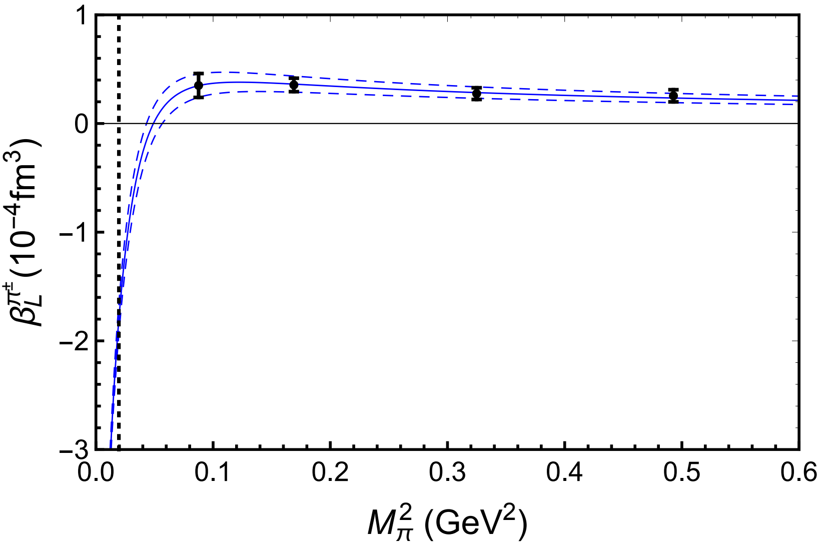
The magnetic polarizability of the neutral pion has been calculated in the background magnetic-field formalism of lattice QCD. In this investigation, the chiral extrapolation of these lattice results is considered in a formalism preserving the exact leading nonanalytic terms of chiral perturbation theory. The $n_f = 2 + 1$ numerical simulations are electro-quenched, such that the virtual sea quarks of the QCD vacuum do not interact with the background field. To understand the impact of this, we draw on partially quenched chiral perturbation theory and identify the leading contributions of quark-flow connected and disconnected diagrams. While electro-quenching does not impact the leading-loop contribution to the magnetic polarizability, the loops which generate the leading term have yet to be considered in lattice QCD simulations. Lattice QCD results are used to constrain the analytic terms in the chiral expansion and supplementing those with the two-loop result from chiral perturbation theory enables an evaluation of the polarizability at the physical quark mass. The resulting magnetic polarizability of the neutral pion is $\beta_{\pi^0} = 3.44(19)^{\text{stat}}(37)^{\text{syst}} \times 10^{-4}$ fm^3, which lies just above the 1σ error bound of the experimental measurement.

π^0 Extrapolation



π^+ Extrapolation

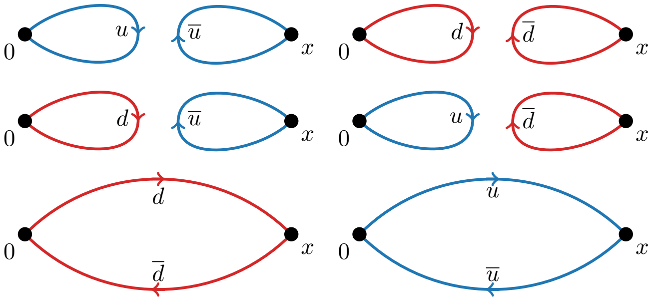


Disconnected Contributions to β^{π^0}

- Neutral pion operator is

$$\pi^0 = \frac{1}{\sqrt{2}} \left(u\overline{u} - d\overline{d} \right)$$

- Hence the following wick contractions are produced



Summary

- Resolved the additive mass renormalisation problem due to the Wilson term in a background magnetic field
- Specialised projection techniques have been used to account for Landau effects
  - Enabling energy shift plateaus
- Performed the first systematic exploration of the mass dependence of the magnetic polarisabilities of the π^0 and π^+ using lattice QCD 2005.10453
- Techniques are applicable to further elements of the hadronic spectrum 2002.07915

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Disconnected Methods in Lattice QCD

- Construct sets of random noise $\{\eta\}$ from $\mathbb{Z}_4$ such that

$$\left\langle \eta_{a\alpha}(x)\, \eta^{\dagger}_{b\beta}(y) \right\rangle = \delta_{xy}\, \delta_{ab}\, \delta_{\alpha\beta}$$

- Corresponding solution vectors are (where M is the fermion matrix)

$$\chi = M^{-1}\, \eta$$

- Hence fermion propagator is

$$S_{ab;\alpha\beta}(x, y) \simeq \left\langle \chi_{a\alpha}(x)\, \eta^{\dagger}_{b\beta}(y) \right\rangle$$

- This is an expensive and noisy process!
  - Necessary as standard point-to-all process for all x is infeasibly expensive

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Connected π^0 Polarisability

- In our mass degenerate lattice simulation

$$E^2_{\pi^0_u}(B/2) = E^2_{\pi^0_d}(B)$$

- and so, allowing $\overline{u}u$ and $\overline{d}d$ to have differing magnetic polarisabilities gives

$$\beta_{\pi^0_u} = 4\,\beta_{\pi^0_d}$$

- Estimate the magnetic polarisability of the full connected π^0 as the average of $\beta_{\pi^0_u}$ and $\beta_{\pi^0_d}$

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Results

Table: Magnetic polarisability values for the pion at each quark mass considered.

m_π (GeV)	a (fm)	β^{π^+} ($\times 10^{-4}$ fm^3)	$\beta^{\pi^0_d}$ ($\times 10^{-4}$ fm^3)	β^{π^0} ($\times 10^{-4}$ fm^3)
0.702	0.1023	0.255(56)	0.900(17)	2.25(5)
0.570	0.1009	0.275(54)	0.872(16)	2.18(4)
0.411	0.0961	0.355(62)	0.766(33)	1.92(9)
0.296	0.0951	0.35(11)	0.754(35)	1.89(9)

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Pion Extrapolation

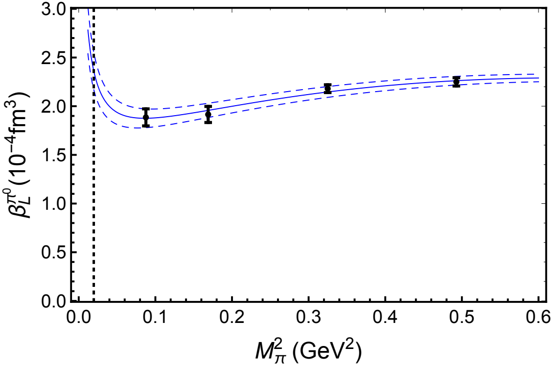

Figure: A description of lattice QCD results for the magnetic polarisability of the neutral pion, $\beta_L^{\pi^0}$, in terms of the leading tree-level terms of chiral effective field theory. Figure from 2010.01580.

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π^0 Extrapolation

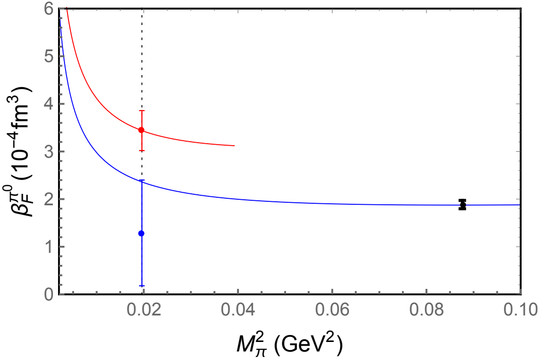

Figure: The full QCD prediction for the magnetic polarizabil- ity of the neutral pion β_{π^0} (red curve). The previous fit (blue curve) of the lattice QCD simulation results (black points) has been corrected to incorporate pion-loop contributions ab- sent in the current simulation results (red curve).... Figure from 2010.01580.