Exploring SU(3) + Higgs theories

The adjoint case

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30^{th} July 2021

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Physics beyond the SM

SM has its problems $\rightarrow$ Need for physics beyond the SM

One possible extension: **Grand unified theories (GUTs)**

- Unification of forces at some high energy scale
- One large gauge group $\mathcal{G} \supset \mathcal{G}_{SM}$ (at least SU(5))
- Needs to be broken to $\mathcal{G}_{SM}$ at lower energy scales

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Most BSM-theories need additional scalar(s) for symmetry-breaking or other reasons.

$\rightarrow$ Gain a good understanding of **gauge-symmetry breaking**
and **gauge-scalar interactions**

Prototype:

- Gauge group $\mathcal{G}$ with gauge fields W^a_μ in some rep.
- Scalar field(s) $\phi_{(\alpha)}$ in some rep. of $\mathcal{G}$
- Some potential $V(\phi^\dagger\phi)$ $\rightarrow$BEH-effect

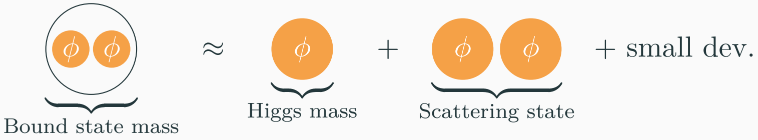
$$\mathcal{L} = -\frac{1}{4} \underbrace{W^a_{\mu\nu}} W_a^{\mu\nu} + (\underbrace{D_\mu}\phi)^\dagger D^\mu\phi - \underbrace{V\left(\phi^\dagger\phi\right)}$$

$$\partial_\mu W^a_\nu - \partial_\nu W^a_\mu - gf^{abc}W^b_\mu W^c_\nu \qquad \partial_\mu - igW^a_\mu t^{(\alpha)}_a \qquad \lambda\left(\phi^\dagger\phi - f^2\right)^2$$

FMS-mechanism

[Fröhlich et al., **PL** B97 (1980) and **NP** B190 (1981)/ **REVIEW:** Maas **PPNP** 106 (2019), arxiv:1712.04721]

- Physical observables in gauge-scalar theories:
  - PT uses gauge-dependent elementary fields → **unphysical**
  - **Correct**: use gauge-invariant objects (i.e. bound states)
- **FMS**: gauge-invariant objects → Higgs-split → usual PT
- **Prediction**: SM-spectrum stays qualitatively unchanged
- e.g.: The mass spectrum of the Higgs-boson (0_1^+)



- **Confirmed**: via Lattice spectroscopy

[Maas, **MPL** A28 (2013) / Maas and Mufti, **JHEP** (2014)]

The SU(3) + adj. theory

- Setup: SU(3) + adjoint scalar ($\Sigma(x) \in \mathfrak{su}(3)$)
- Potential: $V(\Sigma) = -\mu^2 \operatorname{tr}\Sigma^2 + \lambda\left(\operatorname{tr}\Sigma^2\right)^2 + \rho \operatorname{tr}\Sigma^3 + \lambda_2 \operatorname{tr}\Sigma^4$ (with $\rho \to 0$, $\lambda_2 \to 0$)
- Additional global $\mathbb{Z}_2$-symmetry

What to expect:

- Different spectra for PT and FMS for SU($N > 2$)
  [see SU(3) + fun.: Maas and Törek, **AoP** 397 (2018) / previous talk by Dobson]
- Composite massless vector boson(s) for adjoint scalars
  [see SU(2) + adj.: Afferrante, Maas and Törek **PRD** 101 (2020)]
- Two different breaking patterns

Breaking patterns

[Maas, Sondenheimer and Török **AoP** 402 (2019)/ Sondenheimer **PRD** 101 (2020)]

Symmetry-breaking ⇔ **non-vanishing vev** ⇔ preferred direction

- Two possible vev directions + linear-combinations

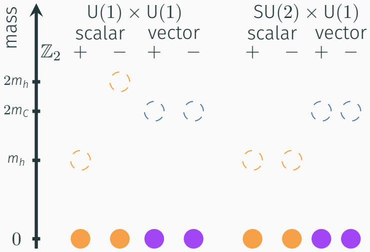
$\Sigma = v\Sigma_0 + \sigma(x)$ with $\Sigma_0 = \Sigma_3 t_3 + \Sigma_8 t_8$

$$t_3 \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad t_8 \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$U(1) \times U(1)$	$SU(2) \times U(1)$
3 distinct eigenvalues	2+1 distinct eigenvalues
almost any direction	6 directions for $\Sigma \propto t_8$-*perms*.

Perturbation theory

Gauge-invariant



- FMS predicts huge differences in spectra
- Expects **composite massless** scalar- and vector-particles

Scanning the phase diagram

First task: Scan for global- and gauge-symmetry breaking

using volume behavior of suitable order parameters

Global $\mathbb{Z}_2$ symmetry:

- $\mathbb{Z}_2$ order parameter: e.g. $\left\langle\left(\frac{1}{V}\sum_x \mathrm{tr}\left[\Sigma(x)^3\right]\right)^2\right\rangle$

SU(3) gauge symmetry:

- Needs gauge-fixing[1] to find phases
- **Unitary Gauge**: $G(x)\Sigma(x)G(x)^\dagger \to \Sigma(x)'$ s.t. $\Sigma(x)'$ is diagonal
- Gauge-fixed order parameter: e.g. $\left\langle \mathrm{tr}\left[\left(\frac{1}{V}\sum_x \Sigma(x)'\right)^2\right]\right\rangle$

[1] Other methods e.g. [Greensite, Matsuyama **PRD** 98 (2018)]

Simulation Details

Adjoint action contains **two links** in interaction see Appendix

Link: Heatbath-Algorithm with Cabibbo-Marinari-trick for
SU($N > 2$) + Metropolis step for interaction

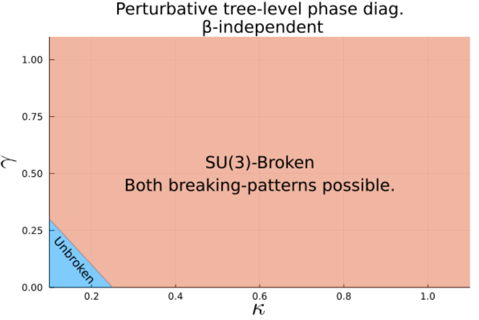
Pseudo-Link-OR: Usual OR + Metropolis step

Scalar: Adapted approximate Heatbath-Algorithm for SU(3)
see for SU(2): [Knechtli **arxiv**:hep-lat/9910044]

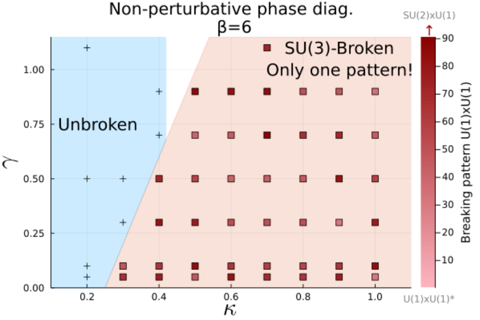
Scalar-OR: Random rotation around interaction vector see Appendix

Gauge-fixing: Analytic diagonalisation at each lattice site

PRELIMINARY



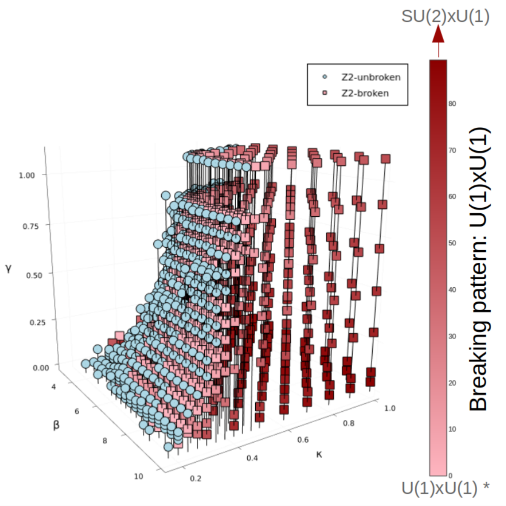
Perturbative (tree-level) phase diag.



Non-perturbative gauge-fixed phase diag.

Comparison of predicted and obtained phase diagram.
Non-perturbative slice taken from **full gauge-fixed phasediagram**.

PRELIMINARY



Global $\mathbb{Z}_2$ symmetry (blue = unbroken/red = broken). Colorbar describes the direction of the vev.
Further order parameters can be found here and gauge-fixed.

Summary & Outlook

- Gauge-scalar theories provide a huge playground for BSM-physics
- Be careful when comparing (Lattice-)spectroscopy results with PT $\rightarrow$ FMS-mechanism
- First look at the phase diagram of the SU(3) $+$ adj. theory
  - Agreement of gauge-phases and global-phases
    $\rightarrow$ Needs to be understood
  - Hints at different breaking patterns
    $\rightarrow$ Needs to be clarified
  - **TODO**: Check spectroscopy and FMS prediciton

Thank you!

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Gauge-invariant Perturbation Theory

[Fröhlich et al., PL **B97** (1980) and NP **B190** (1981) / A. Maas, PPNP **106** (2019), arXiv: 1712.04721]

1. Formulate gauge-invariant operator and correlator e.g.:

$$0^+ \text{ singlet:} \quad H(x) = \left(\phi_i^\dagger \phi_i\right)(x)$$

$$\langle H(x)H(y)\rangle = \left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle$$

2. Expand Higgs field in fixed gauge $\phi_i = vn_i + h_i$

$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = dv^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle +$$
$$+2v\left\{\left\langle \left(h_i^\dagger h_i\right)(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle + (x \leftrightarrow y)\right\} + \left\langle \left(h_i^\dagger h_i\right)(x)\left(h_j^\dagger h_j\right)(y)\right\rangle$$

3. Perform standard Perturbation Theory

$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = d'v^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}} +$$
$$+ \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}}^2 + \mathcal{O}(g^2, \lambda)$$

Lattice Action

SU(N)-Gauge-Higgs-Theory on the Lattice

[I. Montvay and G. Münster, Quantum Fields on a Lattice (1994)]

$$S = \sum_{x\in\Lambda} \left[\beta \left(1 - \frac{1}{N} \sum_{\mu<\nu} \mathrm{Re}\{\mathrm{Tr}\{U_{\mu\nu}(x)\}\} \right) + \right.$$
$$+ \gamma \left(\phi^\dagger(x)\phi(x) - 1 \right)^2 + \phi^\dagger(x)\phi(x) -$$
$$\left. - \kappa \sum_{\pm\mu} \phi^\dagger(x) U_\mu(x) \phi(x + e_\mu) \right]$$

Lattice parameters: β, γ and κ

Continuum parameters: g, λ and f

Lattice Action

Lattice version of a SU(3)-Gauge-Scalar-Theory with the scalar field in the adjoint representation $\Sigma(x) = \Sigma_a(x)t_a$

$$S = \sum_{x\in\Lambda}\Bigg[\beta\left(1 - \frac{1}{3}\sum_{\mu<\nu}\mathrm{Re}\{\mathrm{tr}\,\{U_{\mu\nu}(x)\}\}\right) + \\ + \gamma(2\,\mathrm{tr}\,\{\Sigma(x)\Sigma(x)\} - 1)^2 + 2\,\mathrm{tr}\,\{\Sigma(x)\Sigma(x)\} - \\ - 2\kappa\sum_{\mu=1}^{4}\mathrm{tr}\,\left\{\Sigma(x)U_\mu(x)\Sigma(x+e_\mu)U_\mu^\dagger(x)\right\}\Bigg]$$

Lattice parameters: β, γ and κ

Scalar-OR

1. Local change under $\Sigma(x) \to \Sigma'(x)$ should be 0:

$$\begin{aligned}\Delta S =& \gamma(\mathrm{tr}\left\{\Sigma(x)\Sigma(x)\right\} - 1)^2 + \mathrm{tr}\left\{\Sigma(x)\Sigma(x)\right\} - \\ & -\gamma\left(\mathrm{tr}\left\{\Sigma'(x)\Sigma'(x)\right\} - 1\right)^2 - \mathrm{tr}\left\{\Sigma'(x)\Sigma'(x)\right\} - \\ & -\left(\Sigma_a(x) - \Sigma'_a(x)\right) \underbrace{\kappa \sum_\mu \mathrm{tr}\left\{t_a U_\mu(x)\Sigma(x+e_\mu)U^\dagger_\mu(x)\right\}}_{k_a}\end{aligned}$$

2. Choose: $\Sigma'_a = \Sigma_a - \sigma_a$ with $|\Sigma'|^2 = |\Sigma|^2$ and $\sigma_a k_a = 0$
3. Draw $n-1$ random numbers and set $\sigma'_a = (0, r_1, \ldots, r_{n-1})$
4. Project σ'_a orthogonal to k_a by $\sigma_a = \sigma'_a - \frac{(\sigma'_b k_b)k_a}{k_c k_c}$
5. Rescale σ_a by $2\frac{\Sigma_a \sigma_a}{\sigma_b \sigma_b}$ to preserve norm