Sp(4) lattice gauge theory with fermions in multiple representations

Jong-Wan Lee

(Pusan National University)

Work in progress with E. Bennett, J. Holligan, D. Hong, H. Hsiao, D. C.-J. Lin, B. Lucini, M. Mesiti, M. Piai, D. Vadacchino

The 38th International Symposium on Lattice Field Theory

July 30, 2021

4D UV models for Comp. Higgs & partial-top

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1
	SO(9)			5/12		M2
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3
	SO(9)			5/3		M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12		M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3		M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12

Composite Higgs + partial top compositeness

Cacciapaglia, Ferretti, Flacke & Serodio (2019)

4D UV models for Comp. Higgs & partial-top

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1
	SO(9)			5/12		M2
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3
	SO(9)			5/3		M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12		M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3		M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12

Composite Higgs + partial top compositeness

$SU(4)/Sp(4)$

\# of pNGBs = 5

4 of 5 PNGBs: Higgs doublets

$SU(2)_L \times U(1)_Y \subset Sp(4)$

SM EW

$SU(3)_c \times U(1)_Y \subset SO(6)$

SM Strong

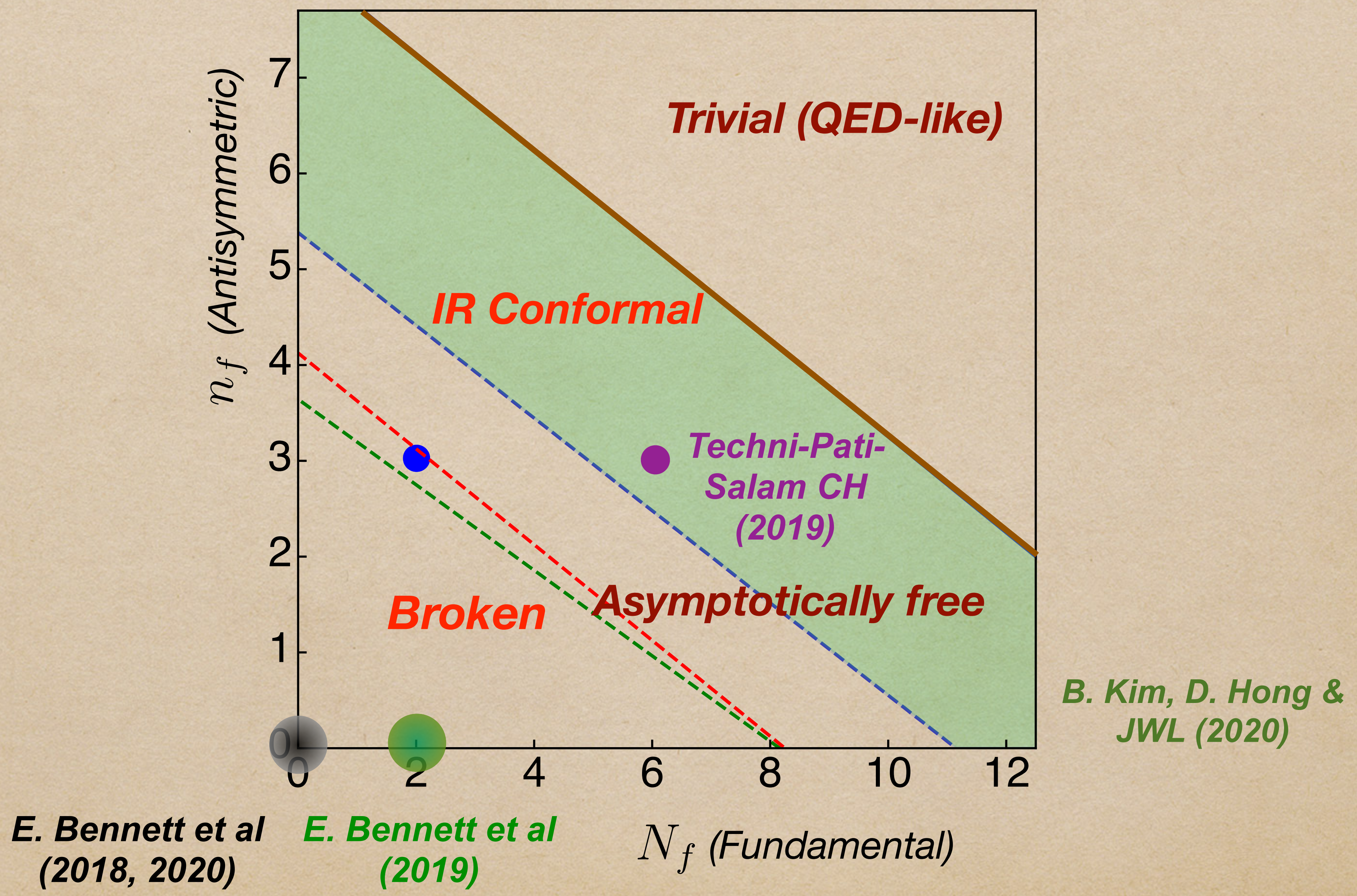
e.g. top partner

$\hat{\Psi}^{a\alpha b} \equiv \left(\psi^a \chi^\alpha \psi^b\right)$

carry color charge

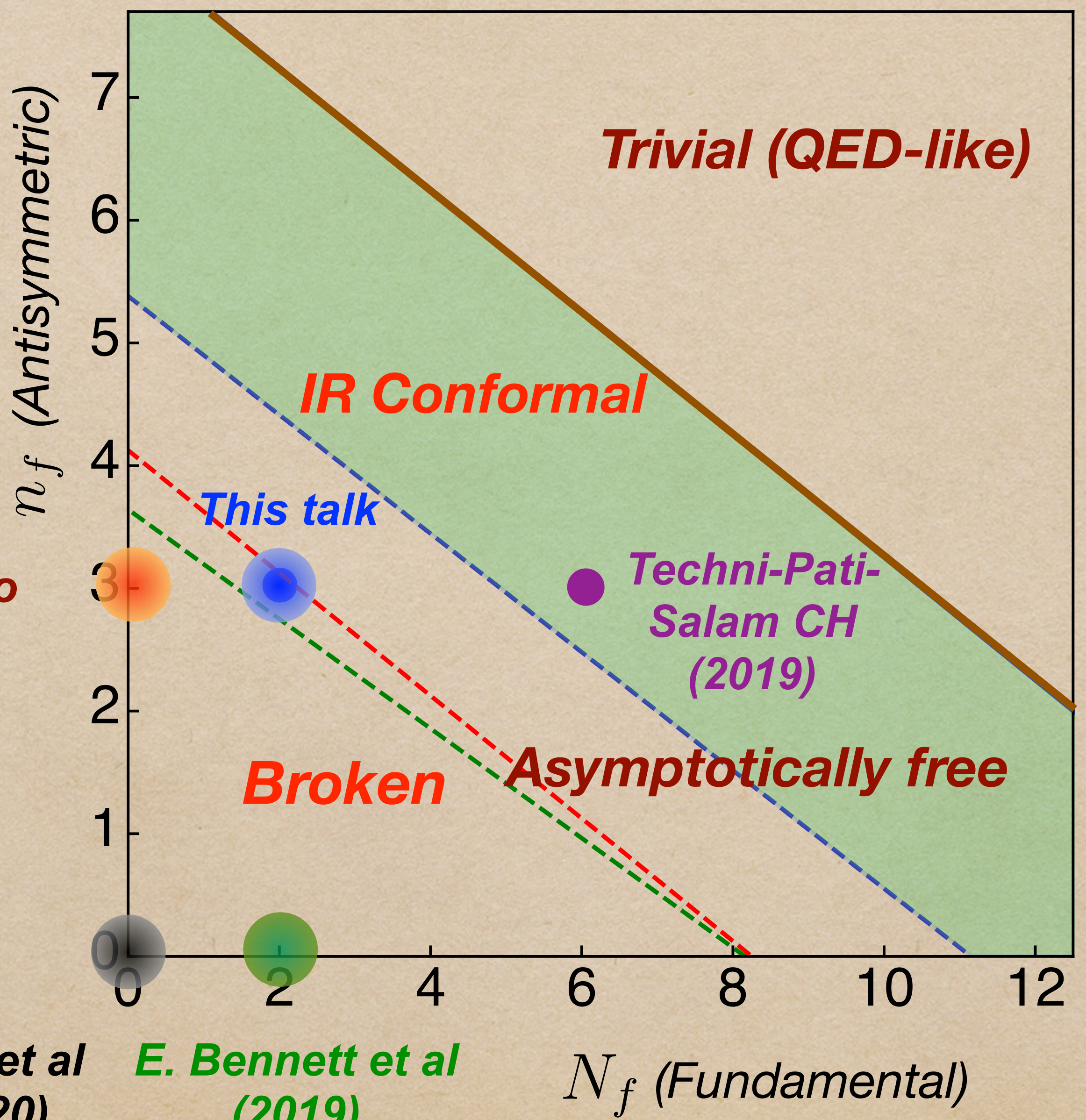
Cacciapaglia, Ferretti, Flacke & Serodio (2019)

Theory space of Sp(4) gauge group



Theory space of Sp(4) gauge group

Talk by H. Hsiao



Talk by G. Cacciapaglia

B. Kim, D. Hong & JWL (2020)

E. Bennett et al (2018, 2020)

E. Bennett et al (2019)

Talk by J. Holligan & D. Vadacchino

Sp(4) on the lattice

- Lattice formulation with the standard Wilson gauge & fermion actions

$$S \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$

Gauge action

Fermion action

$$+a^4 \sum_x \overline{Q}_j(x) D^F Q_j(x) + a^4 \sum_x \overline{\Psi}_k(x) D^{AS} \Psi_k(x)$$

fundamental (F) antisymmetric (AS)

$$D^F Q_j(x) \equiv (4/a + m_0^f) Q_j(x) - \frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^F(x) Q_j(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^F(x-\hat{\mu}) Q_j(x-\hat{\mu}) \right\},$$

$$D^{AS} \Psi_k(x) \equiv (4/a + m_0^{as}) \Psi_k(x) - \frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^{AS}(x) \Psi_k(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^{AS}(x-\hat{\mu}) \Psi_k(x-\hat{\mu}) \right\}$$

where $U_\mu(x) = U_\mu^F(x) \in Sp(4)$ and

e.g. Del Debbio, Patella & Pica (2008) for SU(N)

$$\left(U_\mu^{AS}\right)_{(ab)(cd)}(x) \equiv \mathrm{Tr}\left[(e_{AS}^{(ab)})^\dagger U_\mu(x) e_{AS}^{(cd)} U_\mu^{\mathrm{T}}(x)\right], \quad \text{with } a<b,\ c<d.$$

Here, e_{AS} is antisymmetric and Ω-traceless, where $\Omega = \Omega_{jk} = \Omega^{jk} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

Sp(4) on the lattice

- Lattice formulation with the standard Wilson gauge & fermion actions

$$S \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$

$$+a^4 \sum_x \overline{Q}_j(x) D^F Q_j(x) + a^4 \sum_x \overline{\Psi}_k(x) D^{AS} \Psi_k(x)$$

$$D^F Q_j(x) \equiv (4/a + m_0^f) Q_j(x) - \frac{1}{2a} \sum_\mu \left\{(1-\gamma_\mu) U_\mu^F(x) Q_j(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^F(x-\hat{\mu}) Q_j(x-\hat{\mu})\right\},$$

$$D^{AS} \Psi_k(x) \equiv (4/a + m_0^{as}) \Psi_k(x) - \frac{1}{2a} \sum_\mu \left\{(1-\gamma_\mu) U_\mu^{AS}(x) \Psi_k(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^{AS}(x-\hat{\mu}) \Psi_k(x-\hat{\mu})\right\}$$

where $U_\mu(x) = U_\mu^F(x) \in Sp(4)$ and

e.g. Del Debbio, Patella & Pica (2008) for SU(N)

$$(U_\mu^{AS})_{(ab)(cd)}(x) \equiv \mathrm{Tr}\left[(e_{AS}^{(ab)})^\dagger U_\mu(x) e_{AS}^{(cd)} U_\mu^{\mathrm{T}}(x)\right], \quad \text{with } a<b,\ c<d.$$

Here, e_{AS} is antisymmetric and Ω-traceless, where $\Omega = \Omega_{jk} = \Omega^{jk} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

Simulation details

- HiRep code with appropriate modifications:

Del Debbio, Patella & Pica (2008)

  - resymplectisation *E. Bennett el al (2017)*
  - reduced matrix *E. Bennett el al (2018)*
  - antisymmetric representation *E. Bennett el al (2019)*
  - multiple representations (fund. + two-index rep.) ***This work***

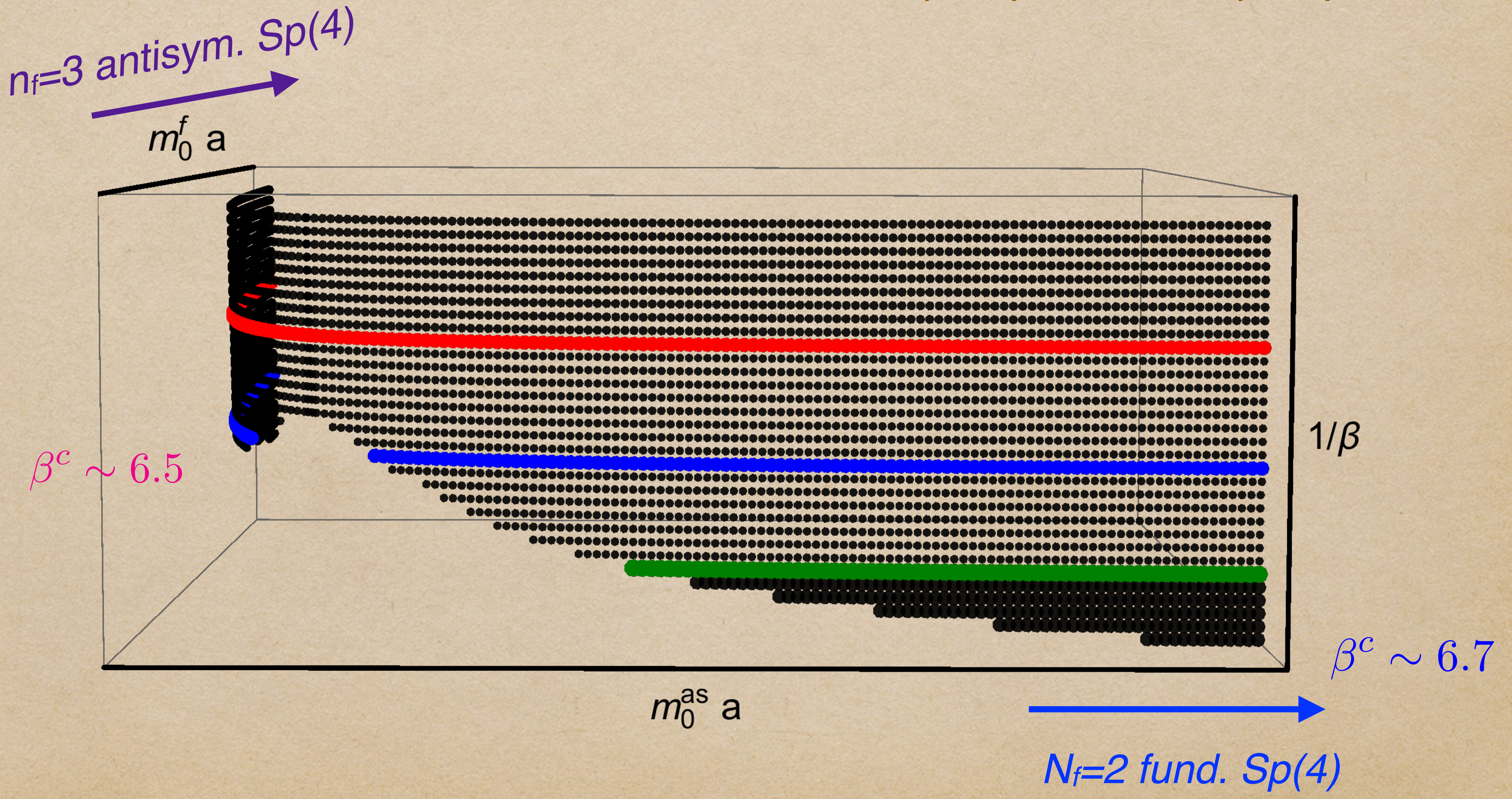
- Using HMC (RHMC) algorithms, we simulate lattice *Sp(4)* theory coupled to both $N_f=2$ F & $n_f=3$ AS Dirac fermions.

- For the exploratory studies of hadron spectrum we have used point sources while leaving more sophisticated measurements in our future work.

Results I: bare parameter space

- From our previous studies of N_f=2 F Sp(4) & n_f=3 AS Sp(4) we learned that 1st order bulk phase transitions exist for $\beta \lesssim 6.7$ & $\beta \lesssim 6.5$, respectively.

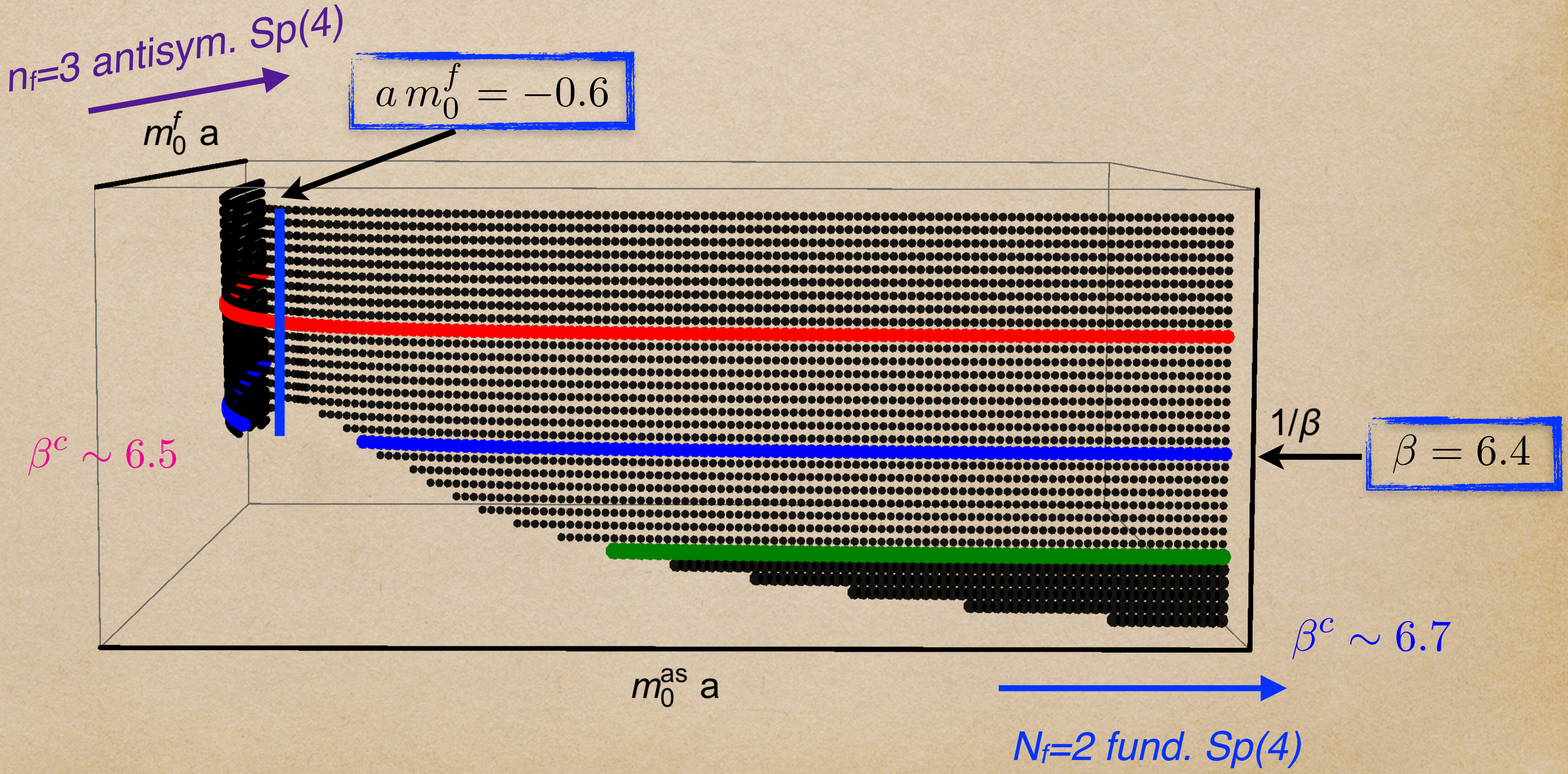
E. Bennett el al (2018) JWL el al (2019)



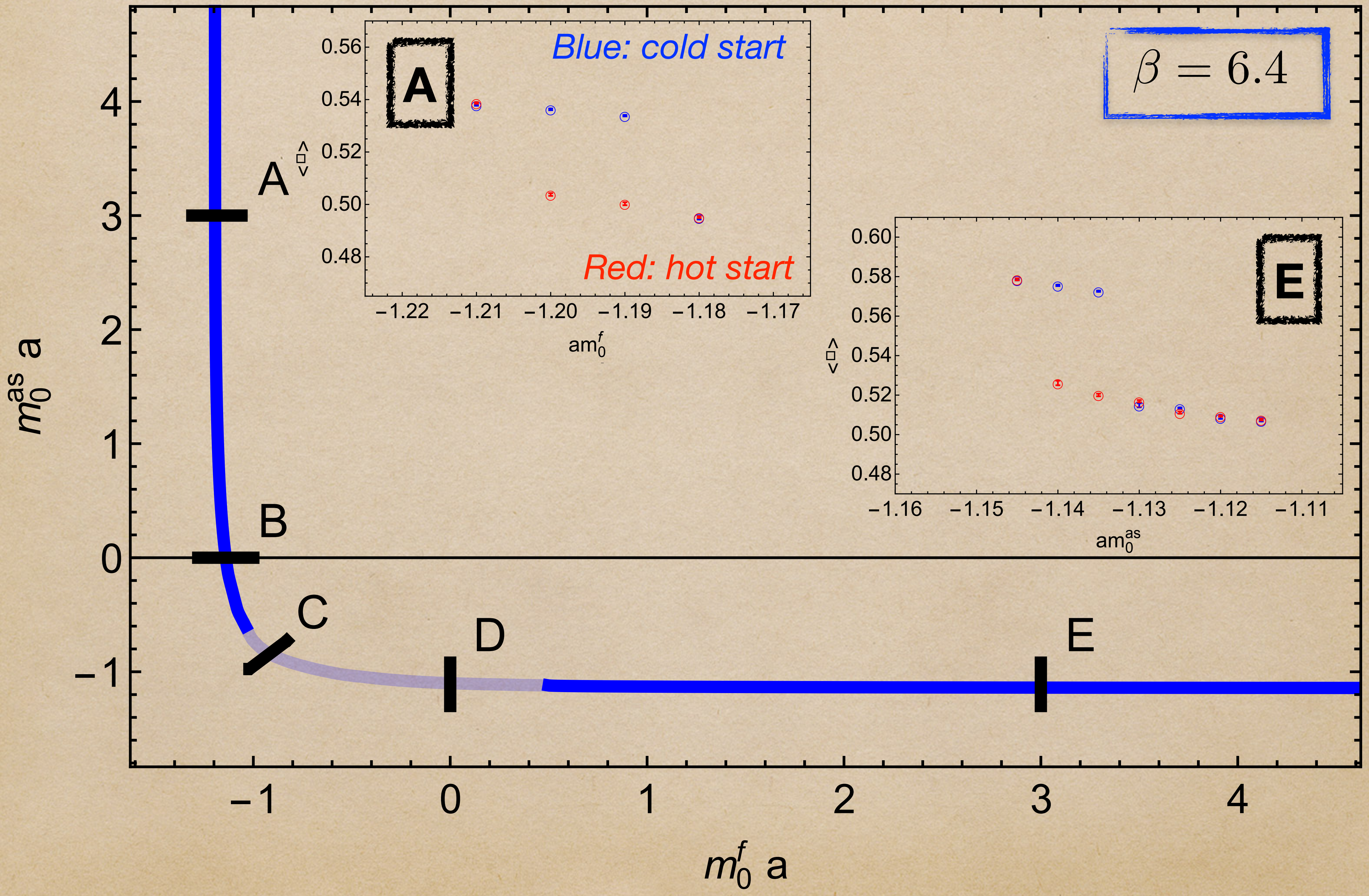
Results I: bare parameter space

- From our previous studies of *N_f=2 F Sp(4)* & *n_f=3 AS Sp(4)* we learned that 1st order bulk phase transitions exist for $\beta \lesssim 6.7$ & $\beta \lesssim 6.5$, respectively.

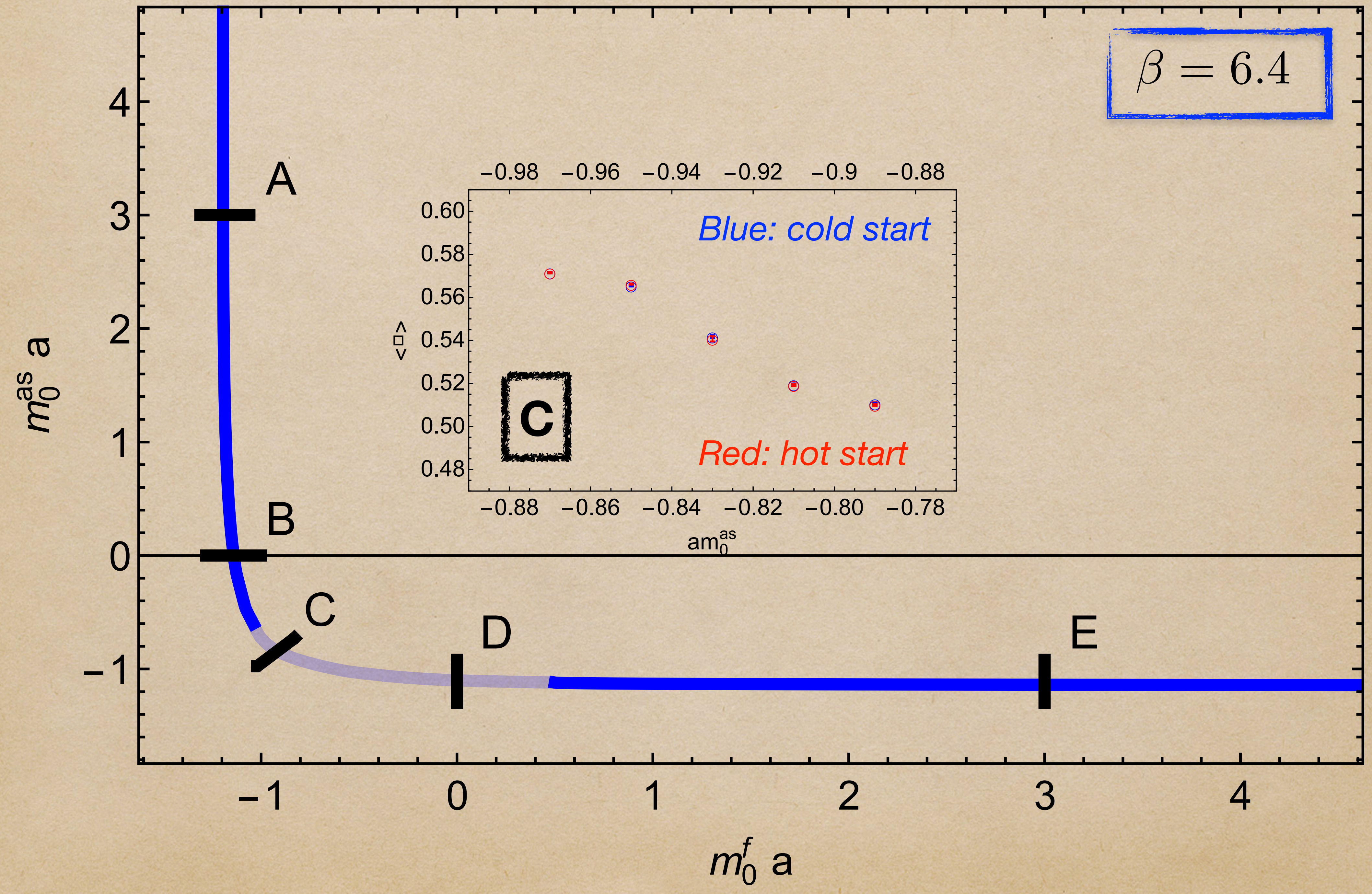
E. Bennett el al (2018) ***JWL el al (2019)***



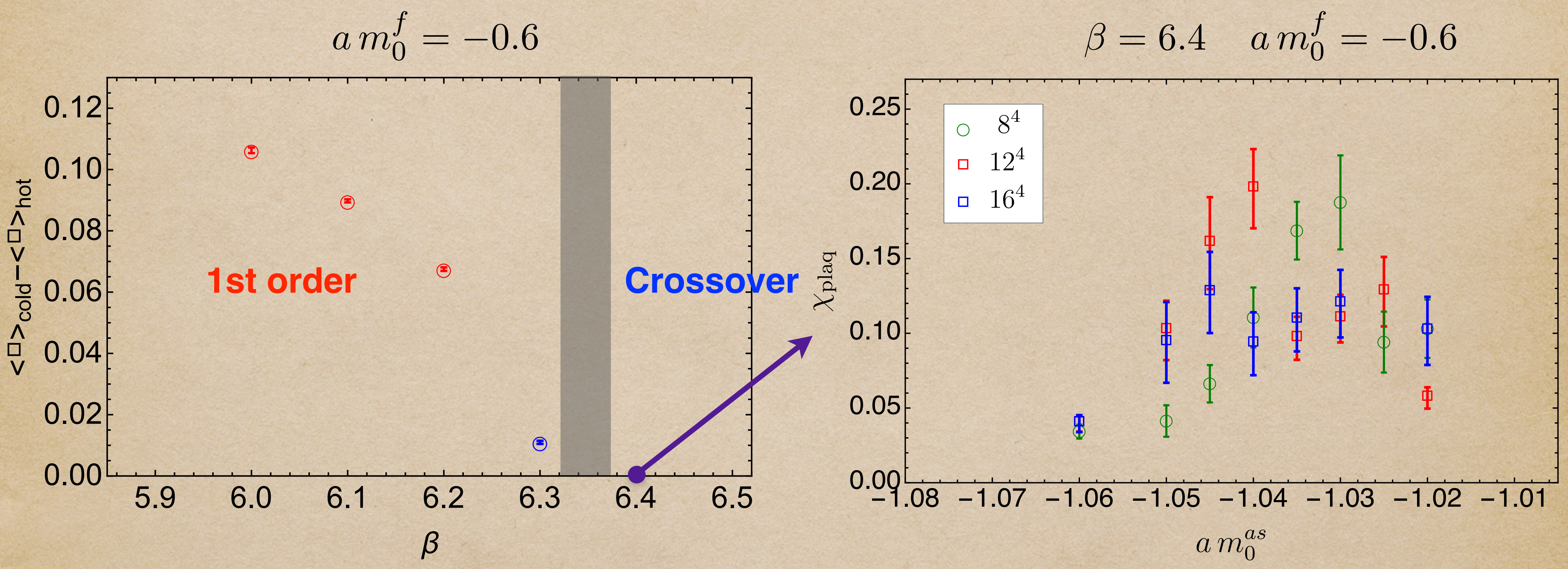
Results I: bare parameter space



Results I: bare parameter space

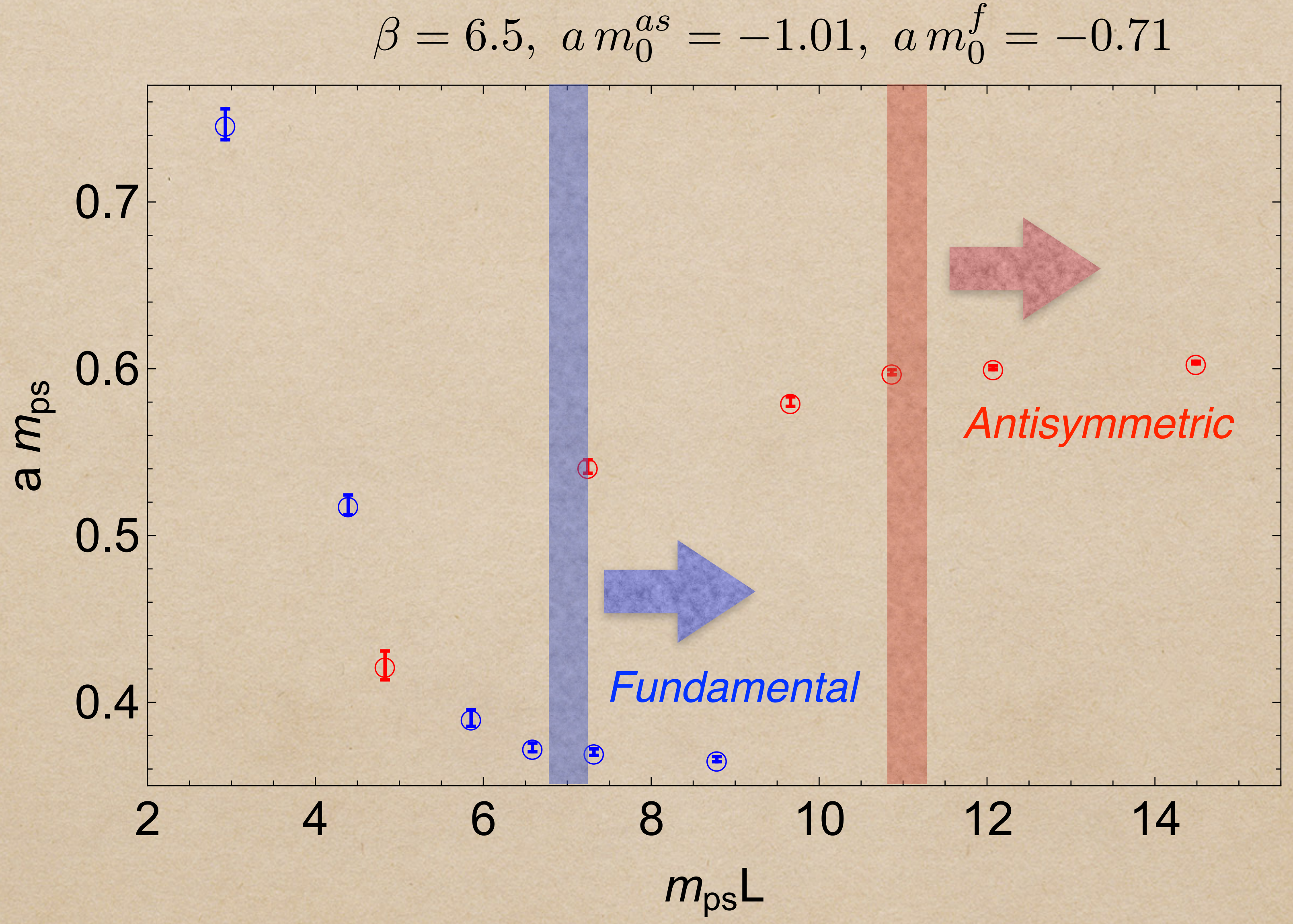


Results I: bare parameter space



- If both $N_f=2$ *F* & $n_f=3$ *AS* Dirac fermions are present, the weak coupling region is extended to the smaller beta value of $\beta \sim 6.4$.

Results II: Finite Volume effects



- Finite volume effects are expected to be negligible if $7 \lesssim m_{\rm PS}^{f}\, L$ & $11 \lesssim m_{\rm PS}^{as}\, L$.

Results II: Finite Volume effects

- The different signs of finite volume effects can be understood from the low-energy effective field theory.

Bijnens & Lu (2009)

$$m_{\rm PS}^2 = M^2\left(1 + a_M \frac{A(M) + A_{\rm FV}(M)}{F^2} + b_M(\mu)\frac{M^2}{F^2} + \mathcal{O}(M^4)\right)$$

$$A(M) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2} \qquad A_{\rm FV}(M) \overset{ML\gg 1}{\longrightarrow} -\frac{3}{4\pi^2}\left(\frac{M\pi}{2L^3}\right)^{1/2}\exp[-ML]$$

$SU(2N_f)\to Sp(2N_f)$	$SU(2N_f)\to SO(2N_f)$	$SU(N_f)\times SU(N_f)\to SU(N_f)$
$a_M = -\frac{1}{2} - \frac{1}{N_f}$	$a_M = \frac{1}{2} - \frac{1}{2N_f}$	$a_M = -\frac{1}{N_f}$
-1 (N_f=2 F)	1/3 (n_f=3 AS)	

Chimera baryon as top partner

- Recall the global symmetry and its spontaneous breaking

$$SU(4)/Sp(4) \otimes SU(6)/SO(6)$$

where $SO(4)$ subgroup of $Sp(4)$ ~ $SU(2)_L$ gauge group in SM &
$SU(3)$ subgroup of $SO(6)$ ~ $SU(3)_c$ gauge group in SM

- Then, the top partner can be sourced by the operators

$$\begin{aligned}
\mathcal{O}^{L,R}_{\mathrm{CB},1} &= \left(\overline{Q^{1\,a}}\gamma^5 Q^{2\,b} + \overline{Q^{2\,a}}\gamma^5 Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,,\\
\mathcal{O}^{L,R}_{\mathrm{CB},2} &= \left(-i\overline{Q^{1\,a}}\gamma^5 Q^{2\,b} + i\,\overline{Q^{2\,a}}\gamma^5 Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,,\\
\mathcal{O}^{L,R}_{\mathrm{CB},4} &= -i\left(\overline{Q^{1\,a}}Q^{2\,b}_C + \overline{Q^{2\,a}_C}Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\\
\mathcal{O}^{L,R}_{\mathrm{CB},5} &= i\left(-i\,\overline{Q^{1\,a}}Q^{2\,b}_C + i\overline{Q^{2\,a}_C}Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,.
\end{aligned}$$

which transform 3 of $SU(3)_c$ and 4 of $SO(4)$.

Chimera baryon as top partner

- Recall the global symmetry and its spontaneous breaking

$$SU(4)/Sp(4) \otimes SU(6)/SO(6)$$

where $SO(4)$ subgroup of $Sp(4)$ ~ $SU(2)_L$ gauge group in SM &
$SU(3)$ subgroup of $SO(6)$ ~ $SU(3)_c$ gauge group in SM

- We also consider the $U(1)_A$ counterparts ($\mathbb{1} \rightarrow i\gamma^5$, expected to be heavier)

$$\begin{aligned}
\mathcal{O}'^{L,R}_{\text{CB},1} &= i\left(\overline{Q^{1\,a}}Q^{2\,b} + \overline{Q^{2\,a}}Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,, \\
\mathcal{O}'^{L,R}_{\text{CB},2} &= \left(\overline{Q^{1\,a}}Q^{2\,b} - \overline{Q^{2\,a}}Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,, \\
\mathcal{O}'^{L,R}_{\text{CB},4} &= \left(\overline{Q^{1\,a}}\gamma^5 Q^{2\,b}_C + \overline{Q^{2\,a}_C}\gamma^5 Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,, \\
\mathcal{O}'^{L,R}_{\text{CB},5} &= i\left(\overline{Q^{1\,a}}\gamma^5 Q^{2\,b}_C - \overline{Q^{2\,a}_C}\gamma^5 Q^{1\,b}\right)\Omega_{bc}P_{L,R}\Psi^{k\,ca}\,.
\end{aligned}$$

which transform 3 of $SU(3)_c$ and 4 of $SO(4)$.

See Talk by H. Hsiao

Interpolating operators for Chimera baryon

- Analogous to the Lambda baryon in QCD, we construct the interpolating operator of a spin-1/2 Chimera baryon as

$$\mathcal{O}^{k}_{\mathrm{CB},\,\alpha}(x) = -i(C\gamma^5)^{\beta\gamma}\Omega_{ac}\Omega_{bd}Q^{2a}_{\beta}(x)Q^{1b}_{\gamma}(x)\Psi^{k\,cd}_{\alpha}(x)$$

- Then, the 2-point correlation function is

$$\langle \mathcal{O}_{\mathrm{CB}}(t)_{\gamma}\overline{\mathcal{O}_{\mathrm{CB}}(0)}_{\gamma'}\rangle = \sum_{\vec{x}}\Omega_{da}\Omega_{bc}\Omega^{c'b'}\Omega^{a'd'}S_{\Psi}(t,\vec{x})^{ca,c'a'}_{\gamma,\gamma'}\,S^{2}_{Q}(t,\vec{x})^{d,b'}_{\alpha,\alpha'}(C\gamma^5)_{\alpha\beta}S^{1}_{Q}(t,\vec{x})^{b,d'}_{\beta,\beta'}(C\gamma^5)_{\alpha'\beta'}$$

where the fermion propagators in given representations are

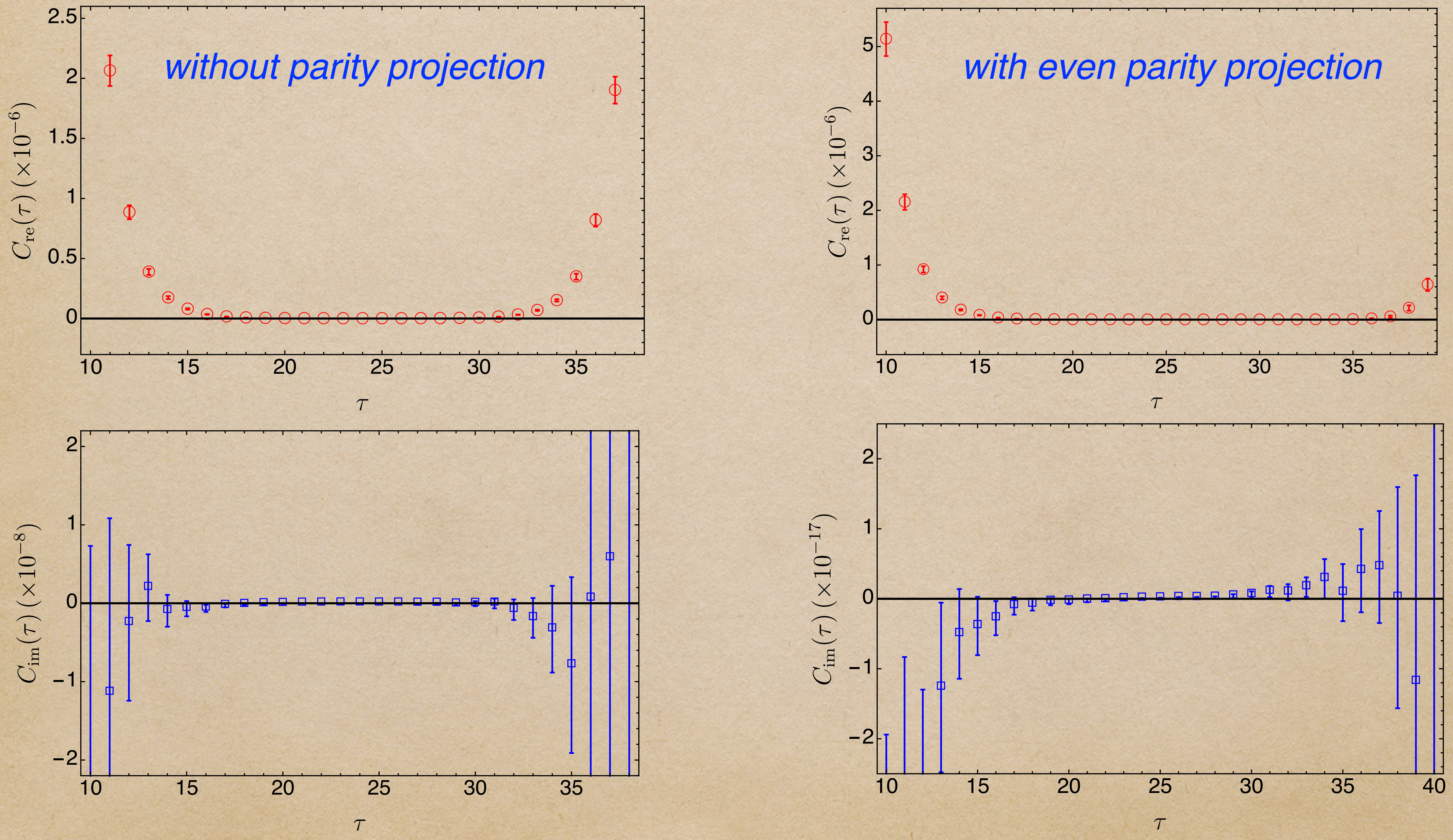
$$S_{Q}(t,\vec{x})^{a,b}_{\alpha,\beta} = \langle Q(t,\vec{x})^{a}_{\alpha}\overline{Q(0)^{b}}_{\beta}\rangle \text{ and } S_{\Psi}(t,\vec{x})^{ab,cd}_{\alpha,\beta} = \langle \Psi(t,\vec{x})^{ab}_{\alpha}\overline{\Psi(0)^{cd}}_{\beta}\rangle.$$

- We also consider the parity projections in the nonrelativistic limit.

$$\mathcal{O}^{\pm}_{\mathrm{CB}}(x) = P_{\pm}\,\mathcal{O}_{\mathrm{CB}}(x) \quad \text{with} \quad P_{\pm} = \frac{1}{2}(\mathbb{1}\pm\gamma_0)$$

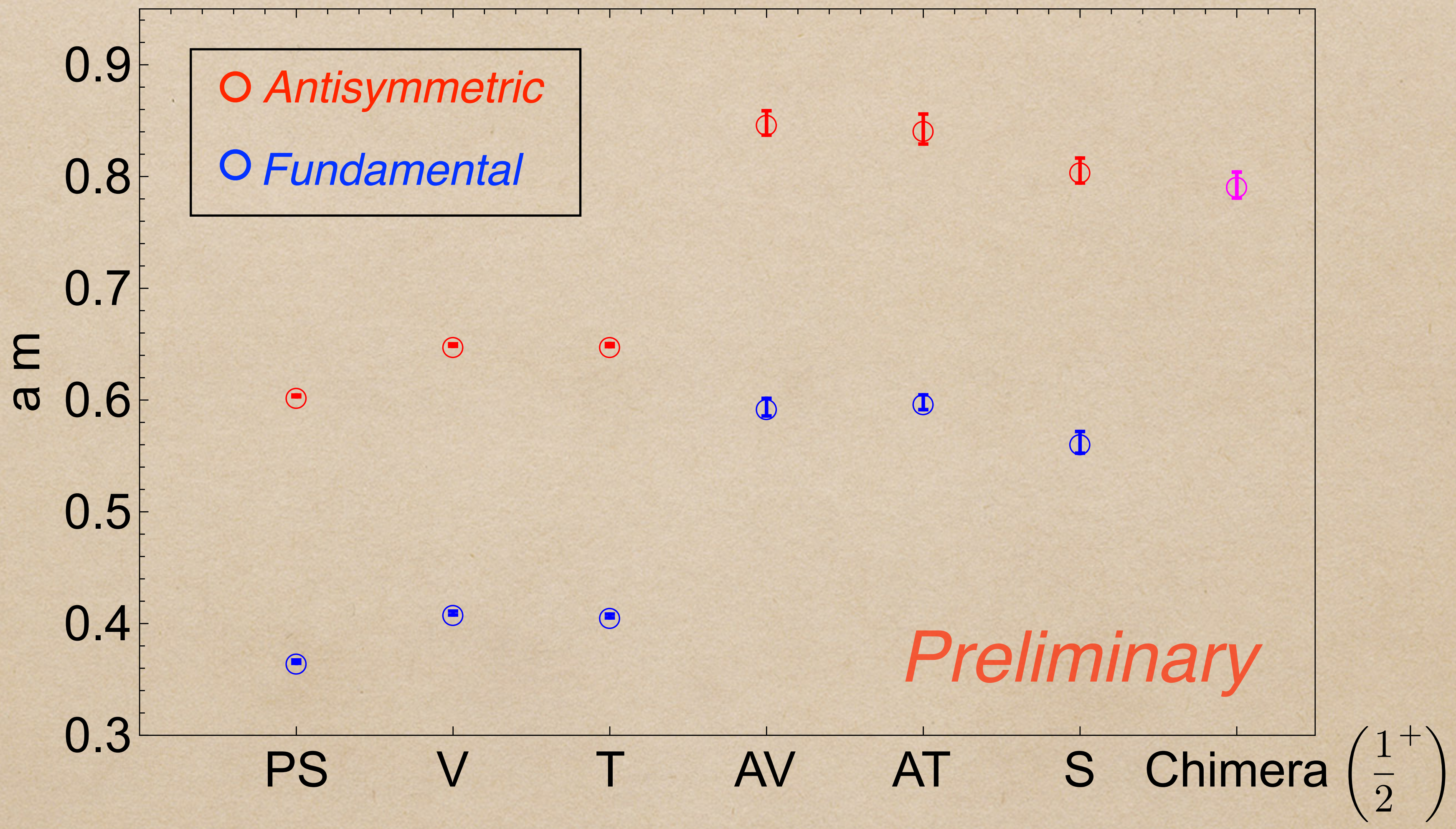
Results III: Chimera baryon

- Real & imaginary part of 2-point correlation functions of the Chimera baryon



Results IV: Masses of mesons & Chimera baryon

$$\beta = 6.5,\ a\,m_0^{as} = -1.01,\ a\,m_0^{f} = -0.71,\ T \times L^3 = 48 \times 24^3$$



- We considered spin-0 & 1 flavored mesons: pseudoscalar(PS), vector(V), tensor(T), axial-vector(AV), axial-tensor(AT) & scalar(S).

Summary & outlook

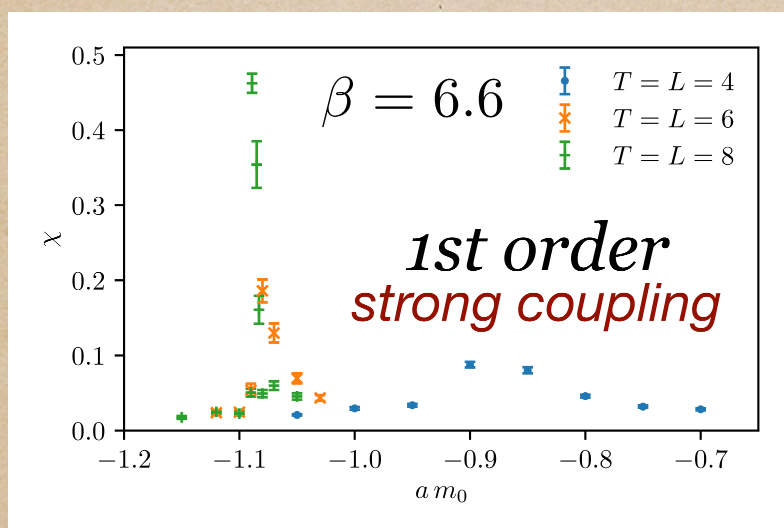
- We have developed numerical techniques to simulate *Sp(2N)* lattice gauge theories coupled to fermions in the multiple representations.
- The first lattice studies of the VU model with the exact flavor content required for CH & top-partial comp.: *Sp(4)* with *N_f=2 F & n_f=3 AS* Dirac fermions.
  - Weak coupling region: $\beta \lesssim 6.4$
  - FV effects are under control: $7 \lesssim m_{\rm PS}^{f} L$ & $11 \lesssim m_{\rm PS}^{as} L$
  - Chimera baryon (top partner): parity projection, smearing & variational method
- To do list
  - Generate ensembles at various values of β , m_0^f , m_0^{as} and calculate the low-lying spectra of composite states: mass dependence & lattice artifacts
  - Compute the (low-lying) Dirac eigenvalues

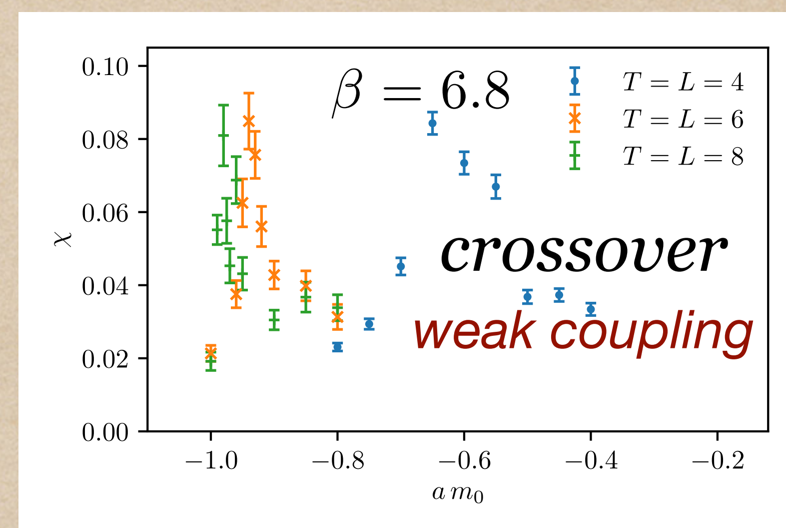
Chirally broken or conformal? ***How light is the chimera baryon?***

Thank you for your attention!

Backup I

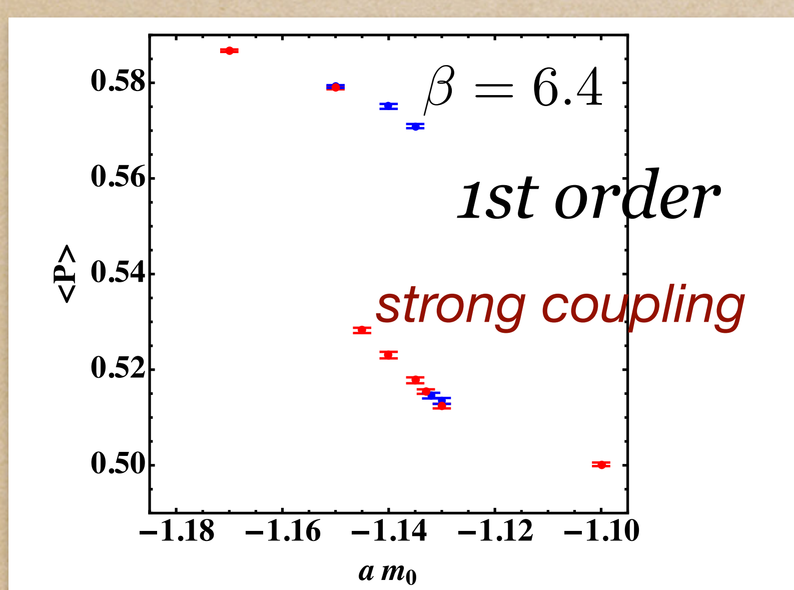
- From our previous studies of N_f=2 F Sp(4) & n_f=3 AS Sp(4) we learned that 1st order bulk phase transitions exist for $\beta \lesssim 6.7$ & $\beta \lesssim 6.5$, respectively.

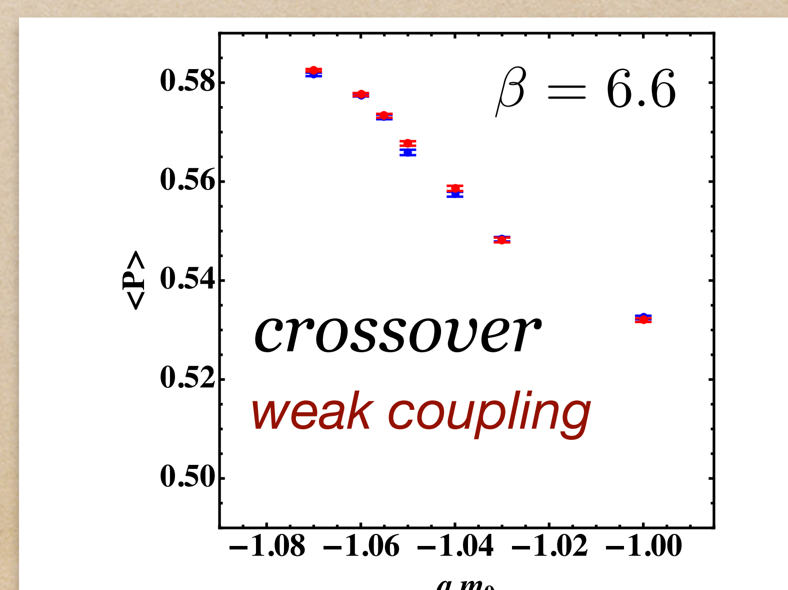




N_f=2 fund. Sp(4)

E. Bennett el al (2018)





N_f=3 anti-sym. Sp(4)

JWL el al (2019)