Sp(4) lattice gauge theory with fermions in multiple representations

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Work in progress with E. Bennett, J. Holligan, D. Hong, H. Hsiao, D. C.-J. Lin, B. Lucini, M. Mesiti, M. Piai, D. Vadacchino

The 38th International Symposium on Lattice Field Theory July 30, 2021

4D UV models for Comp. Higgs & partial-top IF LAND MODELS FOR COMP. FILTERS & DAM

TABLE I. Model details. The first column shows the EW and QCD colour cosets, respectively, followed

by the representations under the confining hypercolour (HC) gauge group of the EW sector fermions

bounds on the singlet participant participant participation in Section IV. We our conclusions in Section V. We

Composite Higgs + partial top compositeness

Cacciapaglia, Ferretti, Flacke & Serodio (2019)

In unitary gauge, besides the heavy vectors only the physical pions are retained. They

PS = *m*(*v*³ + *mv*²

Compared to the original EFT results in terms of *m^f* in [55], the above linearized ansatz

= 7*.*62*,* 7*.*7*,* 7*.*85*,* 8*.*0*,* 8*.*2 (2.4)

invloues 10 unknown Lectro and The remaining two LECs to be determined from 5 measurements. The remaining two LECs to be determined from 5 measurements. The remaining two LECS to be a stream of the remaining two LECS of th

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In order to construct the Dirac operator *DAS* for fermion fields *ab* in the 2-index

N(2*N* 1) 1 such matrices have the following non-vanishing entries. For *b* = *N* + *a* and

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 -1 0 0 0

 $0 -100$

(3.3)

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The symmetries of the system are more transparent in the two-component notation, yet it is

3 Lattice setup

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3.1 Lattice action

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gauge sector: plaquette action

3 Lattice setup

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3.1 Lattice action

lattice parameters: lattice coupling , bare fermion masses *m*^f

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4 Observables

HMC + RHMC

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3.2 (Rational) Hybrid Monte Carlo

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and $\sum_{i=1}^{n}$ for $\frac{1}{2}$ in $\frac{1}{2}$, we follow the presentation in $\frac{1}{2}$, we define $\frac{1}{2}$, we define $\frac{1}{2}$, we define $\frac{1}{2}$

an orthonormal basis *e* (*ab*) *AS* (with the multi-index (*ab*) running over ordered pairs with 1

a<b 2*N*) for the appropriate vector space of 2*N* ⇥ 2*N* antisymmetric matrices. The

Lattice formulation with the standard Wilson gauge & fermion actions e **1** *Lattice formulation with the standard Wilson gauge & fermion actions* **in the standard Wilson**gauge sector: player

fermion sector: Wilson-Dirac formulation for fermions in two distinct representations

$$
S \equiv \beta \sum_{x} \sum_{\mu < \nu} \left(1 - \frac{1}{4} \text{Re Tr} \, U_{\mu}(x) U_{\nu}(x + \hat{\mu}) U_{\mu}^{\dagger}(x + \hat{\nu}) U_{\nu}^{\dagger}(x) \right)
$$
\nFermion action

\n
$$
+ a^{4} \sum_{x} \overline{Q}_{j}(x) D^{F} Q_{j}(x) + a^{4} \sum_{x} \overline{\Psi}_{k}(x) D^{AS} \Psi_{k}(x)
$$
\nfundamental (F)

\n
$$
D^{F} Q_{j}(x) \equiv (4/a + m_{0}^{f}) Q_{j}(x) - \frac{1}{2a} \sum_{\mu} \left\{ (1 - \gamma_{\mu}) U_{\mu}^{F}(x) Q_{j}(x + \hat{\mu}) + (1 + \gamma_{\mu}) U_{\mu}^{F}(x - \hat{\mu}) Q_{j}(x - \hat{\mu}) \right\},
$$

$$
D^{AS}\Psi_k(x) \equiv (4/a + m_0^{as})\Psi_k(x) - \frac{1}{2a} \sum_{\mu} \left\{ (1 - \gamma_{\mu}) U_{\mu}^{AS}(x)\Psi_k(x + \hat{\mu}) + (1 + \gamma_{\mu}) U_{\mu}^{AS}(x - \hat{\mu})\Psi_k(x - \hat{\mu}) \right\}
$$

 $U(AS)$ $\qquad \qquad (x \rightarrow \mathbb{F} \left[(ab) \setminus \dagger_{\mathbf{I}^T} (x) (cd) \mathbf{I}^T \right]$ $\qquad \qquad (1 \rightarrow 0)$ $\left\{ \begin{array}{c} \n\mu \quad \text{(a0)(ca)} \quad \text{(a)} \quad \text{(b) } \quad \text{(c) } \quad \text{(d) } \quad \text{(e)} \quad \text{(f) } \quad \text{(g)} \quad \text{(g)} \quad \text{(h) } \quad \text{(i)} \quad \text{(i)} \quad \text{(ii)} \quad \text{(iii)} \quad \text{(iv) } \quad \text{(v) } \quad \text{(v) } \quad \text{(vi) } \quad \text{(v) } \quad \text{(vi) } \quad \text{(v) } \quad \text{(vi) } \quad \$ antisymmetric representation, we follow the prescription in [12]. For *Sp*(2*N*), we define Here, e_{AS} is antisymmetric and Ω -traceless, where $\Omega = \Omega_{ik} = \Omega^{jk} =$ where $U_{\mu}(x) = U_{\mu}^{F}(x) \in Sp(4)$ and $\mathcal{L}_\mu^F(x)\in Sp(4)$ and e.g. Del Debbio, Patella & Pica (2008) for SU(N) $(U_{\mu}^{AS})_{(ab)(cd)}(x) \equiv \text{Tr}\left[(e_{AS}^{(ab)})^{\dagger}U_{\mu}(x)e_{AS}^{(cd)}\right]$ antisymmetric representation, we follow the prescription in [12]. For *Sp*(2*N*), we define an orthonormal basis *e* (*ab*) *AS* (with the multi-index (*ab*) running over ordered pairs with 1 **FINDERE**, e_{AS} is antisymmetric and *M*-traceless, **Where** $\Omega = \Omega$ $\int_{(ab)(cd)} (x) \equiv \text{Tr} \left[(e_{AS}^{(ab)})^{\dagger} U_{\mu}(x) e_{AS}^{(cd)} U_{\mu}^{T}(x) \right]$ **i** *,* with $a < b$, $c < d$. $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 &$ Here, e_{AS} is antisymmetric and Ω -traceless, where $\Omega = \Omega_{jk} = \Omega^{jk} \equiv$ defined by $\sqrt{2}$ BBB@ 0 0 10 0 0 01 $\overline{1}$ $\overline{}$ where $U_{\mu}(x) = U_{\mu}^{\dagger}(x) \in Sp(4)$ and H Greg, EAS is all E *D_{<i>A*} $U_{\mu}(x) = U_{\mu}^{F}(x) \in$ (4) and *Uµ*(*x*) = *U^F ^µ* (*x*) 2 *Sp*(4) (3.2) In the property of the Dirac operator operator in the 2000 minutes and 2-index $P \cdot P$ 1 2 (**1** + 0) (3.16) *e.g. Del Debbio, Patella & Pica (2008) for SU(N)* $(0 \t 0 \t 10)$ e_{AS} is antisymmetric and Ω -traceless,

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⇠ 6*.*5 (3.9)

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Lattice formulation with the standard Wilson gauge & fermion actions gauge sector: player e p 1 ² *^a* (*a*1) *,* for *c < a,* Wilson gauge & fermion actions

fermion sector: Wilson-Dirac formulation for fermions in two distinct representations

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3 Lattice setup

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4 Observables

HMC + RHMC

2 *a N*

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S = \bigotimes_{x} \sum_{\mu < \nu} \left(1 - \frac{1}{4} \text{Re Tr} \, U_{\mu}(x) U_{\nu}(x + \hat{\mu}) U_{\mu}^{\dagger}(x + \hat{\nu}) U_{\nu}^{\dagger}(x) \right) + a^4 \sum_{x} \overline{Q}_j(x) D^F Q_j(x) + a^4 \sum_{x} \overline{\Psi}_k(x) D^{AS} \Psi_k(x) D^F Q_j(x) \equiv (4/a + m_0^f) Q_j(x) - \frac{1}{2} \sum_{x} \left\{ (1 - \gamma_{\mu}) U_{\mu}^F(x) Q_j(x + \hat{\mu}) + (1 + \gamma_{\mu}) U_{\mu}^F(x - \hat{\mu}) Q_j(x - \hat{\mu}) \right\},
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where $U_{\mu}(x) = U_{\mu}^{\dagger}(x) \in Sp(4)$ and $U(AS)$ $\qquad \qquad (x \rightarrow \mathbb{F} \left[(ab) \setminus \dagger_{\mathbf{I}^T} (x) (cd) \mathbf{I}^T \right]$ $\qquad \qquad (1 \rightarrow 0)$ $\left\{ \begin{array}{c} \n\mu \quad \text{(a0)(ca)} \quad \text{(a)} \quad \text{(b) } \quad \text{(c) } \quad \text{(d) } \quad \text{(e)} \quad \text{(f) } \quad \text{(g)} \quad \text{(g)} \quad \text{(h) } \quad \text{(i)} \quad \text{(i)} \quad \text{(ii)} \quad \text{(iii)} \quad \text{(iv) } \quad \text{(v) } \quad \text{(v) } \quad \text{(vi) } \quad \text{(v) } \quad \text{(vi) } \quad \text{(v) } \quad \text{(vi) } \quad \$ antisymmetric representation, we follow the prescription in [12]. For *Sp*(2*N*), we define *D_{* μ *}(<i>x*) = $U_{\mu}^{F}(x) \in$ (4) and *^µ* (*x*) *^k*(*x* + ˆ*µ*) + (1 + *µ*)*UAS ^µ* (*x µ*ˆ) *^k*(*x µ*ˆ) *, Uµ*(*x*) = *U^F ^µ* (*x*) 2 *Sp*(4) (3.2) where $U_{\mu}(x) = U_{\mu}^{F}(x) \in Sp(4)$ and $\mathcal{E}_{\mu}^{F}(x)\in Sp(4)$ and e.g. Del Debbio, Patella & Pica (2008) for SU(N) $(U_{\mu}^{AS})_{(ab)(cd)}(x) \equiv \text{Tr}\left[(e_{AS}^{(ab)})^{\dagger}U_{\mu}(x)e_{AS}^{(cd)}\right]$ antisymmetric representation, we follow the prescription in [12]. For *Sp*(2*N*), we define $\int_{(ab)(cd)} (x) \equiv \text{Tr} \left[(e_{AS}^{(ab)})^{\dagger} U_{\mu}(x) e_{AS}^{(cd)} U_{\mu}^{T}(x) \right]$ **i** *,* with $a < b$, $c < d$. $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 &$ defined by $\sqrt{2}$ 0 0 10 $\overline{1}$ 1 2 (**1** + 0) (3.16) *e.g. Del Debbio, Patella & Pica (2008) for SU(N)* $(0 \t 0 \t 10)$

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Simulation details

HiRep code with appropriate modifications: *Del Debbio, Patella & Pica (2008)*

- $\sum_{i=1}^{n}$ resymplectisation *E. Bennett el al (2017)*
- \mathcal{C} reduced matrix *E. Bennett el al (2018)*
- $\sum_{i=1}^{n}$ antisymmetric representation *E. Bennett el al (2019)*
- $\sum_{i=1}^{n}$ multiple representations (fund. + two-index rep.) *This work*
- Using HMC (RHMC) algorithms, we simulate lattice *Sp(4)* theory coupled to both *Nf=2 F* & *nf=3 AS* Dirac fermions.
- For the exploratory studies of hadron spectrum we have used point sources while leaving more sophisticated measurements in our future work.

Results I: bare parameter space P ults I: bare $hat{E}$ *a a* α *alle to a containeter space* and *a* α

^µ (*x*) descends from the fundamental link variables *Uµ*(*x*), as

(*ab*)(*cd*) (*x*) ⌘ Tr ^h

AS vanishes identically. The explicit form of the antisymmetric link variables

Uµ(*x*)*e*

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

(*cd*)

AS ^U^T

^µ (*x*)

i

, with *a < b, c < d.* (3.5)

⇠ 6*.*5 (3.9)

. 6*.*7 (3.6)

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(*e*

AS)

the matrix *e*

UAS

by replacing (*Uµ*)*ab* by (*UAS*

Gradient flow method

(13)

UAS

From our previous studies of *Nf=2 F Sp(4)* & *nf=3 AS Sp(4)* we learned that 1st order bulk phase transitions exist for $\beta \lesssim 6.7$ & $\beta \lesssim 6.5$, respectively. F_{1} and F_{2} is obtained the $\frac{1}{2}$ index and F_{2} is obtained F_{2} is obtained the $\frac{1}{2}$ is F_{2} *^µ*)(*ab*)(*cd*) and *Q* by in Eq. (??). *E. Bennett el al (2018) JWL el al (2019)*

coupled to *N^f* = 2 fundamental (F) and *n^f* = 3 two-index antisymmetric (AS) Dirac

3.2 (Rational) Hybrid Monte Carlo

interpolating operators for spin-0 and spin-1 flavored mesons

fermions. The three relevant bare parameters are the lattice gauge coupling, , and the

Figure 1: A schematic phase diagram of the bare parameter space of *Sp*(4) gauge theories *Nf=2 fund. Sp(4)* $\frac{1}{2}$

Results I: bare parameter space P ults I: bare $hat{E}$ *a a* α *d. (<i>C*₂) in Eq. (*Co*₂) in Eq. (*Co₂)* in Eq. (*Co2)* in Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained *c* ⇠ 6*.*5 (3.9)

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fermions. The three relevant bare parameters are the lattice gauge coupling, , and the

3.2 (Rational) Hybrid Monte Carlo

Results I: bare parameter space ^c ⇠ 6*.*5 (3.9) narameter cr 0.60

0.58

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0.58

grant funded by the Korea government(MSIT) (NRF-2018R1C1B3001379) and in part by

Korea Research Fellowship programme funded by the Ministry of Science, ICT and Future o

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⇠ 6*.*7 (3.8)

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Figure 2: A schematic phase structure of the bare fermion masses at = 6*.*4 for *Sp*(4)

4.2 Chimera baryons

Results I: bare parameter space ^c ⇠ 6*.*5 (3.9)

Figure 2: A schematic phase structure of the bare fermion masses at = 6*.*4 for *Sp*(4)

4.2 Chimera baryons

Figure 3: Average plaqutte values obtained by starting from cold (unit) and hot (random)

c

⇠ 6*.*7 (3.8)

Results I: bare parameter space rults 1. hare narameter snace = 6*.*4 (3.10)

⇠ 6*.*4 (3.11)

⇠ 6*.*5 (3.9)

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

c

⁰ = 0*.*6 (3.12)

. 6*.*5 (3.7)

⇠ 6*.*5 (3.9)

⇠ 6*.*4 (3.11)

c

● . 6.5 (3.9)

a If both *Nf*=2 F & *nf*=3 AS Dirac fermions are present, the weak coupling The work of CJDL is supported by the Work of CJDL is supported by the Taiwanese MoST grant 105-2628-M-009-003*egion is extended to the smaller beta value of* $\beta \sim 6.4$. $H \equiv 3$ AS Dirac le $Q \neq 2$ **F** & $n \neq 3$ AS Dirac fermions are present the weak coupling

HMC + RHMC + RHMC

3.3 Scale setting

interpolating operators for spin-0 and spin-1 flavored mesons

3.2 (Rational) Hybrid Monte Carlo

4.2 Chimera baryons

(unit) and hot (random) configurations. We fix the bare mass of the fundamental fermion by

c

The work of BL and MP has been supported in part by the STFC Consolidated Grants

Numerical simulations have been performed on the Swansea SUNBIRD system, on

ST/L000369/1 and ST/P00055X/1. The work of BL is further supported in part by the

Royal Society Wolfson Research Merit Award WM170010.

DV acknowledges support from the INFN HPC-HTC project.

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HMC + RHMC

3.3 Scale setting

3.2 (Rational) Hybrid Monte Carlo

Results II: Finite Volume effects *c* ⇠ 6*.*7 (3.8) ⇠ 6*.*4 (3.11) *a m^f*

Finite volume effects are expected to be negligible if $7 \lesssim m_{\rm PS}^f\, L$ & $11 \lesssim m_{\rm PS}^{as}\, L$. HMC + RHMC

HMC + RHMC + RHMC

Gradient flow method

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3.2 (Rational) Hybrid Monte Carlo

 $\beta = 6.5$ *, a* $m_0^{as} = -1.01$ *, a* $m_0^f = -0.71$

. 6*.*5 (3.7)

⇠ 6*.*7 (3.8)

⇠ 6*.*5 (3.9)

= 6*.*4 (3.10)

.4 (3.11)

⁰ = 0*.*6 (3.12)

⁰ = 0*.*71 (3.13)

PS *L* (3.14)

c

⁰ = 0*.*6 (3.12)

= 6*.*4 (3.10)

⁰ = 0*.*71 (3.13)

PS *L* (3.15)

Results II: Finite Volume effects M_{max} i.e. M_{max} i.e. M_{max} i.e. M_{max} **A**, and the sequence of α Results II. Finite Volu

¹⁶⇡² log*M*²

*^F*² ⁺ *^bM*(*µ*)

*M*²

Here is the finite volume correction whose asymptotic form is finite volume correction whose asymptotic form is
In the finite volume correction whose asymptotic form is finite volume of the finite volume of the finite volu

*^F*² ⁺ *^O*(*M*4)

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*^µ*² *,* (A.2)

As discussed in details in Ref. [3], the coefficients are different depending on the sym-

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finite volume correction enhances the masses of pseudoscalar and vetor mesons in the case

of *Sp*(4) with fundamental flavors. Therefore, the results shown in Fig. 4 are consistent

with the PT prediction in which now the finite volume correction in which now the masses of masses.

, for *SU*(2*N^f*) ! *Sp*(2*N^f*)*.* (A.6)

, (A.1)

✓*M*⇡

² ¹

 $B_{\rm eff}$ comparing those coefficients, we immediately notice that \mathbb{R} has different signals signals signals signals signals.

with Eq. (A.6) if α is greater than or equal to unity. As we also units and the unity. As we also use that

finite volume correction enhances the masses of pseudoscalar and vetor mesons in the case

of *Sp*(4) with fundamental flavors. Therefore, the results shown in Fig. 4 are consistent

2*N^f*

. (A.5)

2*L*³

◆1*/*²

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, for *SU*(2*N^f*) ! *Sp*(2*N^f*)*.* (A.6)

A(*M*)

^A(*M*) = *^M*²

exp[*ML*]*.* (A.4)

*m*²

The finite volume correction to the pseudoscalar mass can be understood in the framework

of chiral perturbation theory (PT). We start with the infinite volume version of continuum

with \mathcal{H} the renormalization scalar and \mathcal{H} the pseudoscalar decay constant in the chiral limit.

^a^M ⁼ ¹

, for *SU*(*N^f*) ⇥ *SU*(*N^f*) ! *SU*(*N^f*)*,*

In the other two cases, the coefficients are

*m*²

!

3

✓*M*⇡

2*L*³

◆1*/*²

4⇡²

*^A*FV(*M*) *ML*¹

*m*²

N^f

² ¹

In the other two cases, the coefficients are

^a^M ⁼ ¹

^a^M ⁼ ¹

PS = *M*²

A, a consequence of one-loop calculation, is given as

with the PT prediction in which now the finite volume correction lower the masses.

PS = *M*²

Here *A*FV denotes the finite volume correction whose asymptotic form is [5]

✓

1 + *a^M*

The different signs of finite volume effects can be understood from the low-energy effective field theory. Bijnens & Lu (2009) replaced by a finite sum, the finite-volume version of Eq. (A.1) can be written as PT, where the next-to-leading order (NLO) results are sufficient to our discussion. The pseudoscalar mass at NLO is *M* CHOIGHT CHOOL IN L *Bijnens & Lu (2009)* The different signs of finite volume effects can be understod **details brand breaking participates of the rest of the functional form in Eq. (A.3) remains in Eq. (A.3) remains same.** experience the second the second terms of 3 $\frac{1}{2}$ ✓*M*⇡ 2*L*³ effects can be understood from the **the finite of the finite values** of \mathbb{R}^n \mathcal{A} discussed in Ref. [3], the coefficients are different depending on the symmetric depending on the symmet

$$
m_{\rm PS}^2 = M^2 \left(1 + \widehat{\hbox{a_{M}}} \frac{A(M) + A_{\rm FV}(M)}{F^2} + b_M(\mu) \frac{M^2}{F^2} + {\cal O}(M^4) \right)
$$

$$
A(M) = -\frac{M^2}{16\pi^2} \log \frac{M^2}{\mu^2} \qquad A_{\rm FV}(M) \stackrel{ML \gg 1}{\longrightarrow} -\frac{3}{4\pi^2} \left(\frac{M\pi}{2L^3}\right)^{1/2} \exp[-ML]
$$

^a^M ⁼ ¹

^a^M ⁼ ¹

^a^M ⁼ ¹

finite volume correction enhances the masses of pseudoscalar and vetor mesons in the case

of *Sp*(4) with fundamental flavors. Therefore, the results shown in Fig. 4 are consistent

with the PT prediction in which now the finite volume correction in which now the masses of the masses. In the

² ¹

² ¹

2

, for *SU*(*N^f*) ⇥ *SU*(*N^f*) ! *SU*(*N^f*)*,*

N^f

N^f

Chimera baryon as top partner global constant contractors con *a* $\overline{1}$ 2 B@ $n \ncup 1$ n as top pa ر
اب t n \overline{a} $\overline{}$ with *a* = 1*,* 2*,* 3, ⌧ *^a* the Pauli matrices, and the eight fields *hr,i* and ⇡*^a r*imera The chimera baryons must have the same quantum numbers as the top quark, in such a h Im *e* i Chimera baruon as top partner tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parameters parameters parameters parameters parameters parameters parameters parameters μ lating any of the symmetries. For what concerns *SU*(2)*L*, the aforementioned assignments lating any of the symmetries. For what concerns *SU*(2)*L*, the aforementioned assignments Ed Daigon and Communication sufficients and q $\mathbf{1}$ in Eqs. (2.6) would suffice the quantum numbers of the left-handed and left-handed a right-handed quarks, in terms of the 4 of *SO*(4). In order to add *SU*(3) colour, and to form

The *^T^L* generators satisfy the *SU*(2)*^L* algebra ^h

^L, and similarly ^h

^r ⌧ *^a*

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The chimera baryons must have the same quantum numbers as the same quantum numbers as the top quark, in such a

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^R , T^j

*hi***1**² + *i*⇡*^a*

R

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in Eqs. (2.6) and (2.9) would suffice to give the quantum numbers of the left-handed and

right-handed quarks, in terms of the 4 of *SO*(4). In order to add *SU*(3) colour, and to form

0

^L^B ⁼ *gQ*¯*^µ*⌫*Bµ*⌫*PLQ .* (B7)

^L, and similarly ^h

CA *,* (C1)

CA *.* (C2)

, (C5)

1

*Q*¹ *^a*5*Q*² *^b* + *Q*² *^a*5*Q*¹ *^b*

1

^C + *Q*² *^a*

⁰³ *^B*⇤

*^C ^Q*¹ *^b*

✓ **0**³ **0**³

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

^R , T^j

*Q*¹ *^aQ*² *^b* + *Q*² *^aQ*¹ *^b*

✓ **0**³ **0**³

*ⁱ ^Q*¹ *^aQ*² *^b*

D

i

^C + *iQ*² *^a*

While *L^A* couples the spin-1 field to the LH component only of *Q*, in *L^B* the LH and RH projections are coupled to

i

^R, while

, (2.10) , (2.1

unbroken subgroup *SO*(4) ⇠ *SU*(2)*^L* ⇥ *SU*(2)*^R* is the subset of the unbroken global *Sp*(4) ⇢ *SU*(4) that is generated

= *i*✏*ijk T ^k*

Ti

0

CA *, T*²

ⁱ ⌧ *^a*

1

one another, so that while *L*⁰ and *L*¹ in isolation define the same theory, the addition of *L^A* or *L^B* leaves di↵erent

1

0 *i* 0 0

CA *, T*³

= *i*✏*ijk T ^k*

^L =

^R =

2

1

^L, and similarly ^h

0

lating any of the symmetries. For what concerns *SU*(2)*L*, the aforementioned assignments

way that one can construct bilinear coupling with the standard-model quarks with the standard-model quarks with

The chimera baryons must have the same quantum numbers as the same quantum numbers as the top quark, in such as

Book of the Control of

a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a

0

B. S. A. Maria Bara

0 0 *i* 0

^L , T^j

r,i all real.

Appendix C: About Lie groups, algebras and SM embedding

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special choice of *SU*(4) generators can be found elsewhere [50]—but we explicitly show the embedding of the SM

antisymmetric representation of *Sp*(4) matches the number of colours in the *SU*(3)*^c* gauge group of the standard

, (2.11)

Recall the global symmetry and its spontaneous breaking Here we summarise some group theory notions relevant for models of composite Higgs and top quark compositeness \overline{a} $\overline{\mathsf{d}}$ $\overline{16}$ spontan **POUS Drear**
 P $\dot{\mathsf{n}}$ \overline{a} Recall the global symmetry and its spontaneous breaking in Eqs. (2.6) and (2.9) would suffice to give the quantum numbers of the left-handed and ⇡¹(*x*) + *ⁱ*⇡²(*x*) ⇡³(*x*) *ⁱ*⇡⁴(*x*) ⇡⁵(*x*) 0 $\frac{1}{2}$ ivocal and grobal by initial $\frac{1}{2}$ and its operations one King
Carried Carried Construction of the Second **Example 2018 Recall the global symmetry and its spontaneous breaking** natural *SU*(3)*^L* ⇥ *SU*(3)*^R* subgroup, and that both the mass term and the strong-coupling

by the following elements of the associated algebra:

with *a* = 1*,* 2*,* 3, ⌧ *^a* the Pauli matrices, and the eight fields *hr,i* and ⇡*^a*

*hr***1**² + *i*⇡*^a*

, (2.10)

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

= *i*✏*ijk T ^k*

r,i all real.

p2

the *SU*(4)*/Sp*(4) coset is

⇡³(*x*) ⇡¹(*x*) *ⁱ*⇡²(*x*) 0 *i*⇡⁴(*x*) + ⇡⁵(*x*)

 $SU(4)/Sp(4)\otimes SU(6)/SO(6)$ $\frac{1}{2}$ (*x*) $\frac{1}{2}$ $SU(4)/Sp(4)\otimes SU(6)/SO(6)$ (6) *ⁱ*⇡⁴(*x*) + ⇡⁵(*x*) 0 ⇡¹(*x*) *ⁱ*⇡²(*x*) ⇡³(*x*) $\mathcal{O}(\mathbf{F})/\mathcal{O}(\mathbf{F}) \otimes \mathcal{O}(\mathbf{F})/\mathcal{O}(\mathbf{F})$ $\mathbf{S}U(4)/\mathbf{S}_p(4)\otimes \mathbf{S}U$ $\mathcal{U}(\pm)/\mathcal{U}(\pm)\otimes \mathcal{U}$ $\mathcal{L}(\mathcal{C})$ $\mathcal{L}(\mathcal{C})$ will combine with the *T*₃ generator of *SU*₍₂₎ to yield ordinary hypercharge.

where $SO(4)$ subgroup of $Sp(4) \sim SU(2)_L$ gauge group in SM & The *SU*(4)*/Sp*(4) coset governs the Higgs sector of the Standard Model. Given the form of ⌦ in Eq. (5), the $SII(3)$ subgraun of $SO(6) \approx S$ i = *i*✏*ijk T ^k* $SU(3)$ \sum $SU(3)$ subgroup of $SO(6) \sim SU(3)_c$ gauge group in SM a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a where $SO(4)$ subgroup of $Sp(4) \sim SU(2)L$ gauge group in SM & $\mathcal{L}(G(4), \alpha)$ is a subsequent of $\mathcal{L}(G(4), \alpha)$ is a sun SUSL which $\mathcal{SO}(4)$ subgroup of $\mathcal{O}p(4)$ \cong $\mathcal{SO}(2)_L$ gauge group in one α $Q(t)$ and $Q(t)$ a between of $C_n(1)$ $SII(2)$ ⁻ agrees and the CM θ where $D\sigma(\pm)$ dabyted σ $p(\pm)$ ∞ σ $(-)$ *R* yaayo yield ni ont a where $SO(4)$ subgroup of $Sp(4) \sim SU(2)_L$ gauge group in SM & α and the fermion that the antisymmetric representation α simply α s $SU(3)$ Subgroup of $SU(6) \sim SU(2)$ replace **order in Eq. (2.5)** with a comparison α $f^{(0)}(c)$ gad

Then, the top partner can be sourced by the operators *L* **T**₁ r ner can <mark>l</mark> b 0 0 *i* 0 ne operato 0 0 *i* 0 Then, the cop paruit $\ddot{}$ be source *Figure 1 combine is the sourced by the operators*
 SL^R $\left(\frac{1}{2^{1.5}} \frac{5}{2} \frac{2^h}{2} \frac{5}{2} \frac{2^{1}h}{2} \frac{5}{2} \frac{2^{1}h}{2} \right)$ tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parametrising QCD. We also notice that a *U*(1)*^X* that commutes with *SU*(3)*^V* is also unbroken, and this **partner can be sourced by the operators SU(3)** model. The natural subgroup *SU*(3)*^L* ⇥ *SU*(3)*^R* ⇢ *SU*(6) is generated by 1 ✓ *^B* **0**³ ◆ 1 ✓ **0**³ **0**³ ◆ replace ⌦*ab* in Eq. (2.5) with ⌦*ab* ! ⌦*acPL,R k cd*⌦*db*. Hence, the operators *^OL,R* \blacksquare Then,

Then, the top parameter can be solved by the operations
\n
$$
\mathcal{O}_{\text{CB},1}^{L,R} = \left(\overline{Q^{1a}}\gamma^5 Q^{2b} + \overline{Q^{2a}}\gamma^5 Q^{1b}\right) \Omega_{bc} P_{L,R} \Psi^{kca},
$$
\n
$$
\mathcal{O}_{\text{CB},2}^{L,R} = \left(-i\overline{Q^{1a}}\gamma^5 Q^{2b} + i\overline{Q^{2a}}\gamma^5 Q^{1b}\right) \Omega_{bc} P_{L,R} \Psi^{kca},
$$
\n
$$
\mathcal{O}_{\text{CB},4}^{L,R} = -i\left(\overline{Q^{1a}}Q_C^{2b} + \overline{Q_C^{2a}}Q^{1b}\right) \Omega_{bc} P_{L,R} \Psi^{kca}
$$
\n
$$
\mathcal{O}_{\text{CB},5}^{L,R} = i\left(-i\overline{Q^{1a}}Q_C^{2b} + i\overline{Q_C^{2a}}Q^{1b}\right) \Omega_{bc} P_{L,R} \Psi^{kca}.
$$

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

, (C5)

⌦*bcPL,R k ca*

✓ *^B* **0**³

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¹⁴ ! *ⁱ*⁵ inside the bilinear in *^Q*:

, t^B

*O*⁰ *L,R*

^R =

CB*,*¹ = *i*

1

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✓ **1**³ **0**³

tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parametrising

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model. The natural subgroup *SU*(3)*^L* ⇥ *SU*(3)*^R* ⇢ *SU*(6) is generated by

^L =

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independent, unbroken generator of *SU*(6) is given by

SO(4) = *SU*(2)*^L* ⇥ *SU*(2)*R*. We write explicitly also the operators obtained by replacing

*Q*¹ *^aQ*² *^b* + *Q*² *^aQ*¹ *^b*

which transform 3 of $SU(3)_c$ and 4 of $SO(4)$ *^L , T^j M*ightransform 3 of $SU(3)$, and 4 of $SO(4)$. antisymmetric representation of $SU(3)_c$ and 4 of $SO(4)$ and standard wrich trans α are α of $\mathcal{S}U(3)$ and Λ of $\mathcal{S}O(1)$ which transform 3 of $SU(3)_c$ and 4 of $SO(4)$. $SO(4)$ **c** independent, unbroken generator of *SU*(6) is given by $SO(4)$ **.**

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✓ **1**³ **0**³

*O*⁰ *L,R*

0³ **1**³

CB*,*¹ = *i*

Ti

^C + *iQ*² *^a*

CA *, T*²

^r ⌧ *^a*

The *^T^L* generators satisfy the *SU*(2)*^L* algebra ^h

with *a* = 1*,* 2*,* 3, ⌧ *^a* the Pauli matrices, and the eight fields *hr,i* and ⇡*^a*

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way that one can construct bilinear construction with the standard-model quarks with the standard-model quarks

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The chimera baryons must have the same quantum numbers as the top quark, in such a

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right-handed quarks, in terms of the 4 of *SO*(4). In order to add *SU*(3) colour, and to form

a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a

way that one can construct bilinear couplings with the standard-model quarks with the standard-model quarks with

CA *, T*²

*Q*¹ *^aQ*² *^b*

*ⁱ ^Q*¹ *^aQ*² *^b*

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

C

^C + *Q*² *^a*

*Q*¹ *^a*5*Q*² *^b* + *Q*² *^a*5*Q*¹ *^b*

^L =

*Q*¹ *^a*5*Q*² *^b* + *Q*² *^a*5*Q*¹ *^b*

the first

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

^R =

^C + *Q*² *^a*

^C + *iQ*² *^a*

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by the following elements of the associated algebra:

The *T^L* generators satisfy the *SU*(2)*^L* algebra

L, T^j

*^C ^Q*¹ *^b*

SO(4) = *SU*(2)*^L* ⇥ *SU*(2)*R*. We write explicitly also the operators obtained by replacing

*^C ^Q*¹ *^b*

the *SU*(4)*/Sp*(4) coset is

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tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parametrising

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

⌦*bcPL,R k ca .*

 $\frac{1}{2}$

¹⁴ ! *ⁱ*⁵ inside the bilinear in *^Q*:

The *SU*(4)*/Sp*(4) coset governs the Higgs sector of the Standard Model. Given the form of ⌦ in Eq. (5), the

*hi***1**² + *i*⇡*^a*

ⁱ ⌧ *^a*

Ti

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^L , T^j

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Chimera baryon as top partner global constant contractors con *a* $\overline{1}$ 2 B@ $n \ncup 1$ n as top pa ر
اب t n \overline{a} $\overline{}$ with *a* = 1*,* 2*,* 3, ⌧ *^a* the Pauli matrices, and the eight fields *hr,i* and ⇡*^a r*imera The chimera baryons must have the same quantum numbers as the top quark, in such a h Im *e* i Chimera baruon as top partner tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parameters parameters parameters parameters parameters parameters parameters parameters μ lating any of the symmetries. For what concerns *SU*(2)*L*, the aforementioned assignments Ed Daigon and Communication sufficients and q $\mathbf{1}$ in Eqs. (2.6) would suffice the quantum numbers of the left-handed and left-handed a right-handed quarks, in terms of the 4 of *SO*(4). In order to add *SU*(3) colour, and to form \blacksquare *Chimera baryon as top partner* ⇣ *ad***₂** *d***₂** *d***₂** *ac***_{***d***₂** *d***₂** *d***₂** *d***₂** *d***₂**} *P[±]* = 1 (**1** + 0) (3.17) (3

will combine with the *T3 generator of SU(2)* and SU(2) to yield ordinary hypercharge. The substitution of *SU(2)*

The *^T^L* generators satisfy the *SU*(2)*^L* algebra ^h

^L, and similarly ^h

QCD. We also notice that a *U*(1)*^X* that commutes with *SU*(3)*^V* is also unbroken, and this

¹⁴ ! *ⁱ*⁵ inside the bilinear in *^Q*:

vacuum break *SU*(3)*^L* ⇥ *SU*(3)*^R* ! *SU*(3)*^V* . We identify this *SU*(3)*^V* with the *SU*(3)*^c* of

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1

*O*⁰ *L,R*

*O*⁰ *L,R*

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CB*,*¹ = *i*

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11 . *mas*

^R , T^j

R

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Recall the global symmetry and its spontaneous breaking Here we summarise some group theory notions relevant for models of composite Higgs and top quark compositeness \overline{a} $\overline{\mathsf{d}}$ $\overline{16}$ spontan **POUS Drear**
 P $\dot{\mathsf{n}}$ \overline{a} Recall the global symmetry and its spontaneous breaking in Eqs. (2.6) and (2.9) would suffice to give the quantum numbers of the left-handed and ⇡¹(*x*) + *ⁱ*⇡²(*x*) ⇡³(*x*) *ⁱ*⇡⁴(*x*) ⇡⁵(*x*) 0 $\frac{1}{2}$ ivocal and grobal by initial $\frac{1}{2}$ and its operations one King
Carried Carried Construction of the Second **Example 2018 Recall the global symmetry and its spontaneous breaking and its spontaneous breaking** natural *SU*(3)*^L* ⇥ *SU*(3)*^R* subgroup, and that both the mass term and the strong-coupling μ *metry and its spontaneous breaking* α *C* aneous b $\ddot{}$ *aking* \Box **Pontaneous breaking**

by the following elements of the associated algebra:

, (2.10)

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

= *i*✏*ijk T ^k*

r,i all real.

the *SU*(4)*/Sp*(4) coset is

⇡³(*x*) ⇡¹(*x*) *ⁱ*⇡²(*x*) 0 *i*⇡⁴(*x*) + ⇡⁵(*x*)

 $SU(4)/Sp(4)\otimes SU(6)/SO(6)$ $\frac{1}{2}$ (*x*) $\frac{1}{2}$ $SU(4)/Sp(4)\otimes SU(6)/SO(6)$ (6) *ⁱ*⇡⁴(*x*) + ⇡⁵(*x*) 0 ⇡¹(*x*) *ⁱ*⇡²(*x*) ⇡³(*x*) $\mathbf{S}U(4)/\mathbf{S}_p(4)\otimes \mathbf{S}U$ $\mathcal{O}(\pm)/\mathcal{O}p(\pm)\otimes \mathcal{O}O$ $\mathcal{L}(\mathcal{C})$ $\mathcal{L}(\mathcal{C})$ will combine with the *T*₃ generator of *SU*₍₂₎ to yield ordinary hypercharge. $SU($ $SU(4)/Sp(4)\otimes SU(6)/SO(6)$

where $SO(4)$ subgroup of $Sp(4) \sim SU(2)_L$ gauge group in SM & The *SU*(4)*/Sp*(4) coset governs the Higgs sector of the Standard Model. Given the form of ⌦ in Eq. (5), the $SII(3)$ subgraun of $SO(6) \approx S$ i = *i*✏*ijk T ^k* $SU(3)$ \sum $SU(3)$ subgroup of $SO(6) \sim SU(3)_c$ gauge group in SM a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a where $SO(4)$ subgroup of $Sp(4) \sim SU(2)L$ gauge group in SM & $\mathcal{L}(G(4), \alpha)$ is a subsequent of $\mathcal{L}(G(4), \alpha)$ is a sun SUSL which $\mathcal{SO}(4)$ subgroup of $\mathcal{O}p(4)$ \cong $\mathcal{SO}(2)_L$ gauge group in one α where $SO(4)$ subgroup of $Sp(4) \sim SU(2)_L$ gauge group in SM & α and the fermion that the antisymmetric representation α simply α s $SU(3)$ Subgroup of $SU(6) \sim SU(2)$ replace **order in Eq. (2.5)** with a comparison α $f^{(0)}(c)$ gad p of $Sp(4) \sim SU(2)_L$ gauge group in SM & $SU(3)$ subgroup of $SO(6) \thicksim SU(3)_c$ gauge group in SM replicas, which one expects to become degenerate with those of *^OL,R*

We also consider the $U(1)_A$ counterparts ($1\to i\gamma^5$, expected to be heavier) **T**₁ \overline{c} the $U(1)$ ar $\frac{1}{2}$ $\frac{1}{2}$ $\frac{5}{2}$ $\mathbb{1} \to i\gamma^5$, *a* $\overline{}$ B@ neavier) 0 0 *i* 0 We also d $\boldsymbol{\beta}^{I} L R$ Λ counter \mathbf{f} **SP(4)** Specified with the *V*(1)_{*A*} consider the *V*(1)_{*A*} consider QCD. We also notice that a *U*(1)*^X* that commutes with *SU*(3)*^V* is also unbroken, and this sider the $U(1)_A$ counterparts ($\mathbb{1} \to i \gamma^5$, expected to be heavier) is a subgroup of I model. The natural subgroup *SU*(3)*^L* ⇥ *SU*(3)*^R* ⇢ *SU*(6) is generated by We also consider the $U(1)_A$ counterparts ($1 \rightarrow i \gamma^5$, expected to b \overline{a} *a***5***<i>b* \rightarrow *b* \rightarrow *b* \rightarrow *b* \rightarrow *a*⁵ *SO*(4) = *SU*(2)*^L* ⇥ *SU*(2)*R*. We write explicitly also the operators obtained by replacing **1** We also consider the $U(1)_A$ counterparts ($1 \rightarrow i \gamma^5$, expected to be heavier)

$$
\mathcal{O}_{\text{CB},1}^{l,L,R} = i \left(\overline{Q^{1a}} Q^{2b} + \overline{Q^{2a}} Q^{1b} \right) \Omega_{bc} P_{L,R} \Psi^{kca},
$$

$$
\mathcal{O}_{\text{CB},2}^{l,L,R} = \left(\overline{Q^{1a}} Q^{2b} - \overline{Q^{2a}} Q^{1b} \right) \Omega_{bc} P_{L,R} \Psi^{kca},
$$

$$
\mathcal{O}_{\text{CB},4}^{l,L,R} = \left(\overline{Q^{1a}} \gamma^5 Q_C^{2b} + \overline{Q_C^{2a}} \gamma^5 Q^{1b} \right) \Omega_{bc} P_{L,R} \Psi^{kca},
$$

$$
\mathcal{O}_{\text{CB},5}^{l,L,R} = i \left(\overline{Q^{1a}} \gamma^5 Q_C^{2b} - \overline{Q_C^{2a}} \gamma^5 Q^{1b} \right) \Omega_{bc} P_{L,R} \Psi^{kca}.
$$

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

, (C5)

✓ *^B* **0**³

◆

¹⁴ ! *ⁱ*⁵ inside the bilinear in *^Q*:

, t^B

*O*⁰ *L,R*

tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parametrising

1

model. The natural subgroup *SU*(3)*^L* ⇥ *SU*(3)*^R* ⇢ *SU*(6) is generated by

^L =

CB operators, while the *^O*⁰ *L,R*

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independent, unbroken generator of *SU*(6) is given by

symmetric *k ab* to the singlets in Eqs. (2.7) and (2.8), and their CP partners. The top

which transform 3 of $SU(3)_{c}$ and 4 of $SO(4)$. i = *i*✏*ijk T ^k ^L , T^j* ch transform 3 of $SU(3)_c$ and 4 of $SO(4)$. See Talk by H. Hsiao wrich trans $\lim_{n \to \infty} 3 \text{ of } SU(3)$ and $\lim_{n \to \infty} SO(4)$ See Talk by H Hsiao which transform 3 of $SU(3)_c$ and 4 of $SO(4)$. See Talk by H. Hsiao independent, unbroken generator of *SU*(6) is given by

◆

✓ **1**³ **0**³

partners are hence sourced by the *^OL,R*

0³ **1**³

Ti

^C + *iQ*² *^a*

CA *, T*²

^r ⌧ *^a*

The *^T^L* generators satisfy the *SU*(2)*^L* algebra ^h

with *a* = 1*,* 2*,* 3, ⌧ *^a* the Pauli matrices, and the eight fields *hr,i* and ⇡*^a*

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way that one can construct bilinear construction with the standard-model quarks with the standard-model quarks

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The chimera baryons must have the same quantum numbers as the top quark, in such a

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right-handed quarks, in terms of the 4 of *SO*(4). In order to add *SU*(3) colour, and to form

a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a

way that one can construct bilinear couplings with the standard-model quarks with the standard-model quarks with

CA *, T*²

*Q*¹ *^aQ*² *^b*

*ⁱ ^Q*¹ *^aQ*² *^b*

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

C

^C + *Q*² *^a*

*Q*¹ *^a*5*Q*² *^b* + *Q*² *^a*5*Q*¹ *^b*

^L =

*Q*¹ *^a*5*Q*² *^b* + *Q*² *^a*5*Q*¹ *^b*

the first

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

^R =

^C + *Q*² *^a*

^C + *iQ*² *^a*

B@

by the following elements of the associated algebra:

The *T^L* generators satisfy the *SU*(2)*^L* algebra

L, T^j

*^C ^Q*¹ *^b*

SO(4) = *SU*(2)*^L* ⇥ *SU*(2)*R*. We write explicitly also the operators obtained by replacing

*^C ^Q*¹ *^b*

the *SU*(4)*/Sp*(4) coset is

وي
الدون

⌘

tries of the Higgs potential. Following the notation in Refs. [50, 62], the matrix of the 5 pNGB fields parametrising

= 0. In the vacuum aligned with ⌦ in Eq. (5), this is the natural choice of embedding of the *SO*(4) symme-

⌦*bcPL,R k ca .*

 $\frac{1}{2}$

The *SU*(4)*/Sp*(4) coset governs the Higgs sector of the Standard Model. Given the form of ⌦ in Eq. (5), the

*hi***1**² + *i*⇡*^a*

following:

ⁱ ⌧ *^a*

Ti

⌘

^L , T^j

L

OL,R

OL,R

i

Line See Talk by H. Hsiao *CU*(4) consider the *SU*(3) consider the Standard Model Model Model Model. And Standard Model. And Model Model Model. And Standard Model *SO*(4) **See Talk by H. Hsiao**

^R =

CB*,*¹ = *i*

CB operations source heavier heavier

1

⇣

✓ **1**³ **0**³

While *L^A* couples the spin-1 field to the LH component only of *Q*, in *L^B* the LH and RH projections are coupled to

SO(4) = *SU*(2)*^L* ⇥ *SU*(2)*R*. We write explicitly also the operators obtained by replacing

i

^R, while

unbroken subgroup *SO*(4) ⇠ *SU*(2)*^L* ⇥ *SU*(2)*^R* is the subset of the unbroken global *Sp*(4) ⇢ *SU*(4) that is generated

= *i*✏*ijk T ^k*

Ti

*Q*¹ *^aQ*² *^b* + *Q*² *^aQ*¹ *^b*

0

CA *, T*²

1

*Q*¹ *^a*5*Q*² *^b*

one another, so that while *L*⁰ and *L*¹ in isolation define the same theory, the addition of *L^A* or *L^B* leaves di↵erent

1

^C 5*Q*¹ *^b*

⌘

PS *L* (3.16)

0 *i* 0 0

CA *, T*³

J

CB are the

= *i*✏*ijk T ^k*

^L =

⌦*bcPL,R k ca*

⌦*bcPL,R k ca*

^R =

2

⌦*bcPL,R k ca*

1

^L, and similarly ^h

0

lating any of the symmetries. For what concerns *SU*(2)*L*, the aforementioned assignments

way that one can construct bilinear coupling with the standard-model quarks with the standard-model quarks with

The chimera baryons must have the same quantum numbers as the same quantum numbers as the top quark, in such as

Book of the Control of

a fermion bound state, we use the anti-symmetric *k ab*. We recall that *SU*(6) admits a

0

B. S. A. Maria Bara

⌦*bcPL,R k ca .*

0 0 *i* 0

^L , T^j

Appendix C: About Lie groups, algebras and SM embedding

00 0 0

^C + *Q*² *^a*

special choice of *SU*(4) generators can be found elsewhere [50]—but we explicitly show the embedding of the SM

singlets, obtained from adding the left-handed and right-handed projections of the anti-

antisymmetric representation of *Sp*(4) matches the number of colours in the *SU*(3)*^c* gauge group of the standard

^L^B ⁼ *gQ*¯*^µ*⌫*Bµ*⌫*PLQ .* (B7)

^L, and similarly ^h

CA *,* (C1)

CB operators source heavier

CA *.* (C2)

, (C5)

1

1

^C + *Q*² *^a*

⁰³ *^B*⇤

*^C ^Q*¹ *^b*

✓ **0**³ **0**³

*iQ*¹ *^a*5*Q*² *^b* ⁺ *ⁱ ^Q*² *^a*5*Q*¹ *^b*

^R , T^j

*Q*¹ *^aQ*² *^b* + *Q*² *^aQ*¹ *^b*

✓ **0**³ **0**³

*ⁱ ^Q*¹ *^aQ*² *^b*

D

i

^C + *iQ*² *^a*

Interpolating operators for Chimera baryon $In+$ *Ok* ⁼ *ⁱ ^Q*² *^a ^C ^Q*¹ *^b* ⌦*bc k ca* CB*,* ↵ ↵ = *iQ*² *d T* (*C*5)⌦*daQ*1*^b* ⌦*bc k ca* ↵ iterpolating operators for Chimera baryo ⁼ *i*(*C*5) *T* ⇠ 6*.*5 (3.9) = 6*.*4 (3.10) $\frac{1}{2}$ *derpolating operare* $\frac{1}{2}$ *dones de Chimera baruon* \Box nterpolating operators for Chimera baryon be sufficient to have a part of the operator in Eq. 4.2, where we choose the second term of the second term of

to what we have done for the computation of the 2-point computation of the 2-point correlator for mesons, it might

= *iQ*² *d T* (*C*5)⌦*daQ*1*^b*

*Q*¹ *d T*

*^C ^Q*¹ *^b*

*^C ^Q*¹ *^b*

⇣

⁼ *ⁱ ^Q*² *^a*

= *i*

= *i ca*

be sufficient to have a part of the operator in Eq. 4.2, where we choose the second term of the second term of

To compute the baryon two-point correlation function and extract the mass, analogous

⌦*bc k ca* ↵

interpolate the Chimera baryon having the same quantum number of top-partner. We

⌦*bc k ca*

c

^C ⌦*da*(*C*5)*Q*² *^b*

 \mathcal{A} baryon-type operator in \mathcal{A} baryon-type operator in \mathcal{A} , we consider the operator which would would would will be operator which will be a set of \mathcal{A} , we can see that we consider the operator whi

⇠ 6*.*7 (3.8)

⌦*ac*⌦*bdQ*2*^a*

To compute the baryon two-point correlation function and extract the mass, analogous

↵⌦*cb*⌦*adQ*² *^d*(*C*5)*Q*¹ *^b*

⌦*bc k ca* ↵

k code

↵ *,* (4.3)

⊙.8) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9) (3.9)

^C + *Q*² *d T* ⌦*da*(*C*5)*Q*¹ *^b*

. (4.4)

= 6*.*4 (3.11)

⁰ = 0*.*6 (3.13)

*Q*1*^b*

Analogous to the Lambda baryon in QCD, we construct the interpolating operator of a spin-1/2 Chimera baryon as

and its propagator of a spin-1/2 Chimera baryon as ambda banyon in OCD we construct the internalating baryon at positive Euclidean time *t* and vanishing momentum *p*~ Using this interpolating operator, we find the most generic propagator for the Chimera Analogous to the Lambda baryon in QCD, we construct the interpolating Ω Perator of a spin-1/2 Unimera paryon as opera i *mera baryon as* ⇠ 6*.*4 (3.12)

 $\frac{dE}{dE}$ _{CB}_ca(*x*) = $-i(C\gamma^5)^{\beta\gamma}\Omega_{ac}\Omega_{bd}Q_{\beta}^{2a}(x)Q_{cd}^{2a}$ $(0)^{1b}(x)\Psi^{kcd}(x)$ ^h*O*CB(*t*)*O*CB(0)0ⁱ ⁼ ^X $B, \alpha(x)$ $C \cap f$ $T = 2ac - 5bd - 2b$ $C \cap f$ $\mathcal{O}^k_{\mathrm{CB},\,\alpha}(x) \,=\, -i (C\gamma^5)^{\beta\gamma} \Omega_{ac} \Omega_{bd} Q_\beta^{2a}(x) Q_\gamma^{1b}(x) \Psi_\alpha^{k\,cd}(x)$ *b*0 *d*0 ↵0(*C*5)↵00*Q*1(0)*d*⁰

Then, the 2-point correlation function is baryon at positive Euclidean time *t* and vanishing momentum *p*~ ⇥ (0)*c*0*a*⁰ 0⌦*c*⁰ , the *z*-point correlation function is ⁰ = 0*.*71 (3.13) $\frac{1}{2}$ Then, the 2-point correlation function is

Ok

c

*^C ^Q*¹ *^b*

particularly use *O*CB ⁴ in Eq. 2.11

^C + *Q*² *^a*

CB*,* ↵

Ok

CB*,* ↵

Ok

CB*,* ↵

)⌦*bc k ca*

⁼ *ⁱ ^Q*² *^a*

↵

*O*CB*,*4,

and its Dirac conjugate is

ⁱ(*Q*¹ *^aQ*² *^b*

*Uac*⁰ =

^h*O*CB(*t*)*O*CB(0)0ⁱ ⁼ ^X

3.2 (Rational) Hybrid Monte Carlo

HMC + RHMC

⌦*^T S*²

 \sim XX

^Q(*t,* [~]*x*)⌦*^T*

where the trace and transpose are for the spinor indices.

~*x*

~*x*

~*x*

S (*t,* ~*x*)

 $\langle \mathcal{O}_{\text{CB}}(t)_{\gamma} \overline{\mathcal{O}_{\text{CB}}(0)}_{\gamma'} \rangle = \sum$ \vec{x} $\Omega_{da} \Omega_{bc} \Omega^{c'b'} \Omega^{a'd'} S_\Psi(t,\vec{x})^{ca,c'a'}_{\gamma,\gamma'} S_Q^2(t,\vec{x})^{d,b'}_{\alpha,\alpha'}(C)$ $=\sum\Omega_{da}\Omega_{bc}\Omega^{c'b'}\Omega^{a'd'}S_\Psi(t,\vec{x})^{ca,c'a'}_{\gamma,\gamma'}S^2_Q(t,\vec{x})^{d,b'}_{\alpha,\alpha'}(0)$ \vec{x} $\Omega_{da} \Omega_{bc} \Omega^{c'b'} \Omega^{a'd'} S_\Psi(t,\vec{x})^{ca,c'a'}_{\gamma,\gamma'} S_Q^2(t,\vec{x})^{d,b'}_{\alpha,\alpha'} (C\gamma^5)_{\alpha\beta} S_Q^1(t,\vec{x})^{b,d'}_{\beta,\beta'} (C\gamma^5)_{\alpha'\beta'}$ $\frac{\partial^2 u}{\partial x^2}$ ¹*da*³*bc*¹^{*l*} 1*l*</sup> 1*l* $\partial \Psi(t, x)_{\gamma, \gamma'}$ ∂Q $\int_{Q}^{a} S_{Q}^{2}(t, \vec{x})_{\alpha, \alpha'}^{d, b'}(C\gamma^{5})$ $\partial_{\alpha\beta}S^{\dagger}_{Q}(t,x)_{\beta,\beta'}^{\alpha}(C\gamma^{\circ})_{\alpha'\beta'}$ 7 . *m^f* PS *L* (3.14) $\langle \cos(t) \rangle_{\gamma} \overline{\mathcal{O}_{\text{CB}}(0)}_{\gamma'} \rangle = \sum_i \Omega_{da} \Omega_{bc} \Omega^{c'b'} \Omega^{a'd'} S_\Psi(t,\vec{x})^{ca,c'a'}_{\gamma,\gamma'} S_Q^2(t,\vec{x})^{d,b'}_{\alpha,\alpha'}(C\gamma^5) \rangle$ $\alpha_{\beta}S_{O}^{1}(t,\vec{x})_{\beta\beta'}^{b,d'}(C\gamma^{5})_{\alpha'\beta'}$ ere the fermion propagators in given representations are = 6*.*5*, amas* ⁰ ⁼ 1*.*01*, am^f* ⁰ ⁼ 0*.*71*, T* ⇥ *^L*³ = 48 ⇥ ²⁴³ (3.14)

mion propagators in given representations *n* representatio **d** where the fermion propagators where the fermion propagators in given representations are are the contract of the contra baryon at positive Euclidean time *t* and vanishing momentum *p*~

baryon at positive Euclidean time *t* and vanishing momentum *p*~

$$
S_Q(t, \vec{x})_{\alpha, \beta}^{a,b} = \langle Q(t, \vec{x})_{\alpha}^a \overline{Q(0)^b}_{\beta} \rangle \text{ and } S_{\Psi}(t, \vec{x})_{\alpha, \beta}^{ab, cd} = \langle \Psi(t, \vec{x})_{\alpha}^{ab} \overline{\Psi(0)^{cd}}_{\beta} \rangle.
$$

*a,c*⁰ *,* and *^Dca*⁰ ⁼

⌦*a*⁰

U(*t,* ~*x*)

⌦*^T S*¹

*ca,c*0

*a,c*⁰ ⇣

*,*⁰ *^S*²

,

*a,c*⁰ ⇣

1

S (*t,* ~*x*)

P⁺ =

^Q(*t,* [~]*x*)⌦*^T*

*c,a*0

(*C*5)

*d,b*⁰ ⇣

(*C*5)*D*(*t,* ~*x*)

*c,a*⁰ *.* (4.7)

⌘*^T* ◆

(**1** + 0) (3.18)

(*C*5)*S*¹

,(4.8)

^Q(*t,* ~*x*)

⌘*^T* ◆ (4.5)*,*

*,*0(*C*5)↵00*,*

(*C*5)

(*C*5)

*,*0(*C*5)↵00*,*

*b,d*0

⌘*^T* ◆ (4.5)*,*

(4.2)

⌦*bc k ca*

↵ *.*

(4.2)

⌘

⌘*^T* ◆ (4.5)*,*

~*x* ↵*,* = h*Q*(*t,* ~*x*) i and *S* (*t,* ~*x*) ↵*,* = h (*t,* ~*x*) ↵*Q*(0)*^b* ↵ (0)*cd* i*.* (4.6) σίαση της βαίτες βισμούσιστο πιταιστισιατινισε We also consider the parity projections in the nonrelativistic limit. ϵ narity projections in the poprelativistic li We also consider the parity projections in the nonrelativistic *e* parity projections in the nonrelativistic lim 11 . *mas* PS *L* (3.16)

$$
\mathcal{O}_{CB}^{\pm}(x) = P_{\pm} \mathcal{O}_{CB}(x) \text{ with } P_{\pm} = \frac{1}{2} (\mathbb{1} \pm \gamma_0)
$$

*ca,c*0

⌦*da*⌦*bc*⌦*c*⁰

*a*0 *,*⁰ Tr ✓

Results III: Chimera baryon

Real & imaginary part of 2-point correlation functions of the Chimera baryon

Results IV: Masses of mesons & Chimera baryon *a m^f* ⁰ = 0*.*6 (3.12) = 6*.*4 (3.11) ⇠ 6*.*4 (3.12)

= 6*.*4 (3.10)

c 19.100 (3.10)

⇠ 6*.*4 (3.11)

We considered spin-0 & 1 flavored mesons: pseudoscalar(PS), vector(V), tensor(T), axial-vector(AV), axial-tensor(AT) & scalar(S). \mathbf{R} $H = \frac{1}{\pi} \left(\frac{\Delta T}{\Delta T} \right)$

3.3 Scale setting

Gradient flow method

Gradient flow method

Summary & outlook $Summaru & \omega$ by replacing (*Uµ*)*ab* by (*UAS ^µ*)(*ab*)(*cd*) and *Q* by in Eq. (??). . 6*.*7 (3.6) Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

Contract

- We have developed numerical techniques to simulate *Sp(2N)* lattice gauge theories coupled to fermions in the multiple representations. e multiple representations. .7 (3.8) zu[.] ⇠ 6*.*5 (3.9) **Experience in the contract of the contract of**
- The first lattice studies of the VU model with the exact flavor content required for CH & top-partial comp.: Sp(4) with N_f=2 F & n_f=3 AS Dirac fermions. $c_{f=2}$ F & $n_f=3$ AS Dirac fermions. ⇠ 6*.*7 (3.8) **c** α *i*_f=3 A3 Dirac refinions. *fermions.* t the VU model with the exact flavor content re
	- \bullet Weak coupling region: $\beta \lesssim 6.4$
	- \sim $\frac{1}{2}$ $\frac{1}{$ FV effects are under control: $\text{ }7 \lesssim m_\text{PS}^f\, L$ & $11 \lesssim m_\text{PS}^{as}\, L$ $\leq m_{\rm DC}^{as} I$ \bullet $\frac{as}{\text{PS}}$ *L*

^µ)(*ab*)(*cd*) and *Q* by in Eq. (??).

artner): parity projection, smearing & variational metho \bullet Chimera baryon (top partner): parity projection, smearing & variational method *mf* = 6*.*4 (3.10) ⁰ = 0*.*6 (3.12)

$H_{\rm{tot}}$ To do list

by replacing (*Uµ*)*ab* by (*UAS*

Gradient flow method

3.2 (Rational) Hybrid Monte Carlo Monte C
2.2 (Rational) Hybrid Monte Carlo Mo lying spectra of composite states: mass dependence & lattice artifacts Generate ensembles at various values of $\,\beta$, m_0^f , $m_0^{as}\,$ and calculate the lowvarious values of β , m_0^f , m_0^{as} and calculate the low-

⁰ ⁼ 1*.*01*, m^f*

 ϵ Com Compute the (low-lying) Dirac eigenvalues

= 6*.*5*, mas*

3.3 Scale setting **Chirally broken or conformal?**

4 Observables

How light is the chimera baryon?

7 . *m^f*

⇠ 6*.*4 (3.12)

⇠ 6*.*5 (3.9)

= 6*.*4 (3.10)

. 6*.*7 (3.6)

. 6*.*5 (3.7)

.

PS *L* (3.14)

PS *L* (3.15)

^µ)(*ab*)(*cd*) and *Q* by in Eq. (??).

⁰ = 0*.*6 (3.12)

. 6*.*7 (3.6)

⁰ ⁼ 0*.*71*, T* ⇥ *^L*³ = 48 ⇥ ²⁴³ (3.13)

11 . *mas*

.4 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11 (3.11) 0.11

⁰ ⁼ 0*.*71*, T* ⇥ *^L*³ = 48 ⇥ ²⁴³ (3.13)

PS *L* (3.14)

⇠ 6*.*4 (3.11)

Thank you for your attention!

Backup I (*ab*)(*cd*) (*x*) ⌘ Tr ^h (*e* (*ab*) *AS*) *† Uµ*(*x*)*e* Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained by replacing (*Uµ*)*ab* by (*UAS ^µ*)(*ab*)(*cd*) and *Q* by in Eq. (??).

^µ (*x*) descends from the fundamental link variables *Uµ*(*x*), as

(*ab*)(*cd*) (*x*) ⌘ Tr ^h

AS vanishes identically. The explicit form of the antisymmetric link variables

Uµ(*x*)*e*

(*cd*)

AS ^U^T

^µ (*x*)

i

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

(*e*

AS)

, with *a < b, c < d.* (3.5)

. 6*.*7 (3.6)

, with *a < b, c < d.* (3.5)

the matrix *e*

UAS

by replacing (*Uµ*)*ab* by (*UAS*

HMC + RHMC

(13)

Finally, the Dirac operator for the 2-index antisymmetric representation *DAS* is obtained

UAS

From our previous studies of *Nf=2 F Sp(4)* & *nf=3 AS Sp(4)* we learned that 1st order bulk phase transitions exist for $\beta \lesssim 6.7$ & $\beta \lesssim 6.5$, respectively.

