Sp(4) and *Sp(6)* Quenched Meson Spectrum.

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*38*th Lattice Symposium
30th July 2021

Outline

1. Motivation
2. Definitions & Method
3. Provisional Results
4. Concluding Remarks

Motivation

- Gauge theories based on the symplectic groups, $Sp(2N)$, have the potential to describe a composite Higgs particle[1] which can soften the the fine-tuning problem.
- The model in which we are interested is the breaking of the approximate global symmetry $SU(4)/Sp(4)$[2].

[1] "Composite Higgs Scalars", Kaplan et. al.

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- The model in which we are interested is the breaking of the approximate global symmetry $SU(4)/Sp(4)$[2].
- To gain a better understanding of the symplectic groups generally, it is worth studying them in their own right.

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- A detailed study has been made of the glueball spectrum for finite N and in the limit $N \to \infty$.[3] We now turn our attention to the quenched meson spectrum (QMS).

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- The $Sp(4)$ QMS has been studied[4] and we now explore the QMS for $Sp(2N)$ generally with a view to extrapolating to $N \to \infty$. We will present provisional results for the QMS for $Sp(4)$ and $Sp(6)$ each with fermions in the fundamental, antisymmetric and symmetric representations.

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Definitions

The symplectic groups, $Sp(2N)$, are defined in terms of the special unitary group of odd rank, $SU(2N)$, via

$$Sp(2N) = \{M \in SU(2N) : M^* = \Omega^\dagger M \Omega\} \tag{1}$$

where

$$\Omega = \begin{bmatrix} 0 & \mathbb{1}_N \\ -\mathbb{1}_N & 0 \end{bmatrix} \tag{2}$$

and $\mathbb{1}_N$ is the $N \times N$ identity matrix.

Implementation

We implement $Sp(2N)$ Yang-Mills on the lattice using the standard Wilson action:

$$S = \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{2N} \operatorname{Re} \operatorname{tr}[U_{\mu\nu}(x)] \right) \tag{3}$$

where $\beta = 4N/g^2$ is the inverse coupling and $U_{\mu\nu}(x)$ is the plaquette in the $\mu\nu$-plane originating at lattice site x:

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x). \tag{4}$$

Methodology

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- If the operator $\mathcal{O}_M$ creates a state $|M\rangle$, with mass m_M then the correlator behaves like

$$C_M(t) \to \frac{|\langle 0|\mathcal{O}_M|M\rangle|^2}{m_M L^3}\left[e^{-m_M t} + e^{-m_M(T-t)}\right] \quad \text{as } t \to \infty \tag{5}$$

where L and T are the spatial and temporal extent of the lattice, respectively.

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  where L and T are the spatial and temporal extent of the lattice, respectively.
- Since the matrix elements can be written in terms of the decay-constants and masses of the state $|M\rangle$, these quantities can be measured by curve-fitting.

Interpolating operators

Label	Interpolating Operator $\mathcal{O}_S$	J^P	QCD meson
PS	$\overline{\psi^i}\gamma^5\psi^j$	0^-	π
S	$\overline{\psi^i}\psi^j$	0^+	a_0
V	$\overline{\psi^i}\gamma^\mu\psi^j$	1^-	ρ
AV	$\overline{\psi^i}\gamma^5\gamma^\mu\psi^j$	1^+	a_1
T	$\overline{\psi^i}\gamma^0\gamma^\mu\psi^j$	1^-	ρ
AT	$\overline{\psi^i}\gamma^5\gamma^0\gamma^\mu\psi^j$	1^+	b_1

Table 1: The operators used to construct meson states. A fermion of flavour i is described by the Dirac spinor ψ^i and its corresponding adjoint spinor is $\overline{\psi^i} \equiv \psi^{i\dagger}\gamma^0$. The meson produced by the corresponding operator in QCD is included for concreteness.

Effective mass plot

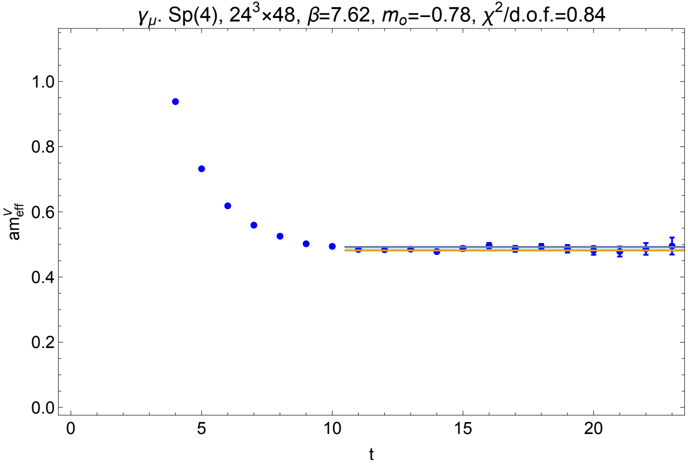

Figure 1: Effective mass plot of the vector meson comprised of antisymmetric fermions in $Sp(4)$. The fit range is the length of the blue strip with its vertical width corresponding to the statistical error in the measurement.

Chiral Extrapolations

- Once we have computed lattice masses for several β values each at several different bare masses am_0, we extrapolate to the chiral limit using chiral perturbation theory for Wilson fermions:

$$\hat{m}_{\mathrm{M}}^{2,\,\mathrm{NLO}} = \hat{m}_{\mathrm{M}}^{2,\chi}(1 + L^0_{m,\mathrm{M}}\hat{m}^2_{PS}) + W^0_{m,\mathrm{M}}\hat{a} \tag{6}$$

$$\hat{f}_{\mathrm{M}}^{2,\,\mathrm{NLO}} = \hat{f}_{\mathrm{M}}^{2,\chi}(1 + L^0_{f,\mathrm{M}}\hat{m}^2_{PS}) + W^0_{f,\mathrm{M}}\hat{a} \tag{7}$$

The quantity $\hat{O}_M^{2,\chi}$ is the observable $\hat{O}^2$ for state M extrapolated to the chiral-limit at next-to-leading order in units of gradient flow, w_0. The quantity $\hat{a} = \frac{a}{w_0}$ where a is the lattice spacing.

Example Chiral Extrapolation

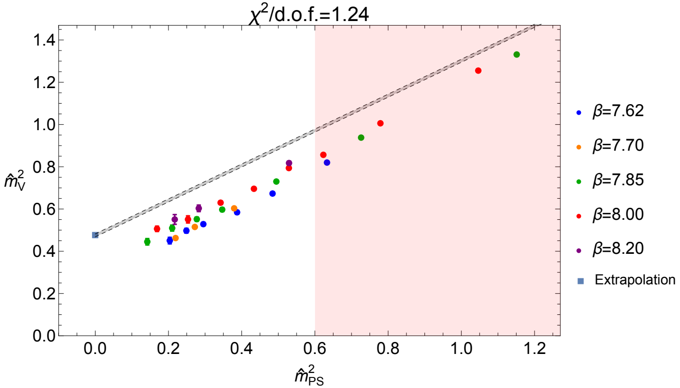

Figure 2: Vector meson mass (squared) for fundamental fermions extrapolated to the chiral limit. Points in the pink region are not included in the extrapolation.

Provisional Results for $Sp(4)$

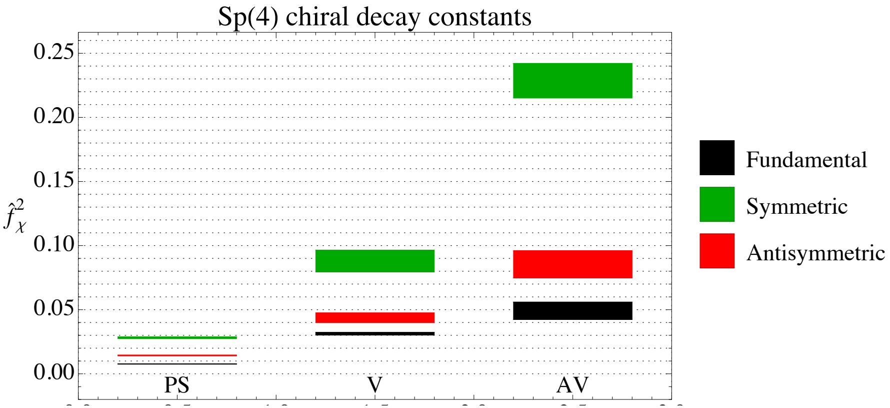

Figure 3: $Sp(4)$ chiral decay constants for each representation.

Provisional Results for $Sp(4)$

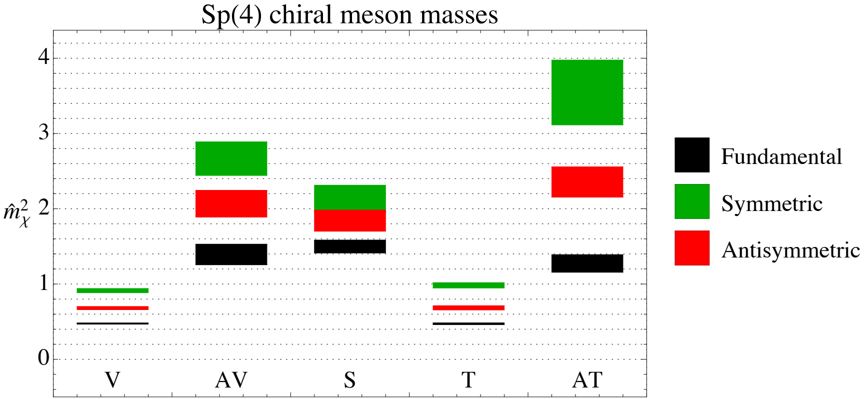

Figure 4: $Sp(4)$ chiral masses for each representation.

Provisional Results for $Sp(6)$

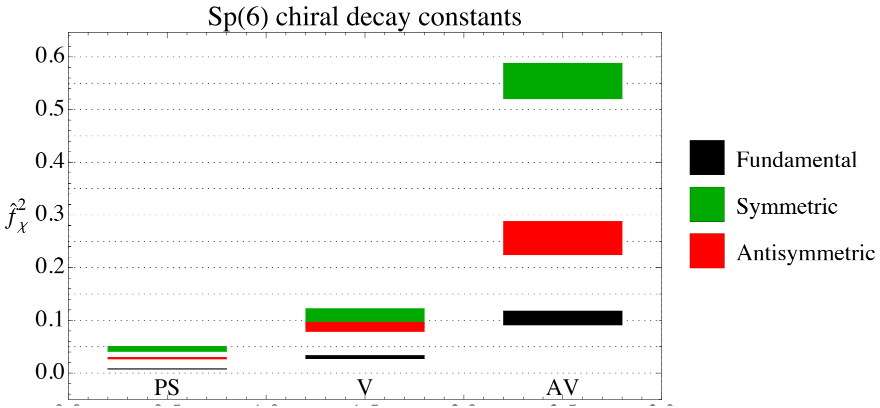

Figure 5: $Sp(6)$ chiral decay constants for each representation.

Provisional Results for $Sp(6)$

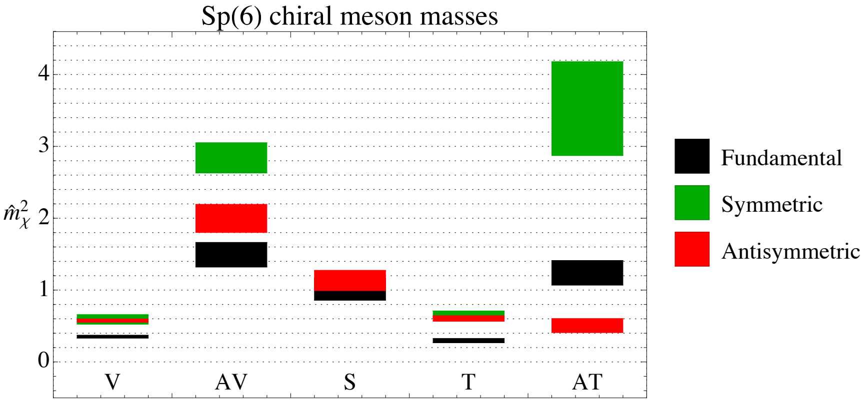

Figure 6: $Sp(6)$ chiral masses for each representation. (The scalar mass is still being finalised.)

Conclusion

- We have stated the motivation for the study of the $Sp(2N)$ gauge theories generally.
- We have presented provisional results for the QMS in $Sp(4)$ and $Sp(6)$ Yang-Mills as a step towards a large-N extrapolation.
- Some states are not under our control yet due to their large mass (e.g. the scalar and axial-tensor channels in $Sp(6)$).
- This work is still ongoing...

Provisional Results for $Sp(4)$

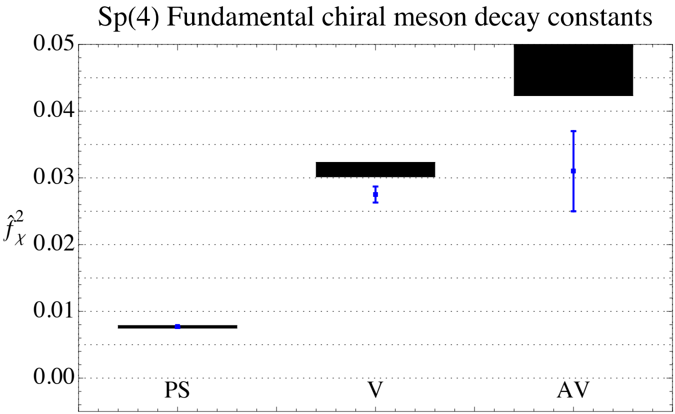

Figure 7: Comparison of my results with those of arXiv:1912.06505. The former are denoted by black boxes and the latter by blue dots.

Provisional Results for $Sp(4)$

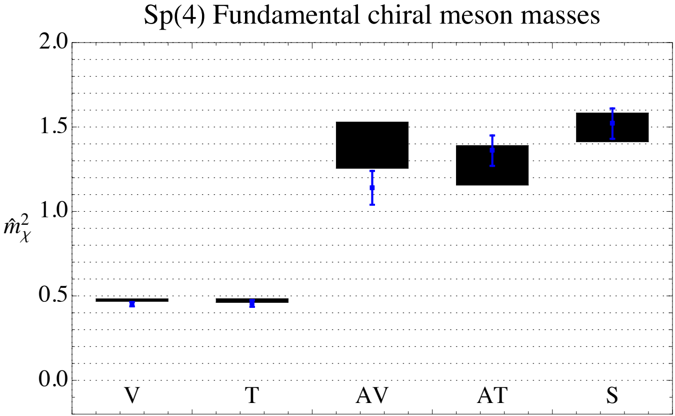

Figure 8: Comparison of my results with those of arXiv:1912.06505. The former are denoted by black boxes and the latter by blue dots.

Provisional Results for $Sp(4)$

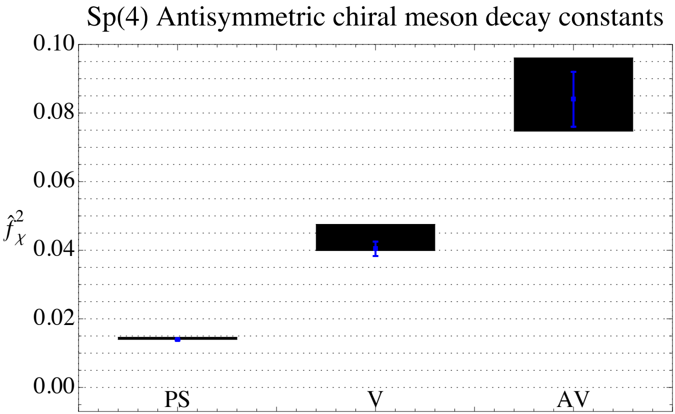

Figure 9: Comparison of my results with those of arXiv:1912.06505. The former are denoted by black boxes and the latter by blue dots.

Provisional Results for $Sp(4)$

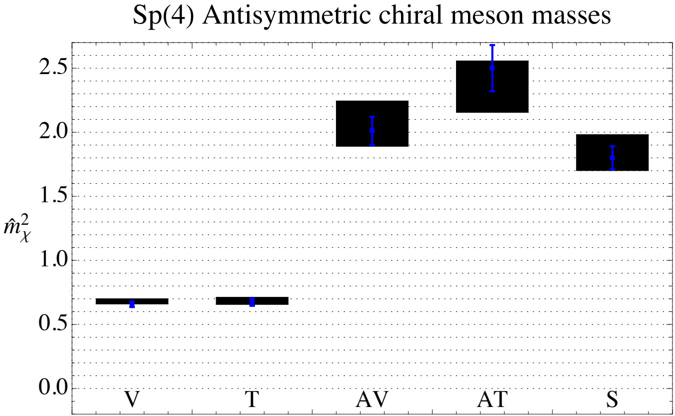

Figure 10: Comparison of my results with those of arXiv:1912.06505. The former are denoted by black boxes and the latter by blue dots.

Provisional Results

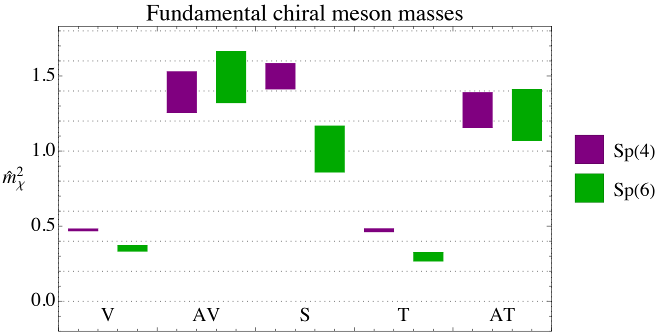

Figure 11: Fundamental meson masses for $Sp(4)$ and $Sp(6)$ both in the chiral limit.

Provisional Results

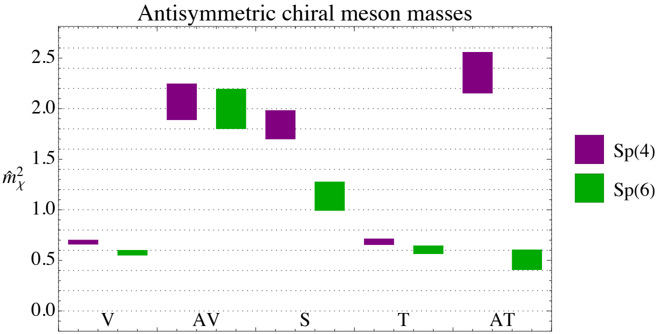

Figure 12: Antisymmetric meson masses for $Sp(4)$ and $Sp(6)$ both in the chiral limit.

Provisional Results

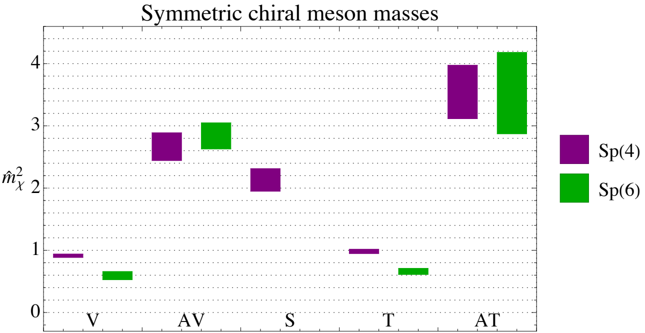

Figure 13: Symmetric meson masses for $Sp(4)$ and $Sp(6)$ both in the chiral limit.

Provisional Results

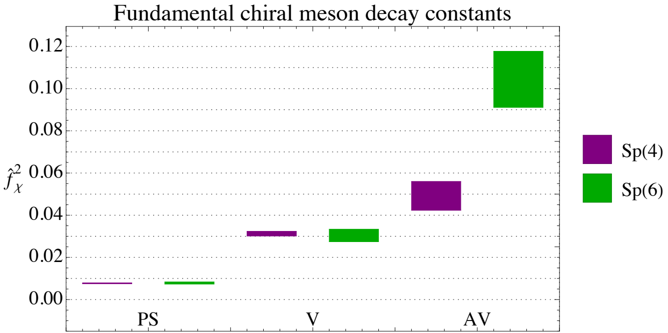

Figure 14: Fundamental decay constants for $Sp(4)$ and $Sp(6)$ both in the chiral limit.

Provisional Results

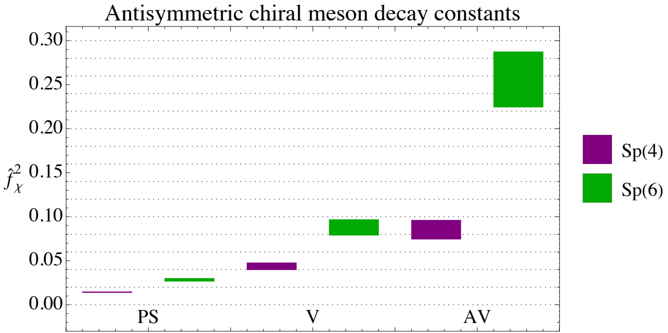

Figure 15: Antisymmetric decay constants for $Sp(4)$ and $Sp(6)$ both in the chiral limit.

Provisional Results

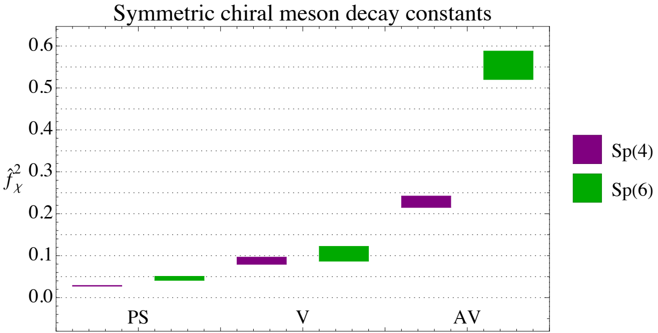

Figure 16: Symmetric decay constants for $Sp(4)$ and $Sp(6)$ both in the chiral limit.