Spectral reconstruction in SU(4) gauge theory with fermions in multiple representations

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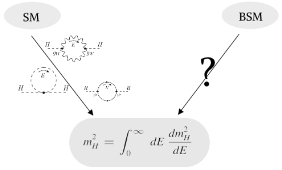
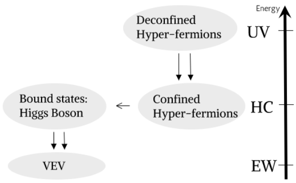
Composite Higgs models

- The identification of the Higgs as a pseudo–NG boson emerging from the breaking of a global symmetry offers a possible solution to the Naturalness problem.
- Such symmetry describes the flavor structure of a new strongly–interacting sector, whose fermions eventually confine into light bound states, including the Higgs boson.
- Strongly–interacting dynamics require a lattice study.





Lattice setup

- A flexible environment for simulating these theories is offered by the libraries Grid and Hadrons (multiple representations, $Nc = 2, 3, 4$)
- A promising model is a $SU(4)$ gauge theory with five fermions in the **6** two-index anti–symmetric and three in the fundamental and anti-fundamental representations. (Ferretti 2014)
- Starting from Del Debbio, Panero *et al* (2019) we simulate two fundamental and two sextet Dirac fermions in a SU(4) gauge group, a simplification of the Ferretti–model first explored by (Ayyar *et al.* 2017).

Hadrons

$$\frac{G}{H} = \frac{SU(5) \times SU(3) \times SU(3)' \times U(1)_X \times U(1)'}{SO(5) \times SU(3)_c \times U(1)_X}$$

$$\left(\frac{SU(5)}{SO(5)}\right) \times \left(\frac{SU(3) \times SU(3)'}{SU(3)_c}\right) \times \left(\frac{U(1)' \times U(1)_X}{U(1)_X}\right) .$$

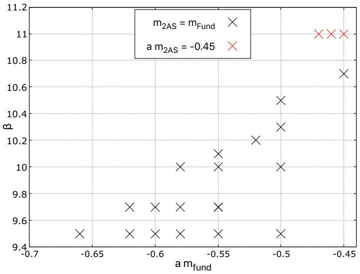
GRID

Lattice setup

Volume $16^3 \times 32$

	β	am_{fund}	am_{2AS}	Accep.	Plaq.	cnfg	$\|\lambda_4\|$	$\|\lambda_6\|$
Z0	10	-0.55	-0.55	89%	0.54977(9)	400	0.06990(36)	0.22517(53)
.	.	.	.					
.	.	.	.					
A1	11	-0.45	-0.45	74%	0.60891(27)	216	0.0365(9)	0.1794(9)
A2	11	-0.46	-0.45	85%	0.60930(25)	633	0.0273(12)	0.1768(9)
A3	11	-0.47	-0.45	85%	0.60942(26)	225	0.0165(10)	0.1769(9)

- We perform HMC simulations by using Wilson fermions with $\mathcal{O}(a)$ clover improvement term on $16^3 \times 32$ to $32^3 \times 64$ lattices
- A – runs simulate similar values explored in (Ayyar *et al.* 2017).



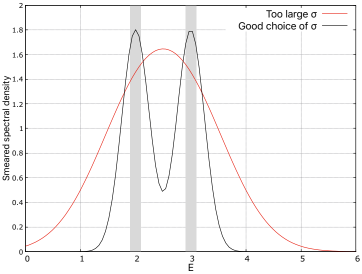
Spectral reconstruction

We reconstruct finite–volume smeared spectral densities from lattice correlators measured on our ensembles by using a variation of the Backus–Gilbert method.

- Finite–volume spectral densities can be smeared from δ–functions to regular functions $\Delta_\sigma(E, E_\star)$ with smearing radius σ

$$\hat{\rho}(E_\star) = \int_0^\infty dE \; \Delta_\sigma(E, E_\star) \; \rho(E)$$

$$\rho(E) = \sum_n w_n \delta(E - E_n) \longrightarrow \sum_n w_n \Delta_\sigma(E, E_n)$$



- We span a smearing kernel with the same functions encoded in the correlators $c(t)$

$$c(t) = \int_0^\infty dE \; \rho(E) \, e^{-tE} \, , \qquad \bar{\Delta}_\sigma(E_\star, E) = \sum_{t=0}^{t_{max}} g_t \, e^{-(t+1)E}$$

- Provided we know the g_t we can compute the smeared spectral density as

$$\hat{\rho}(E_\star) = \sum_{t=0}^{t_{max}} g_t \; c(t+1) = \int_0^\infty dE \; \bar{\Delta}_\sigma(E_\star, E) \; \rho(E)$$

- The coefficients can be found by minimising an appropriate functional $W[g]$

Spectral reconstruction

$$W[g] = \lambda\, A[g] + (1-\lambda)\, B[g]$$

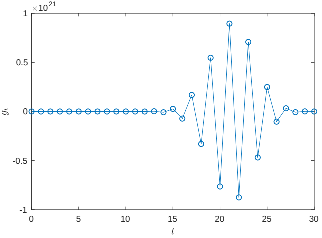
- We use the functional $A[g]$ introduced in (Hansen, Lupo, Tantalo 2019) measuring the difference between the reconstructed and the exact kernel

$$A[g] = \int_0^\infty dE\, |\Delta_\sigma(E_\star, E) - \bar{\Delta}_\sigma(E_\star, E)|^2$$

- The other functional prevents large error propagations from $c(t)$ through the coefficients g_t which can be $\mathcal{O}(10^{20})$

$$B[g] = \boldsymbol{g}^T\, \mathtt{Cov}\, \boldsymbol{g}\,, \qquad \hat{\rho}(E_\star) = \sum_{t=0}^{t_{max}} g_t\, c(t+1)$$

- The shape of the smearing kernel, its width σ and the trade–off parameter λ are inputs of the algorithm.

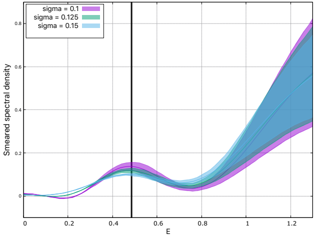


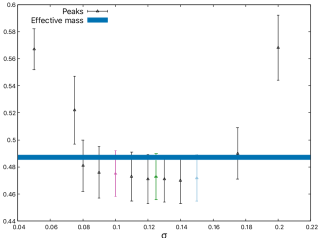
Extracting the ground state

- *A first application*: reconstructing the ground state of the Pseudoscalar–Pseudoscalar channel in a given representation with16 data points (T=32)

- Results are compatible with a standard effective mass calculation at large time separations

- Results are stable by varying the smearing radius σ. Smaller radii can be achieved by increasing T

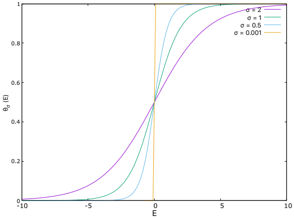




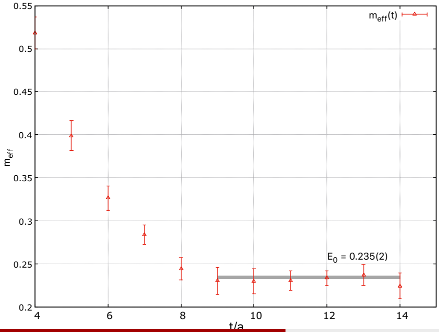
Pseudoscalar channel

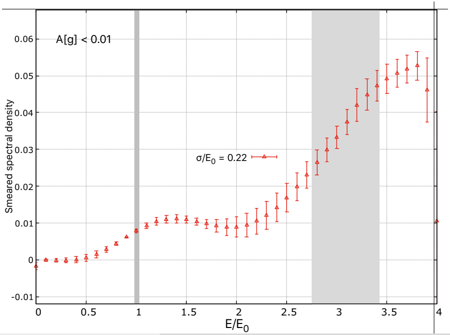
- As a smearing kernel, we use a regularised step function $\theta_\sigma(E - E_\star)$
- $\rho(E) = \sum_n w_n \delta(E - E_n) \implies \tilde{\rho}_\sigma(E) = \sum_n w_n \theta_\sigma(E - E_n)$
  $\rightarrow$ we should see a step at each energy level
- Values for E_0 used for normalisations are obtained by effective mass plots.

σ must be small enough to resolve different energies



Run A3: $16^3 \times 32$, $\beta = 11$, $a\, m_{PCAC}^{fund} = 0.0220(5)$ $m_\pi L = 3.8$



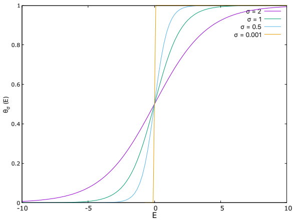


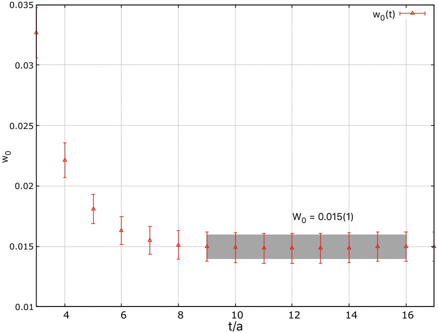
Pseudoscalar channel

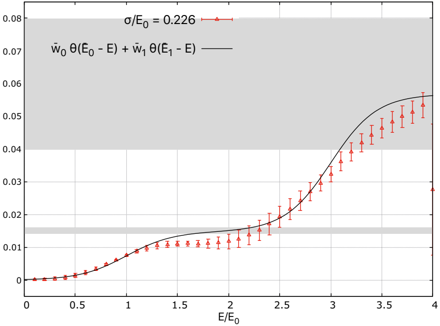
- From the correlators we can also fit the first coefficients w_n in

$$\tilde{\rho}_\sigma(E) = \sum_n w_n \theta_\sigma(E - E_\star)$$

- With fits results for w_n and E_n we can plot the smeared spectral density we would obtain from them: $\tilde{w}_0 \;\; \theta_\sigma(E - \tilde{E}_0) + \tilde{w}_1 \;\; \theta_\sigma(E - \tilde{E}_1)$

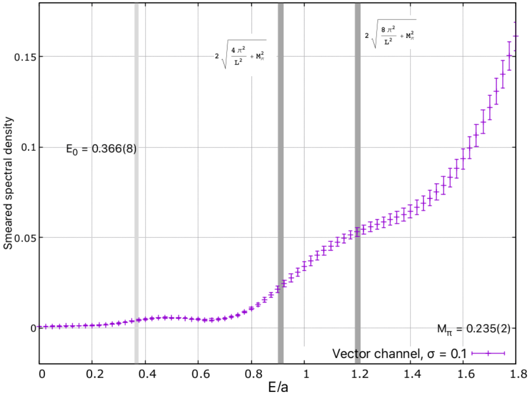






Vector channel

- We computed the spectral density of the vector channel, smeared with a $\theta_\sigma(E - E_\star)$ –kernel
- E_0 is the value obtained from an effective mass plot in this channel
- The other vertical bands are the finite–volume spectrum of two free particle with mass M_π (the ground state of the pseudoscalar channel).



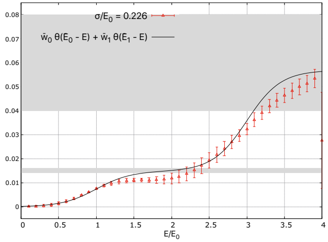
Conclusions

- In our preliminary results, the spectral reconstruction offers a complementary approach which we found to be compatible with other standard methods, matching the expected behaviour also for the first excited states

- The results shown in this work are obtained from only 16 datapoints.

  As we increase the quality of the data and we explore lattices with larger time extents, we aim to gain more insight:

  - Quantitative predictions beyond the ground state
  - More challanging data: lighter masses, scalar channel.



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