Dilaton chiral perturbation theory

Maarten Golterman* with Yigal Shamir

*San Francisco State University

arXiv:1603.04575 PRD94 (2016) 025020

arXiv:1611.04275 PRD95 (2017) 016003

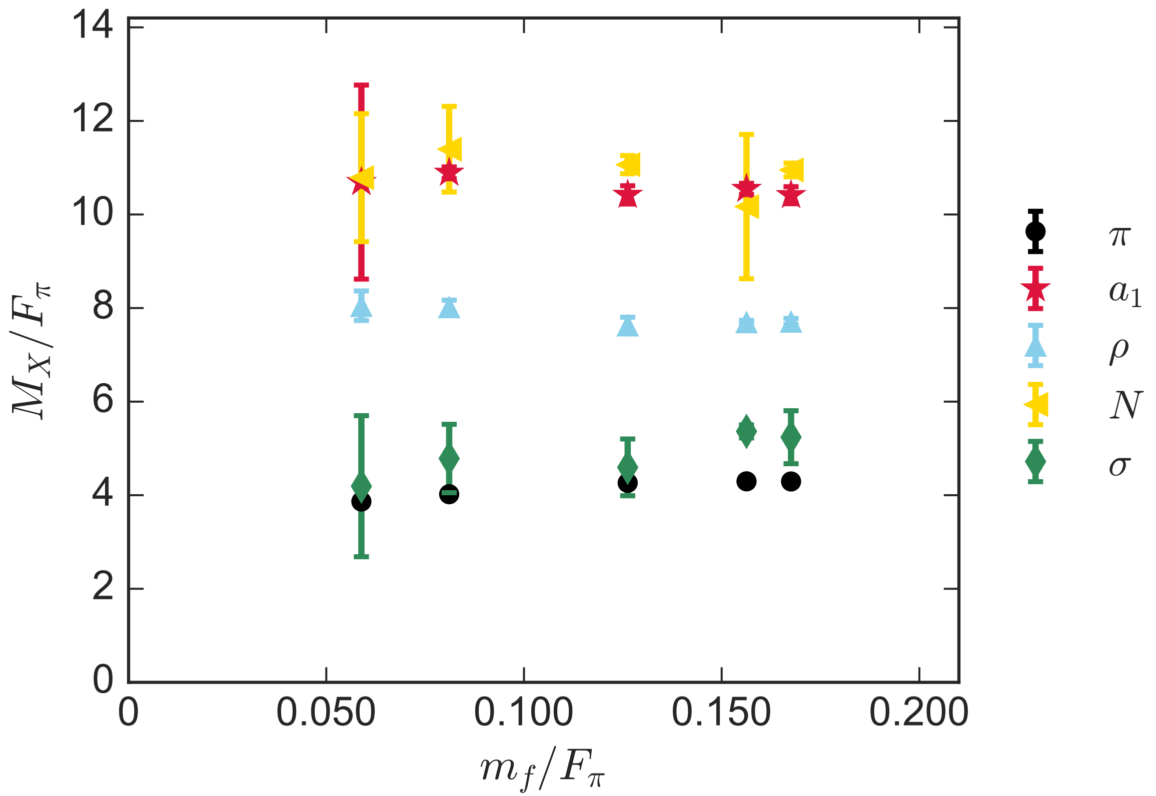
arXiv:1805.00198 PRD98 (2018) 056025

arXiv:1909.10796 PRD100 (2019) 114515
(with Taro Brown, Svend Krøjer, Kim Splittorff)

arXiv:2003.00114 PRD102 (2020) 034515 (with Ethan Neil)

arXiv:2009.13846 PRD102 (2020) 114507

Spectrum of SU(3) with $N_f = 8$ fundamental flavors



LSD collaboration 2018

Different from QCD:

(1) Light scalar

$M_{\text{scalar}} \approx M_\pi$

(2) Approximate

hyperscaling

(3) Staggered fermions:

pattern of

"taste" splittings

- Similar results: LatKMI collaboration (see later)
- Also: SU(3), $N_f = 2$ sextets [LatHC]

Outline

- dChPT$=$ Effective field theory with pions and a dilaton

  Approximate chiral and scale symmetries, power counting

- Tree-level behavior:

  Hyperscaling and the large-mass regime

- Fits of dChPT to $N_f = 8$ lattice data from LSD and LatKMI collaborations

  Staggered fermion taste splittings

Effective Field Theory for pions and a dilaton

Assumptions:

- Theory contains pions = pseudo Nambu-Goldstone (NG) bosons associated with spontaneous chiral symmetry breaking.

- Scale invariance gets restored in the infrared as we approach the conformal window: trace anomaly $\propto$ distance to conformal window.
  In the Veneziano limit: $N_c, N_f \to \infty$, with $n_f = N_f/N_c$ fixed, this happens when n_f approaches a critical value n_f^* from below.
  $n_f - n_f^*$ is a new *small parameter*.

- Theory contains a "dilaton" = pseudo NG boson associated with breaking of scale symmetry, which becomes massless for $N_f \to N_f^*$ (and $m \to 0$).

- (Also need some technical assumptions on the dilaton potential)

Leading-order *dilaton pion* effective field theory (dChPT)

$$
\begin{aligned}
\mathcal{L}^{\rm EFT} \;\; = \;\; & \tfrac{1}{4} f_\pi^2 \, e^{2\tau} \, {\rm tr} \, (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) \\
& + \tfrac{1}{2} f_\tau^2 \, e^{2\tau} (\partial_\mu \tau)^2 \\
& - \tfrac{1}{2} f_\pi^2 B_\pi m \, e^{(3-\gamma_*)\tau} \, {\rm tr} \left(\Sigma + \Sigma^\dagger \right) \\
& + f_\tau^2 B_\tau \, e^{4\tau} \, c_1 (\tau - 1/4)
\end{aligned}
$$

- $\Sigma = e^{2i\pi/f_\pi}$ is usual non-linear pion field, τ is dilaton field
- Shifted τ so that $v = \langle \tau \rangle = 0$ for $m = 0$
- Assumed small: $p^2 \;\sim\; m \;\sim\; c_1 \propto |n_f - n_f^*|$
- $\mathcal{L}^{\rm EFT}$ is order-p^2 lagrangian with LECs $f_{\pi,\tau}$, $B_{\pi,\tau}$,
  $\gamma_* =$ mass anomalous dimension at IRFP, free parameter in dChPT

Leading order predictions

- Minimize potential: $$\frac{m}{c_1\mathcal{M}} = v e^{(1+\gamma_*)v}\,, \qquad \mathcal{M} = \frac{4f_\tau^2 B_\tau}{f_\pi^2 B_\pi N_f(3-\gamma_*)}$$
- Pion mass: $$M_\pi^2 = 2B_\pi m e^{(1-\gamma_*)v(m)} = 2B_\pi c_1 \mathcal{M}\, v(m)\, e^{2v(m)}$$
- Dilaton mass: $$M_\tau^2 = 4B_\tau c_1 e^{2v(m)}\big(1+(1+\gamma_*)v(m)\big)$$
- Decay constants: $$F_{\pi,\tau} = f_{\pi,\tau} e^{v(m)}$$
- Other hadron masses: $$M_{\mathrm{h}} = M_0 e^{v(m)}$$

Ratio $\frac{m}{c_1} = O(1)$ parametrically, but can be large or small

Chiral (small-mass) regime: $\frac{m}{c_1\mathcal{M}} \ll 1 \;\Rightarrow v \propto m$ small, $e^v \approx 1$

Pion mass: $$M_\pi^2 = 2B_\pi m$$

Dilaton mass: $$M_\tau^2 = 4B_\tau c_1 \qquad \propto |n_f - n_f^*|$$

Large-mass regime: $\frac{m}{c_1 \mathcal{M}} \gg 1$

$$\frac{m}{c_1 \mathcal{M}} = v e^{(1+\gamma_*)v} \quad \Longrightarrow \quad e^{v(m)} \sim \left(\frac{m}{c_1 \mathcal{M}}\right)^{\frac{1}{1+\gamma_*}}$$

- Approx. hyperscaling: $M_\pi \sim M_\tau \sim F_\pi \sim F_\tau \sim M_{\rm h} \sim \cdots \sim m^{1/(1+\gamma_*)}$

  Behaves like a mass deformed conformal theory!

- pNG bosons still lighter: $\frac{M_\pi}{M_{\rm h}} \sim \frac{M_\tau}{M_{\rm h}} \sim c_1 \propto |n_f - n_f^*|$

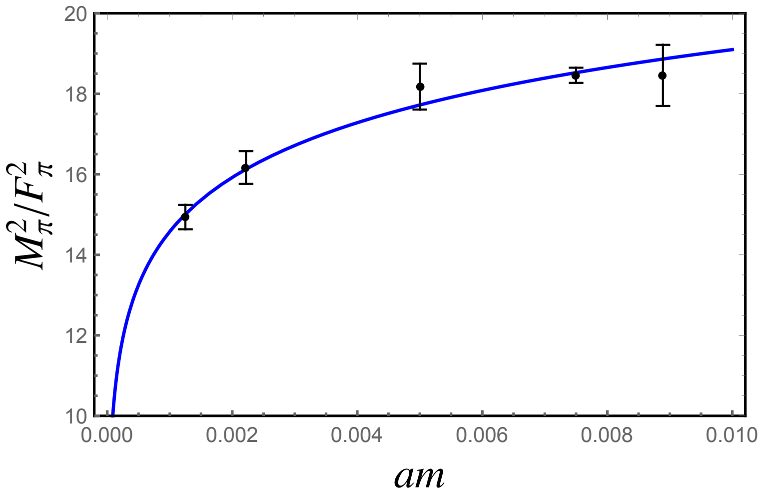
- Loop-expansion parameter: $\frac{M_\pi^2}{(4\pi F_\pi)^2} \sim c_1 v(m) \sim c_1 \log\left(\frac{m}{c_1 \mathcal{M}}\right)$

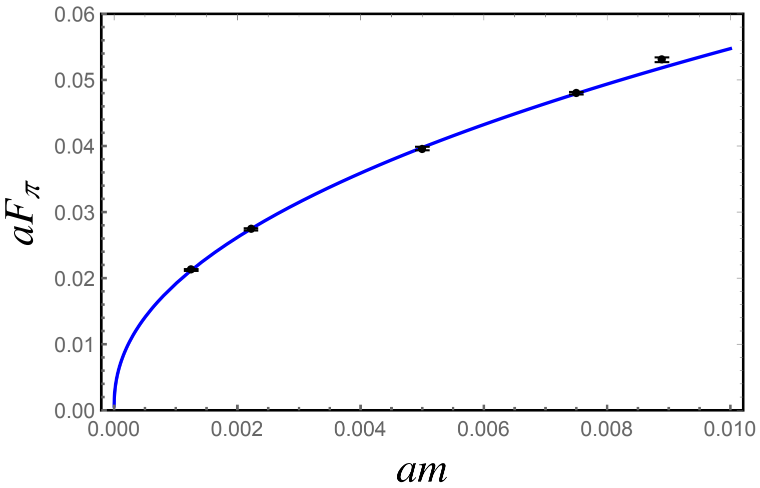
$\Longrightarrow$ Still systematic expansion provided $c_1 \log\left(m/(c_1 \mathcal{M})\right) \ll 1$

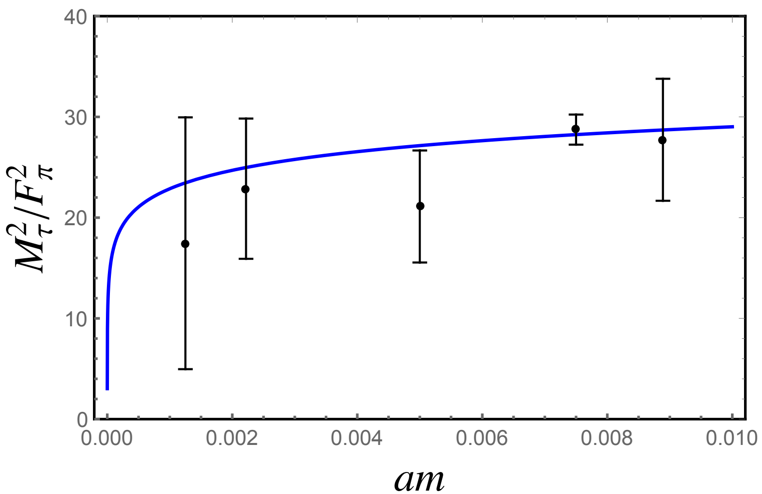
By contrast: $m/\mathcal{M} \ll 1$ required in ordinary ChPT!

Fits to $N_f = 8$ data from LSD collaboration

PRD99 (2019) 014509







Mass values:

$$10^3 am = (1.25, 2.22, 5.00, 7.50, 8.89)$$

5 ens.: $\frac{\chi^2}{\mathrm{dof}} = \frac{11.9}{10}$, p-value $= 0.29$

4 ens.*: $\frac{\chi^2}{\mathrm{dof}} = \frac{2.9}{7}$, p-value $= 0.89$

* shown

Tree level parameter values

- Parameters controlling the mass dependence (well determined):

  mass anom. dim.: $\gamma_* = 0.94(2)$, $\quad aB_\pi = 2.1(1)$

- chiral limit pion decay constant (long extrapolation): $af_\pi = 0.0006(3)$

  vs. values in simulation: $0.02 \lesssim aF_\pi(m) \lesssim 0.06$ (recall: $F_\pi = f_\pi e^{v(m)}$!)

  $\Rightarrow$ $F_\pi(m)L \gtrsim 1$ in simulation
  but having $f_\pi L \gtrsim 1$ requires unrealistically large lattices

  $\Rightarrow$ could be sensitive to higher orders in dChPT

- excellent fits for this mass range

  See also Appelquist *et al.* '17, '18, '19, Fodor *et al.* '19, '20

- Appelquist, *et al.*: Generalize dilaton pot. to $V_\Delta \propto \frac{e^{4\tau}}{4-\Delta}\left(1 - \frac{4}{\Delta}\, e^{(\Delta-4)\tau}\right)$.

  $\Delta \to 4$ recovers dChPT ($\Delta = 2$ is σ-model)

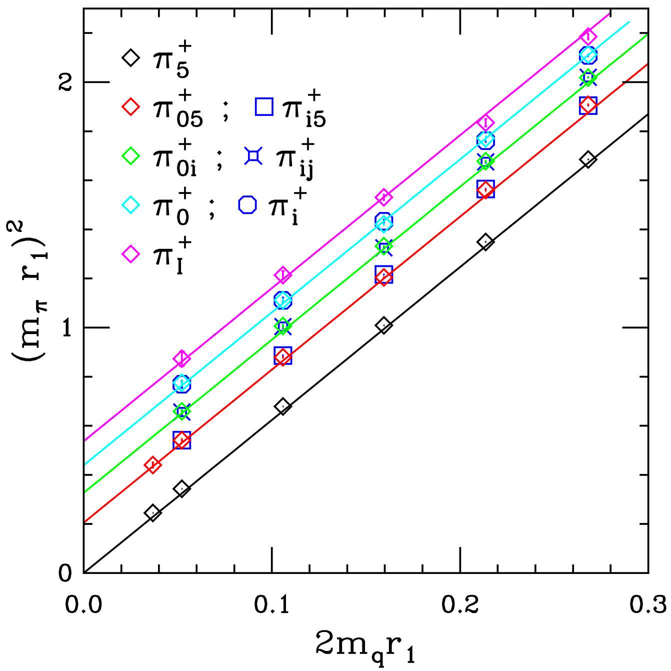
  $\Rightarrow$ No power counting for Δ not close to 4!

Staggered fermions “taste” splittings

Staggered fermion $=$ 4 quarks with remnant of flavor (“taste”) symmetry

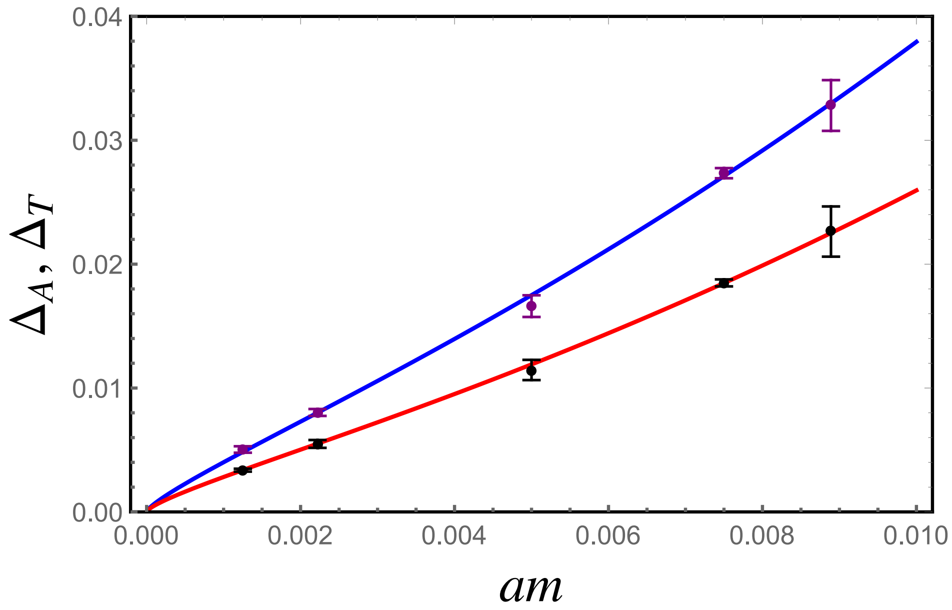
Pions: tastes P (exact NGB), A, T, V, S; $\Delta_A = M_A^2 - M_P^2$, *etc.*

MILC 0903.3598



QCD: $M_\Gamma^2 = Bm + c_\Gamma a^2$
($c_\Gamma = 0$ for NG pion)

LSD



$N_f = 8$: tree-level splittings depend on m through $e^{v(m)}$

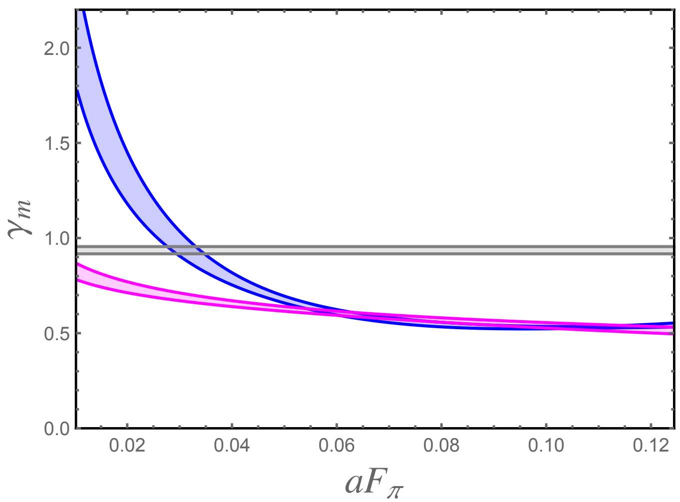
Fits to $N_f = 8$ data from LatKMI collaboration PRD96 (2017) 014508

- Same theory, different lattice action, different (bare) coupling, different masses: $am = (0.012,\ 0.015,\ 0.02,\ 0.03,\ 0.04,\ 0.05,\ 0.06,\ 0.07,\ 0.08,\ 0.1)$

- Need N(N?)LO dChPT — too many parameters for limited data!

  Instead: model m-dependent γ-function: $\gamma(m) = \gamma_0 - bv(m) + cv(m)^2$

  Still satisfies anomalous Ward–Takahashi identity for scale invariance



Good description of data

Magenta band: $c = 0$, eight masses

Blue band: $c \neq 0$, nine masses

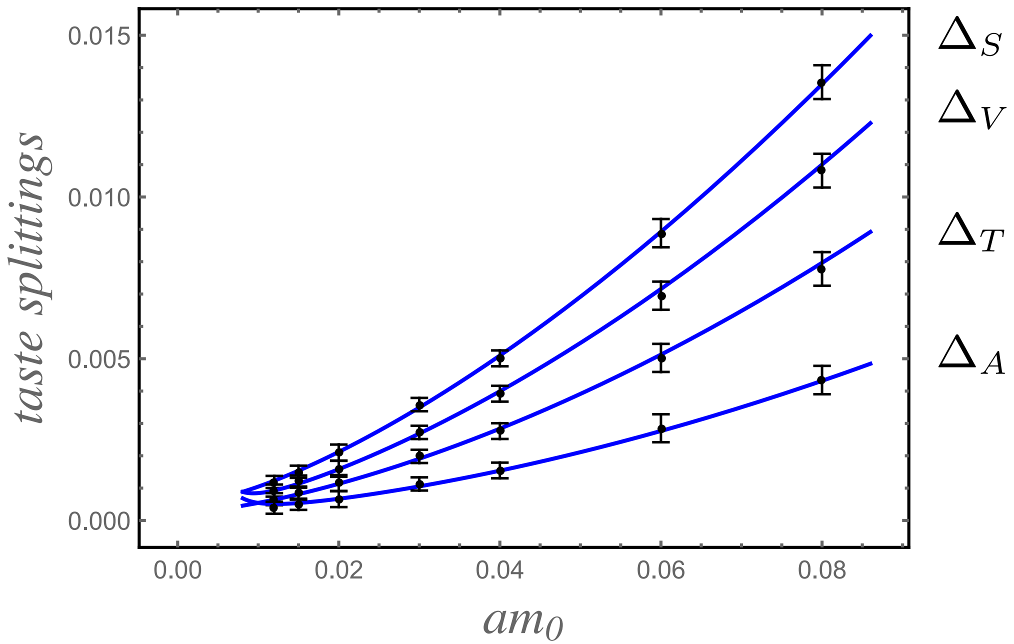
Gray band: LSD value

KMI data: $0.045 \leq aF_\pi \leq 0.12$

Taste splittings (KMI data)

- LatKMI measured all taste splittings!

  smaller set of masses: $am = (0.012,\ 0.015,\ 0.02,\ 0.03,\ 0.04,\ 0.06,\ 0.08)$



- Fit with $\gamma(m) = \gamma_0 - bv(m)$, p-value $= 0.44$

  (Cannot determine (all 8!) taste-breaking pars. from LSD or LatKMI data)

Concluding remarks

- *Large-mass* regime: $c_1 \ll \frac{m}{\mathcal{M}} \ll c_1 e^{1/c_1}$.
  Expansion still systematic, thanks to smallness of $c_1 \propto |n_f - n_f^*|$,
  which measures the distance from the conformal sill.
  Approximate hyperscaling: mass deformation dominates over slow running.

- Current simulations of the SU(3), $N_f = 8$ theory are deep in the
  large-mass regime, $\frac{m}{c_1 \mathcal{M}} = v e^{(1+\gamma_*)v} \gg 1$ for both LSD and LatKMI.

- Chiral-limit $af_\pi \simeq 0.0006(3)$ (LSD) much smaller than $aF_\pi(m) \sim 0.04$.
  Very long extrapolation from large-mass regime
  - $\Rightarrow$ may change with mass range in the fit/higher orders in dChPT
  - $\Rightarrow$ hard to determine whether af_π and c_1 are non-zero!
  - $\Rightarrow$ Does dChPT also–effectively–apply inside the conformal window??

- Explains why it is so hard to distinguish an infrared conformal theory
  from a chirally broken (confining) theory with “walking” coupling.