Applications of the gradient flow beta-function

for the Lattice Higgs Collaboration (LatHC)

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topics of the talk:

- two β-functions, defined on the gradient flow, will be discussed and tested in the ten-flavor model
- the gradient flow on the gauge and fermion fields can be viewed in the framework of renormalization group transformations 1802.07897 1806.01385
- step β-function in finite physical volume and the derivative β-function, $\beta = t \cdot dg^2/dt$ (infinite physical volume), complement each other in model studies
- the derivative beta function $\beta = t \cdot dg^2/dt$ makes contact with Harlander-Neumann infinite volume 3-loop expansion
- contact with HN 3-loop was the goal of the original lattice study of $\beta = t \cdot dg^2/dt$ when LatHC first tested it in p-regime of massless fermions 1711.04833 , various tests in 1910.06408 Anna H.,Oliver W. and in 1912.07653 LatHC

I. step β-function on gradient flow in finite physical volume

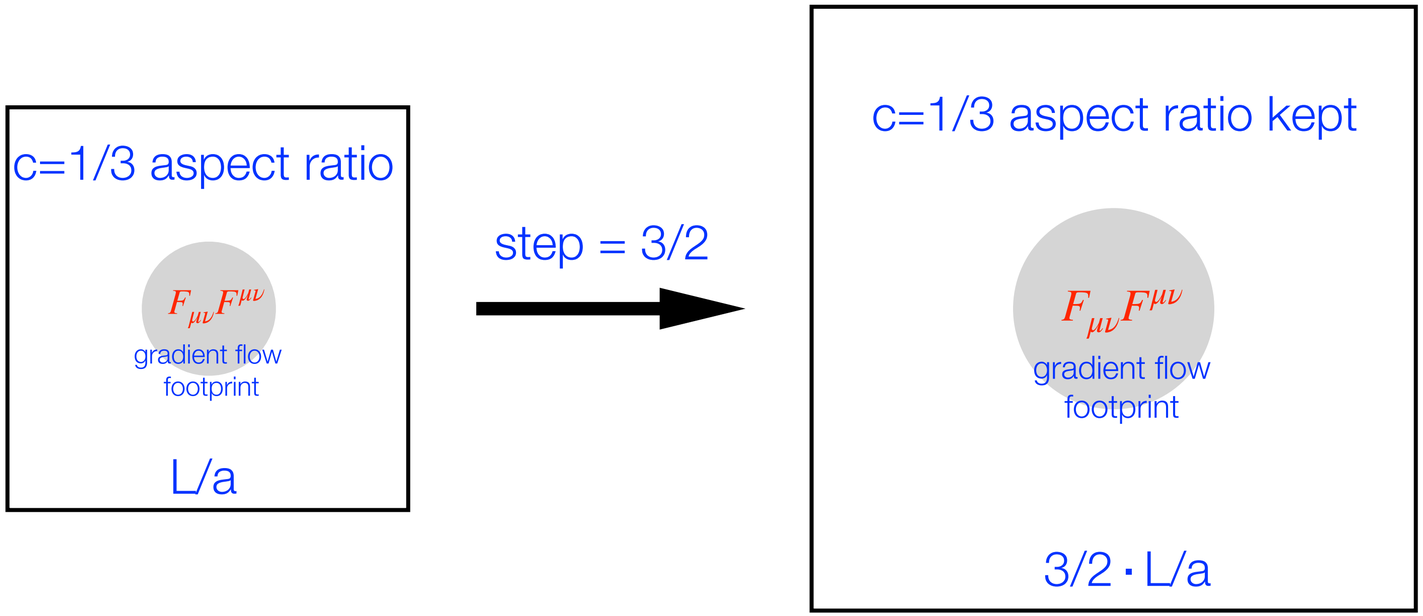
LatHC 2012 1208.1051 [hep-lat]

anti-periodic fermions in the scheme, not SF

operator measured at gradient flow time t
defines the renormalized gauge coupling g(t)

Wilsonflow**S**ymanzikaction**C**loveroperator defines the scheme: **WSC**
(**SSS, SSC, WSS** schemes will also be used)

$F_{\mu\nu}F^{\mu\nu}$ operator measured at gradient flow time t defines the renormalized gauge coupling g(t) which scales with gradient flow time t, or equivalently, with the physical scale L at fixed aspect ratio c in the continuum limit



at fixed c and fixed step: L/a → infinity (a -> 0 continuum limit)

II. derivative β-function on gradient flow (infinite physical volume) 1711.04833 LatHC

also tested in 1910.06408 by Anna H. and O. W.; and LatHC in 1912.07653

- the derivative beta function $\beta = t \cdot dg^2/dt$ makes contact with Harlander-Neumann 3-loop expansion (infinite volume). A potential application is the QCD coupling α_s at the Z-pole (Holland's talk)

- it is used here to study the ten-flavor model

- two different ways to take continuum limit to infinite volume:

$\beta = t \cdot dg^2/dt$

$F_{\mu\nu}F^{\mu\nu}$

gradient flow fixed footprint

L/a

(1)

fixed a^2/t, $a^2/L^2 \to 0$

$a^2/t \to 0$ second step

(2)

or, at fixed c, $a^2/L^2 \to 0$

$c \to 0$ second step

$\beta_c(g^2(t))$ is new scheme

provides consistency check

$\beta = t \cdot dg^2/dt$

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large L/a

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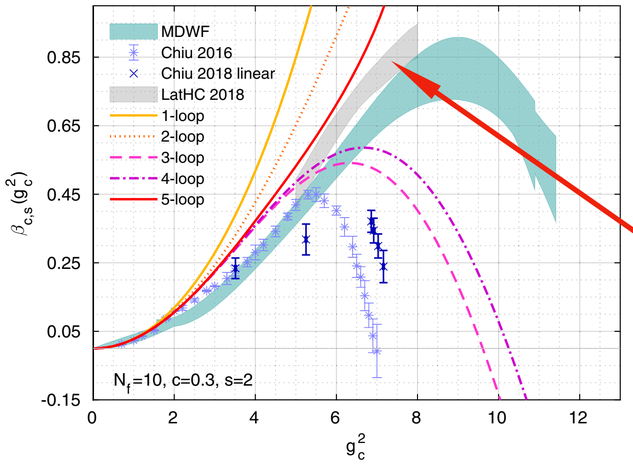
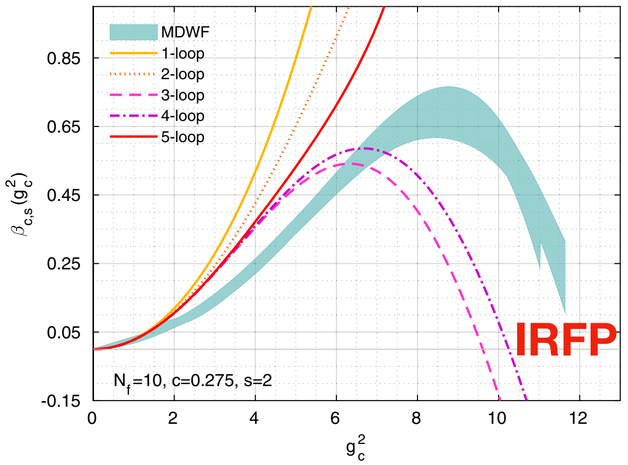


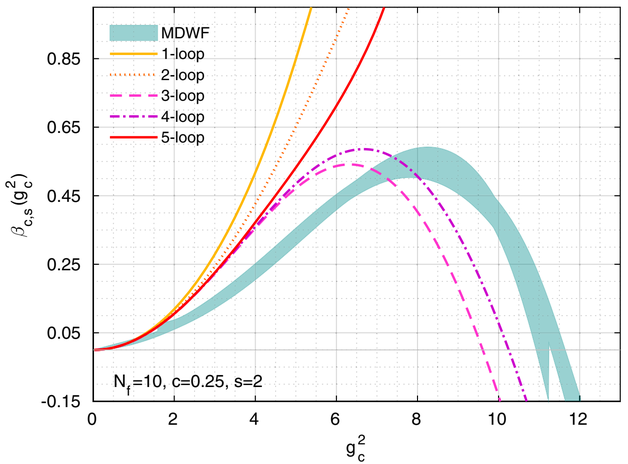
Boulder-BU collaboration

A. HASENFRATZ, C. REBBI, and O. WITZEL
PHYS. REV. D **101,** 114508 (2020)
2004.00754

- nf = 10 s=2 step beta function
- tension with 2018-2019 LatHC results?
- Symanzik improved gauge action
- massless Möbius Domain Wall fermions
- tree-improved WSS gradient flow scheme
- three aspect ratios c=0.25, c=0.275, c=0.30
- largest volume L=32
- gauge coupling extended to $g^2 \approx 11$
- c=0.25,0.275 results **strongly suggest IRFP?**
- consistency with 4+6 composite Higgs model?



IRFP ?



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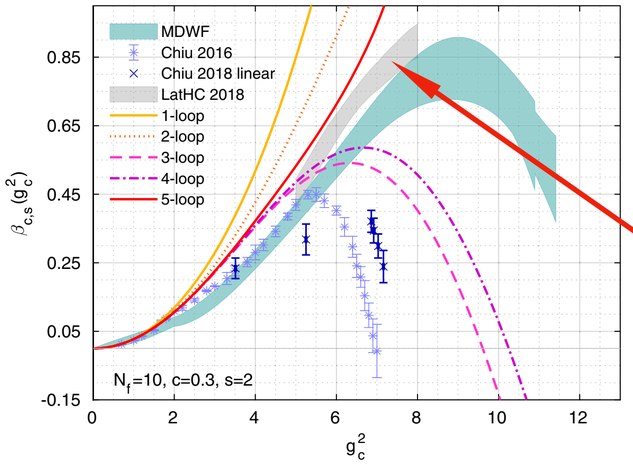

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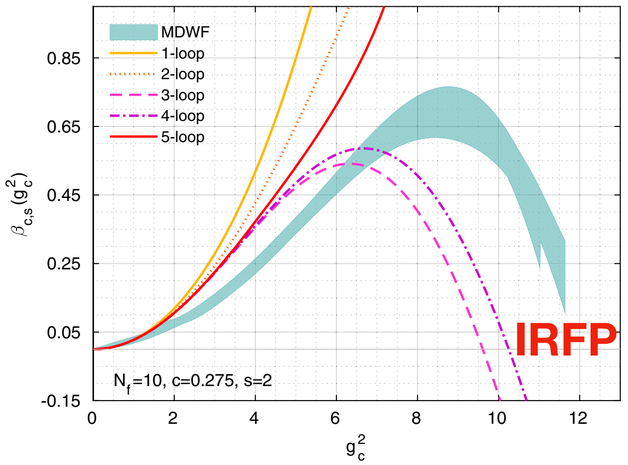

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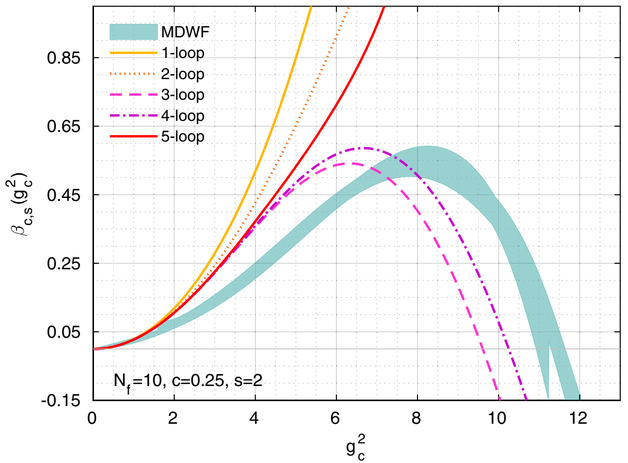
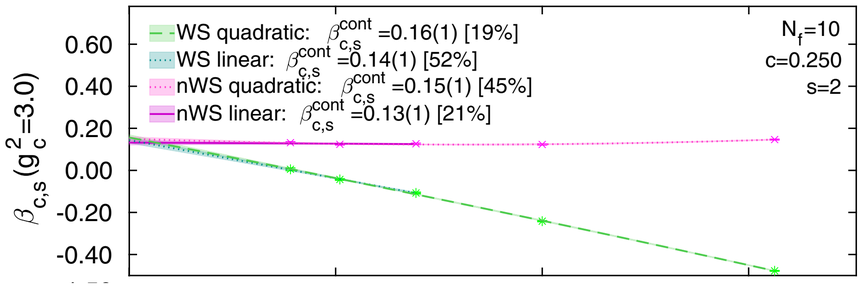
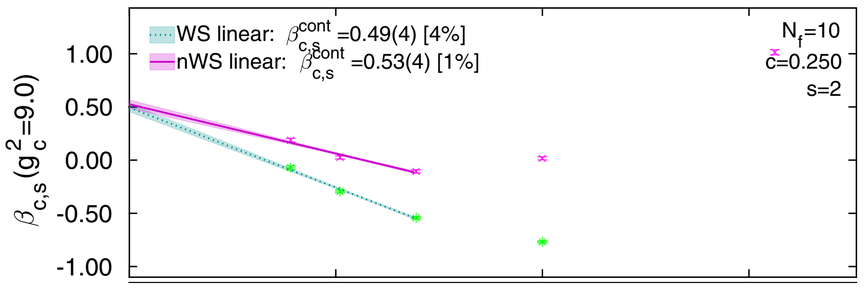


IRFP ?

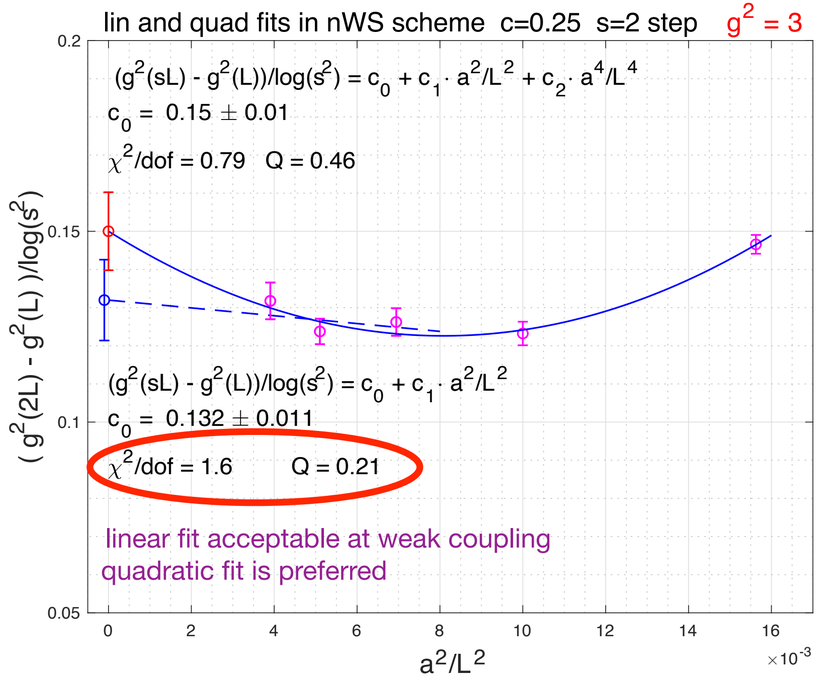
- reading 2004.00754 was puzzling
- the fitting method was puzzling
- history of moving the IRFP to stronger coupling when LatHC could not reproduce it

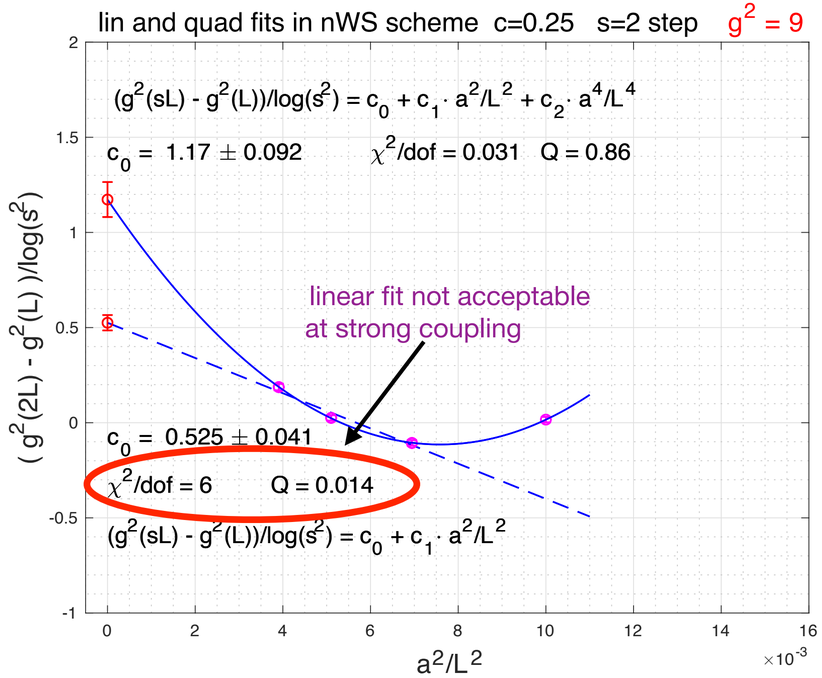
Boulder-BU fits of published data in 2004.00754 :

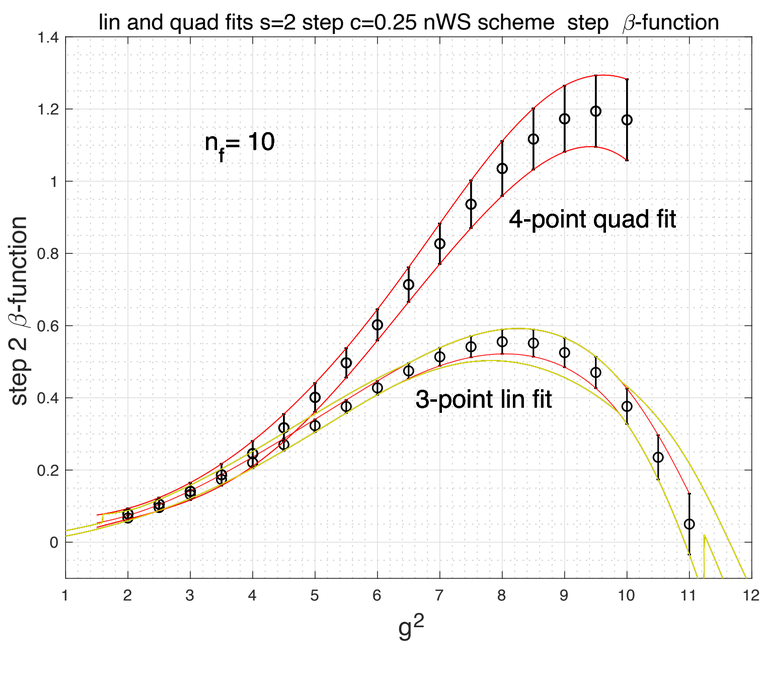


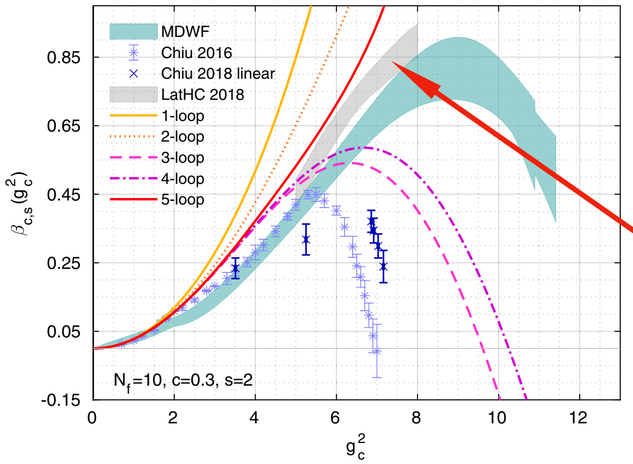


our fits to Boulder-BU published data immediately after the posting of 2004.00754 : (before we extended our 2018-2019 analysis)









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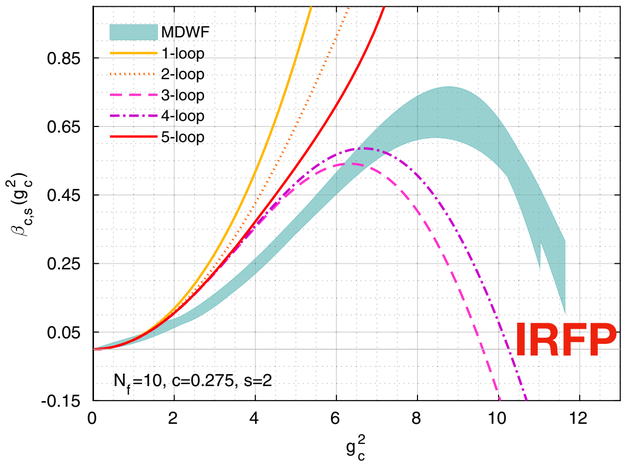

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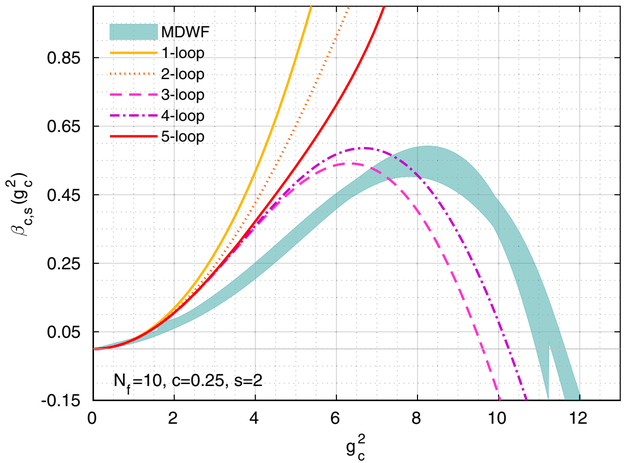


IRFP ?

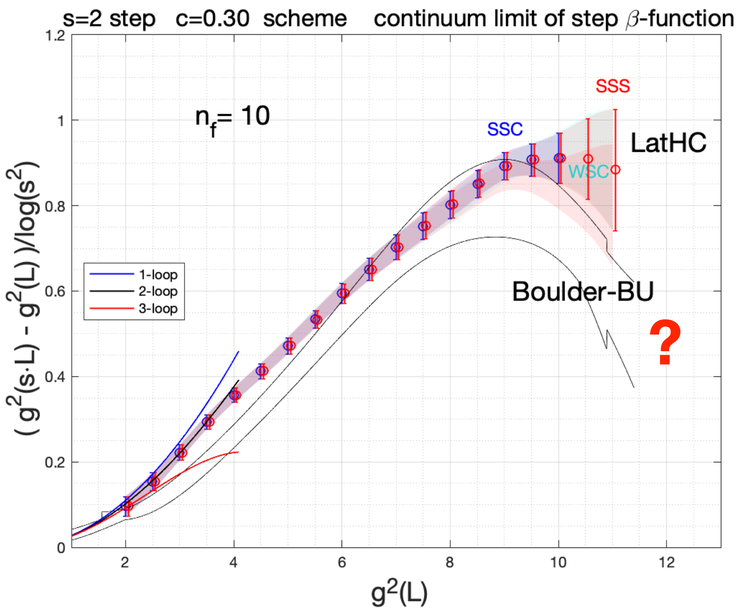
LatHC collaboration

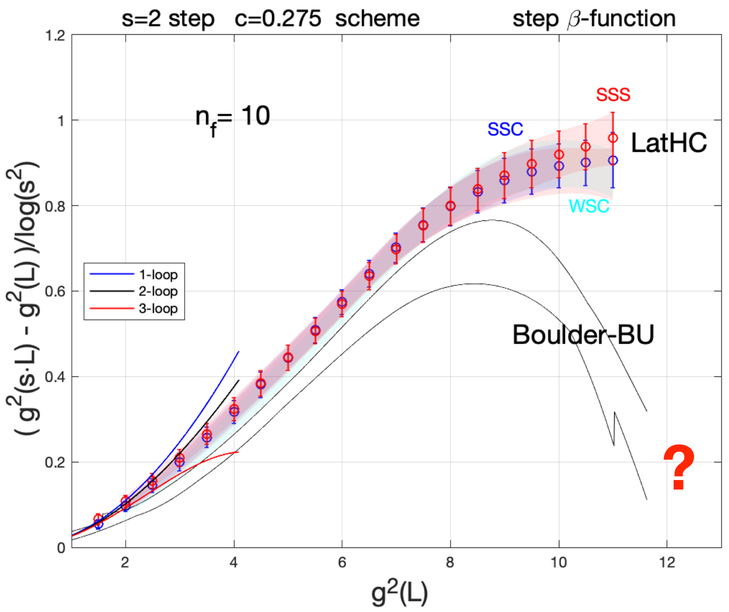
extended analysis now at Lat2021
(Holland on Wed. and in this talk)

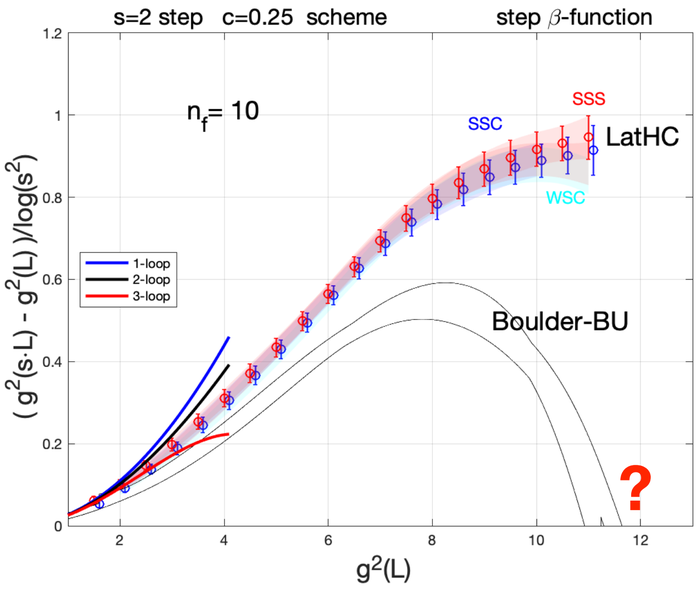
- nf = 10 s=2 step beta function
- 2018-2019 LatHC results largely extended
- Symanzik improved gauge action
- massless staggered fermions
- tree-improved SSS, SSC, WSC, WSS glow
- three aspect ratios c=0.25 c=0.275 c=0.30
- large volumes L=32, L=36, L=40, L=48
- gauge coupling now extended to $g^2 \approx 11$
- **IRFP is not found, not even hinted!**

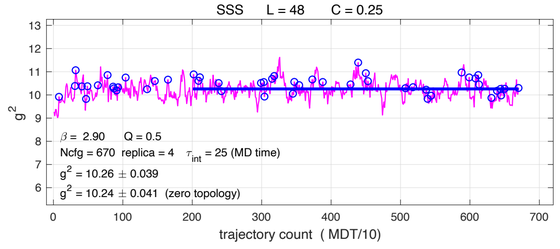
our step β-function results from ~ 200 lattice ensembles in large volumes:

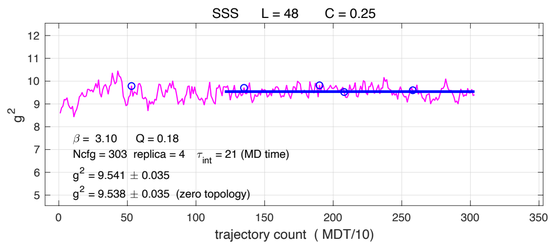
(we do not see any of the trends suggested by the Boulder-BU analysis)

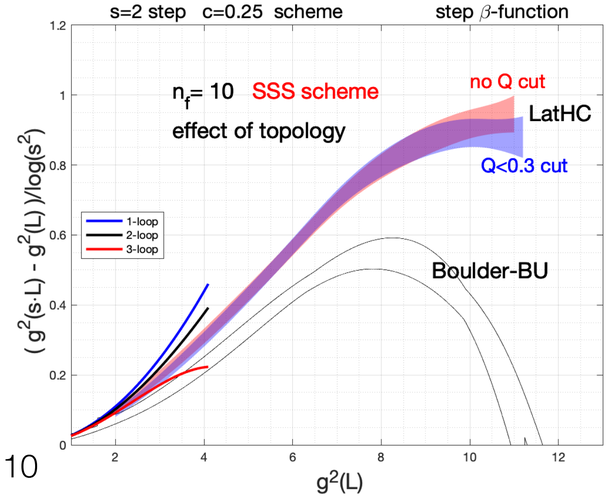






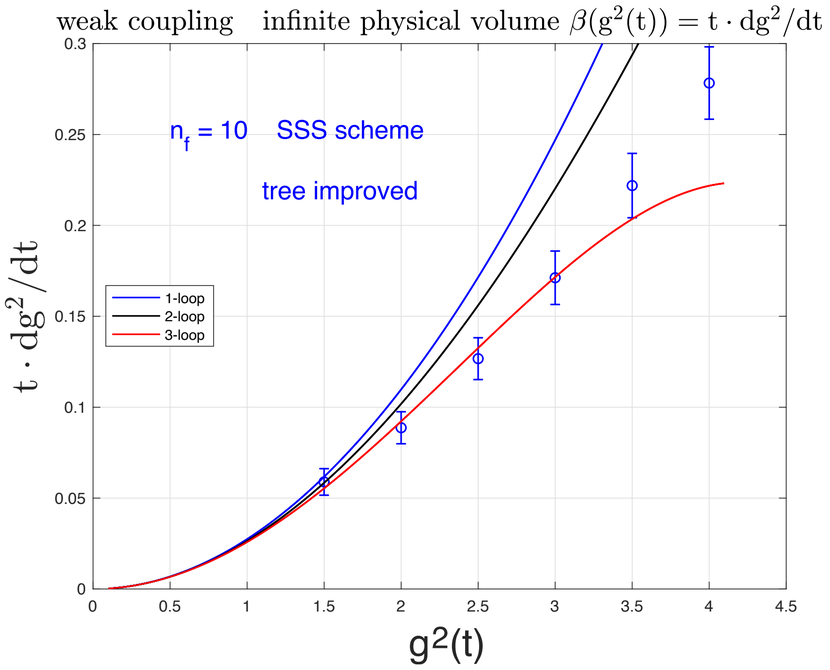




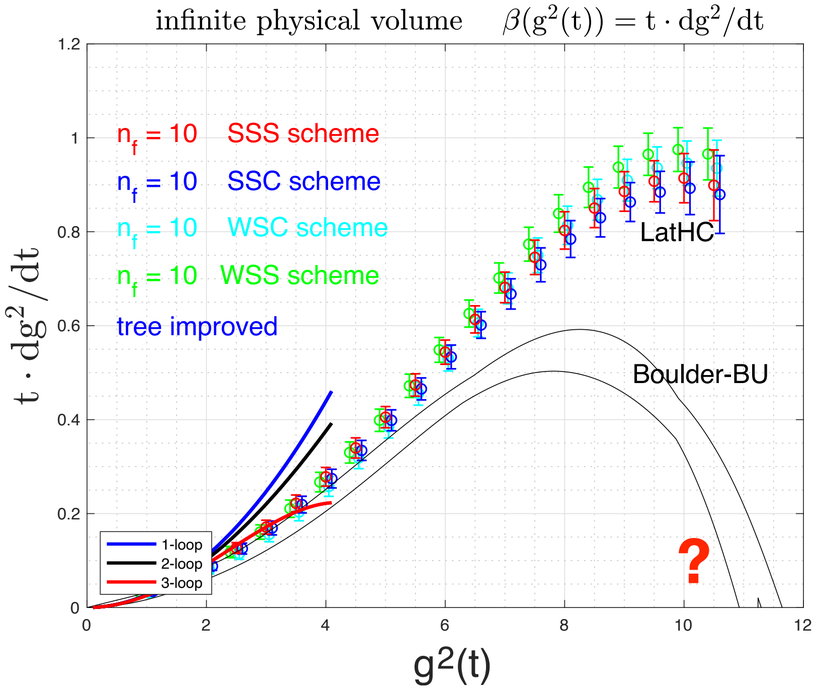


Topology is a nonissue!

our $\beta = t \cdot dg^2/dt$ derivative β-function based results extrapolated from large L=32,36,40,48 volumes to infinite volume:



contact with Harlander-Neumann 3-loop at weak coupling!



consistent results from two different β-functions **without any hint of IRFP!**

Conclusions and outlook

- the two β-functions complement each other well

- $\beta = t \cdot dg^2/dt$ uses the large volumes to reach the continuum limit

- there is no hint for IRFP in the ten-flavor model within lattice reach
  small volumes (limited by DWF) combined with linear fits at strong coupling leads to wrong results
  consistency with 4+6 composite Higgs model, hyperscaling, etc?

- there is a similar story in the twelve-flavor model, for another day …