The non-perturbative β function at the sill of the conformal window

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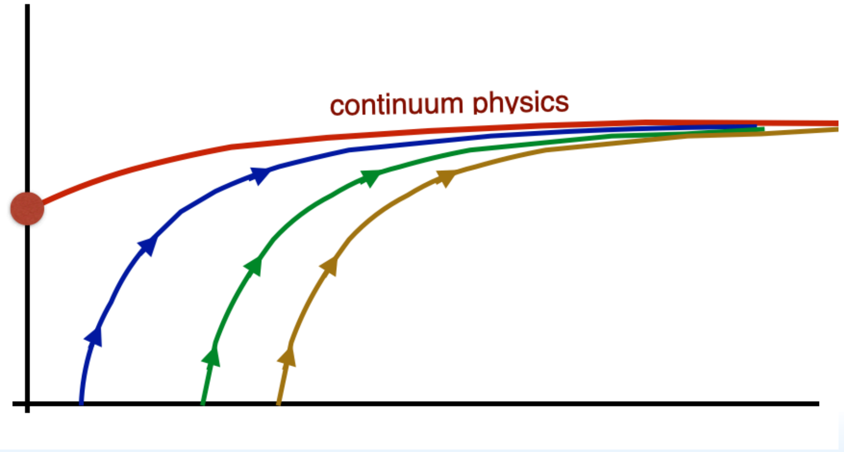
Lattice 2021

Work in collaboration with O. Witzel



The continuous β function from gradient flow

A. Carosso, AH, E. Neil, PRL 121,201601 (2018)

GF can define a real-space Wilsonian RG transformation: $\mu \propto 1/\sqrt{8t}$,

$$\beta(g_{GF}) = -\,t\frac{dg_{GF}^2}{dt}$$

In Wilsonian RG language:

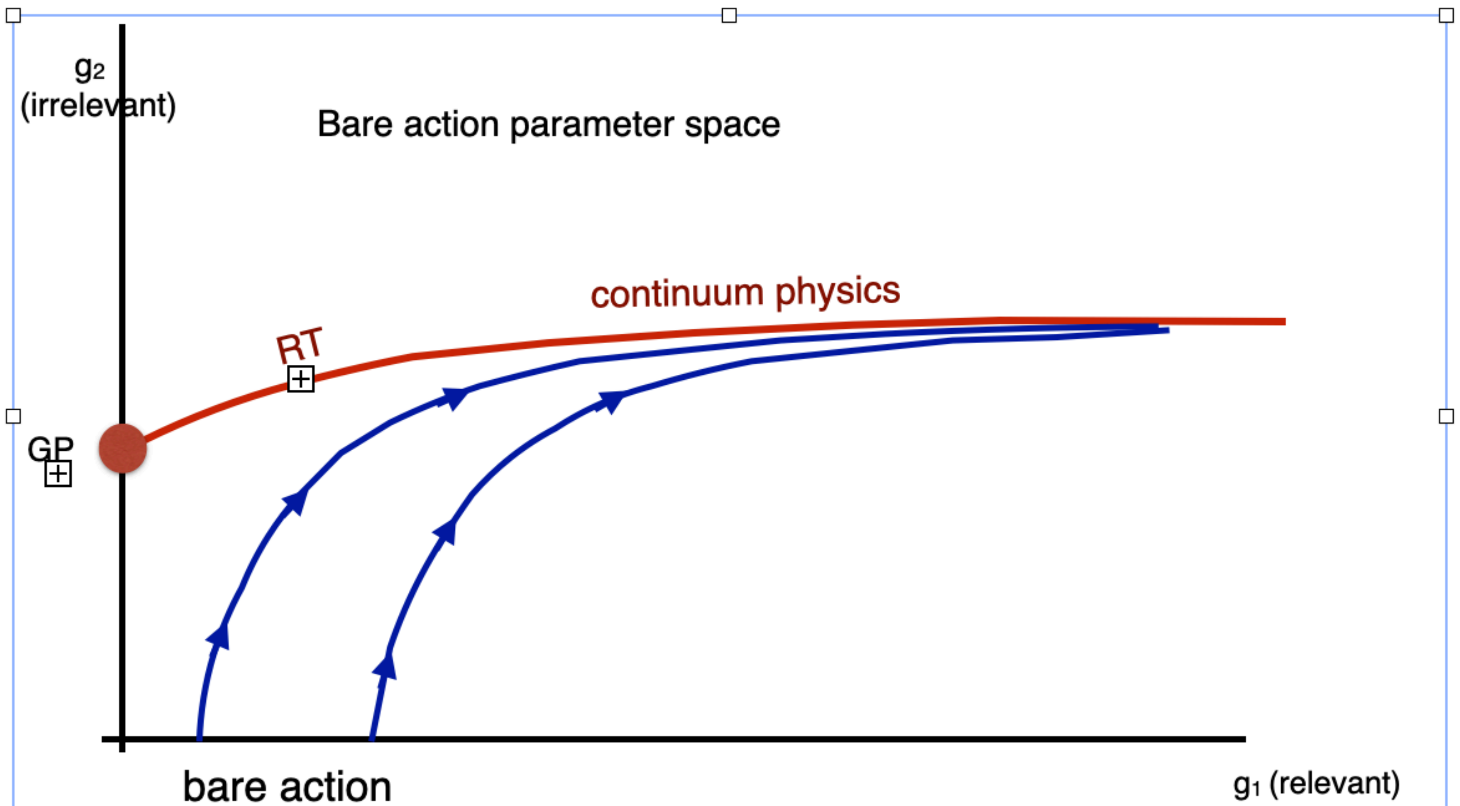
✦GF flow (RG flow): sets the FP and its renormalized trajectory (RT)(continuum physics)

✦Lattice action: starting point of RG flow

-along the RT flows from different bare couplings overlap, and describe continuum physics

✦Operator: $g_{GF}^2 = \mathcal{N} t^2 \langle E \rangle$

- g_{GF}^2 should correspond to scaling operator along the RT



The picture is valid

- in infinite volume
- in $am = 0$ chiral limit

As $t/a^2 \to \infty$

- the flows approach the RT
- the correct scaling operator is projected out
- Remaining cut-off effects are removed by $a^2/t \to 0$

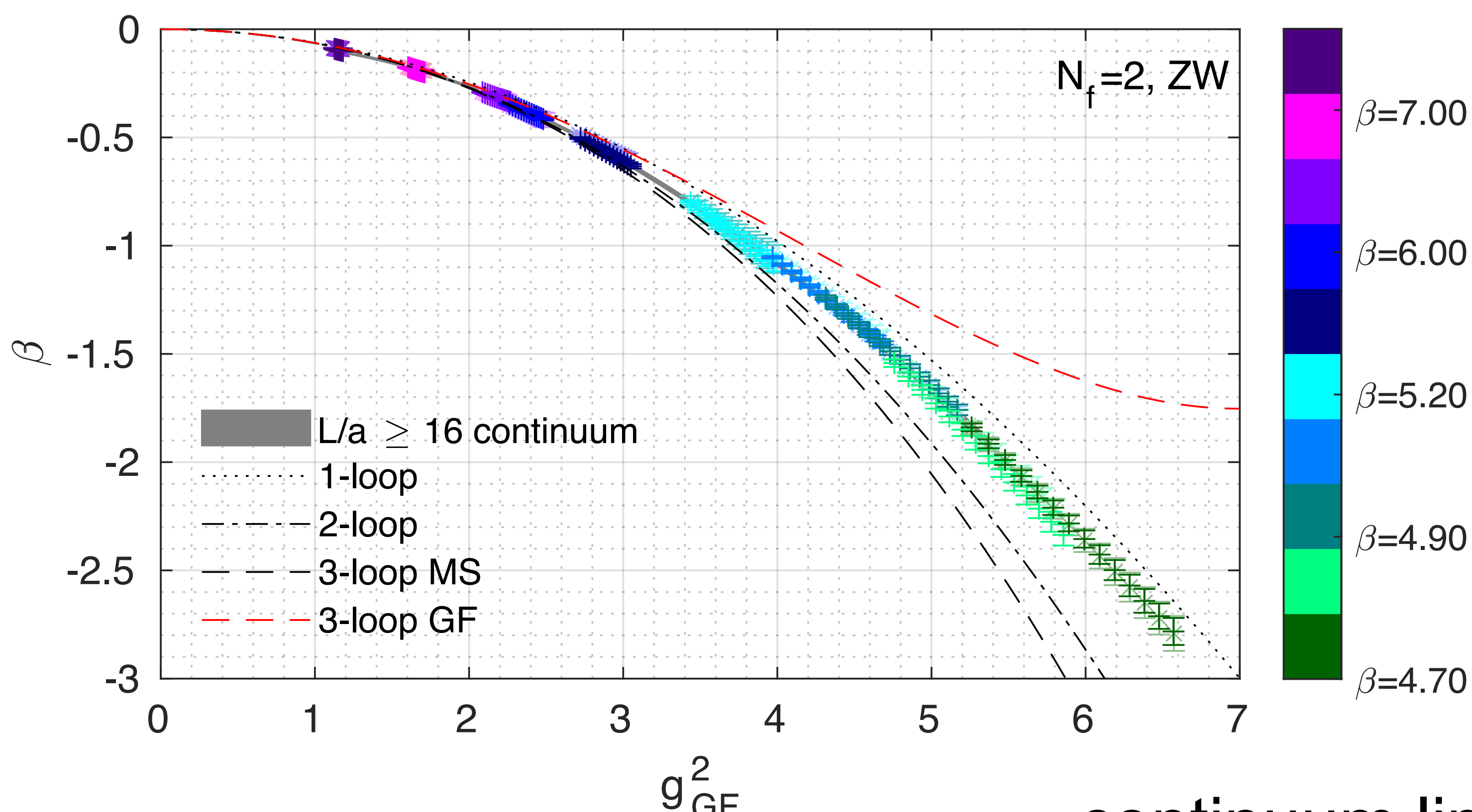
This is the continuum limit

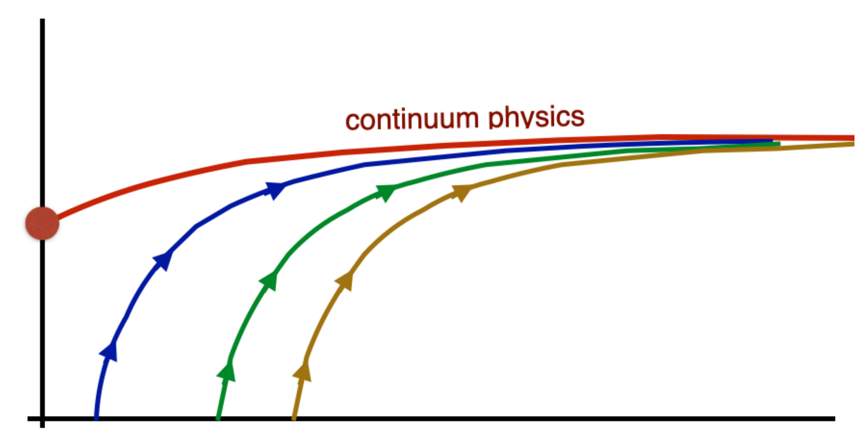
The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

We use
- Symanzik gauge action, Mobius domain wall fermions (Grid)
- Zeuthen (Z), Wilson(W) and Symanzik(S) flows : optimize to pull the RT close
- Wilson plaquette(W), clover(C) and Symanzik(S) operators : combine to optimize for the scaling operator

ZW raw data
- shows minimal finite volume effects and significant overlap





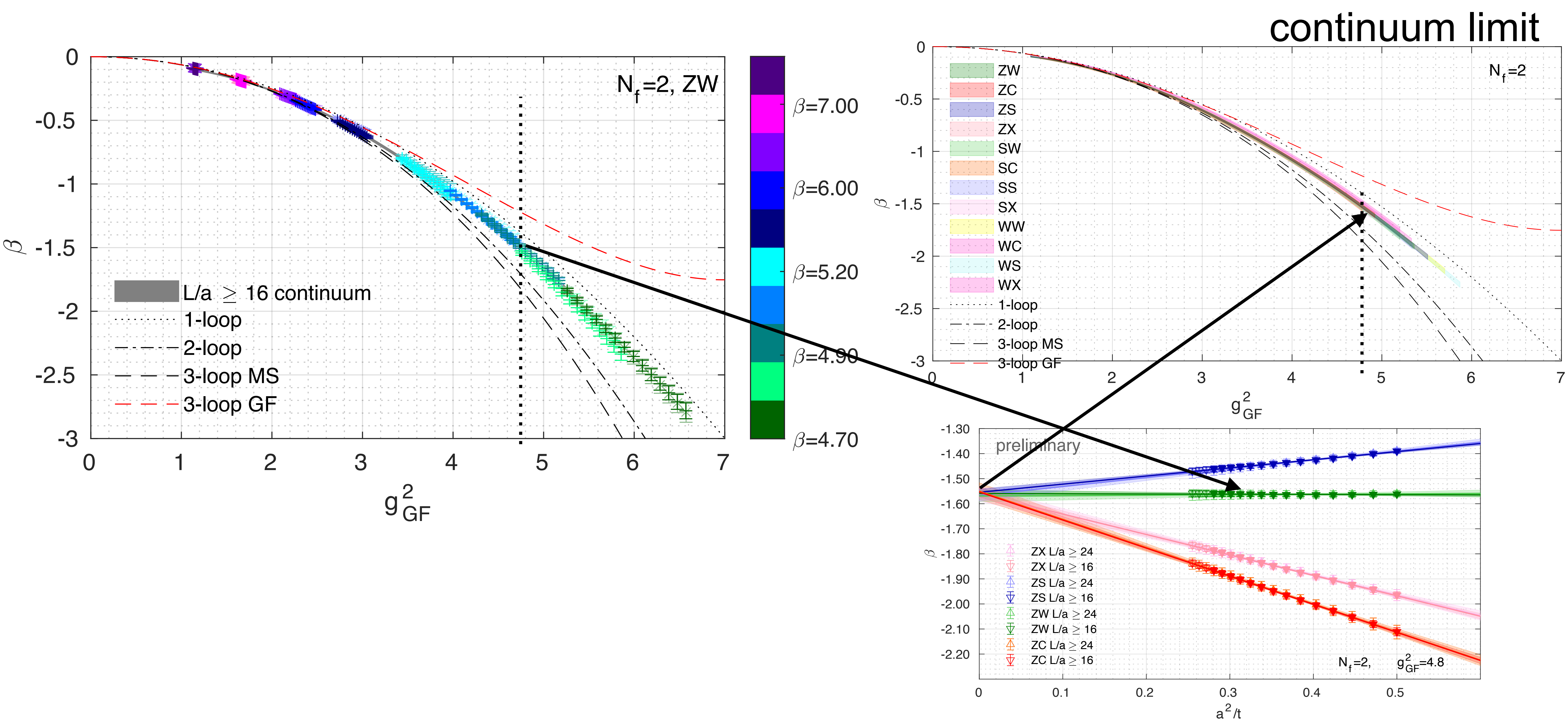
continuum limit (below colored points)

The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

Analysis steps for continuum limit:

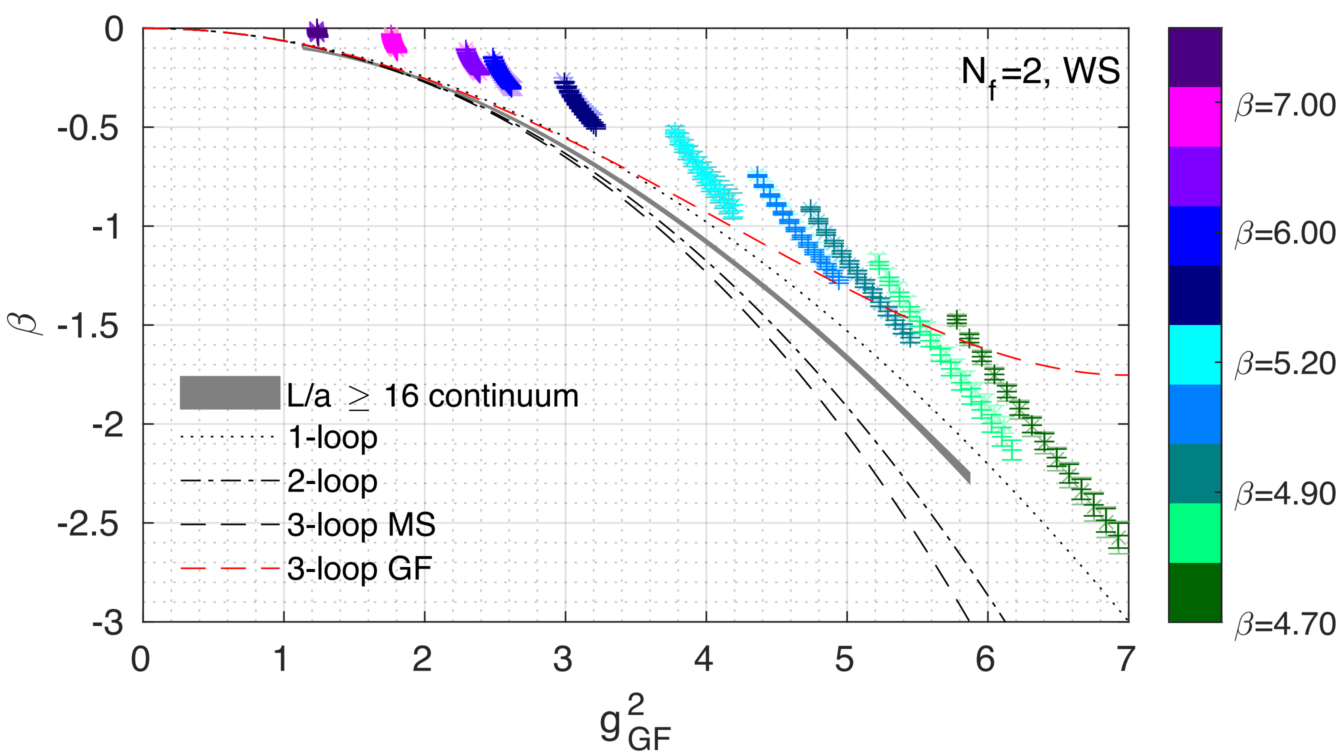
1. Infinite volume extrapolation ($1/L^4$ in the chirally symmetric regime)
2. Infinite flow time extrapolation (a^2/t) at fixed g^2_{GF}

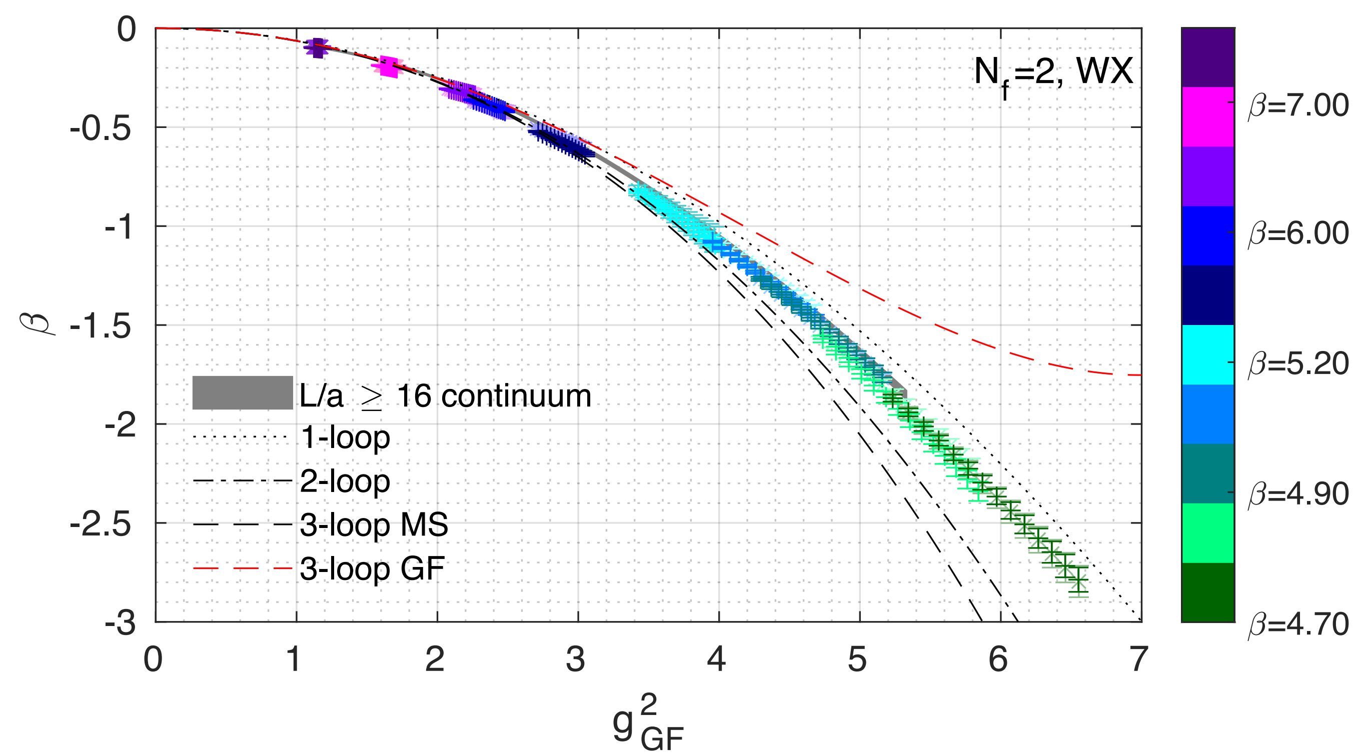


The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

- Other flow/operator combos predict the same continuum limit, sometimes with larger cut-off effects (WS)
- It is possible to “optimize” the operator for different flows and find “scaling operators” (e.g. $X = 0.25W + 0.75C$ is optimal for Wilson flow) (WX)



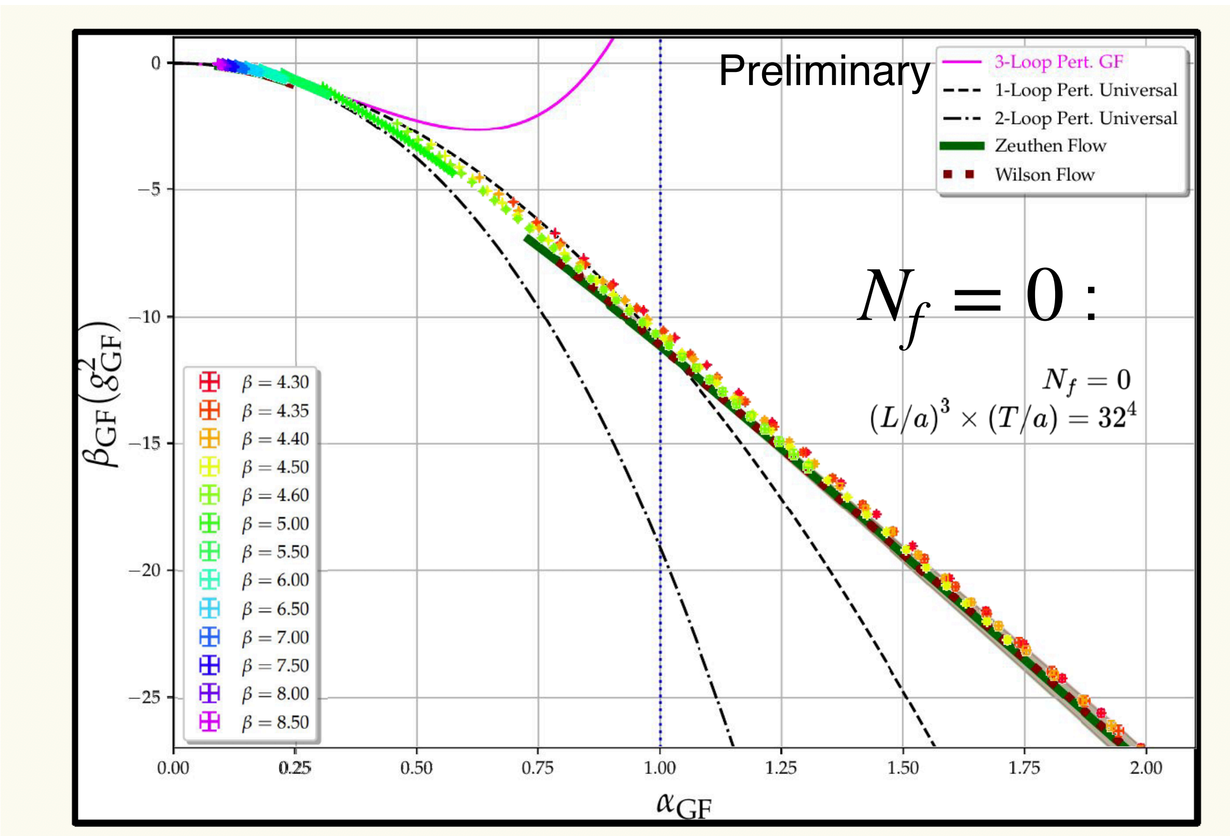


GF β function is closest to 1-loop; consistent with GF 3-loop up to $g^2_{GF} \approx 2.5$

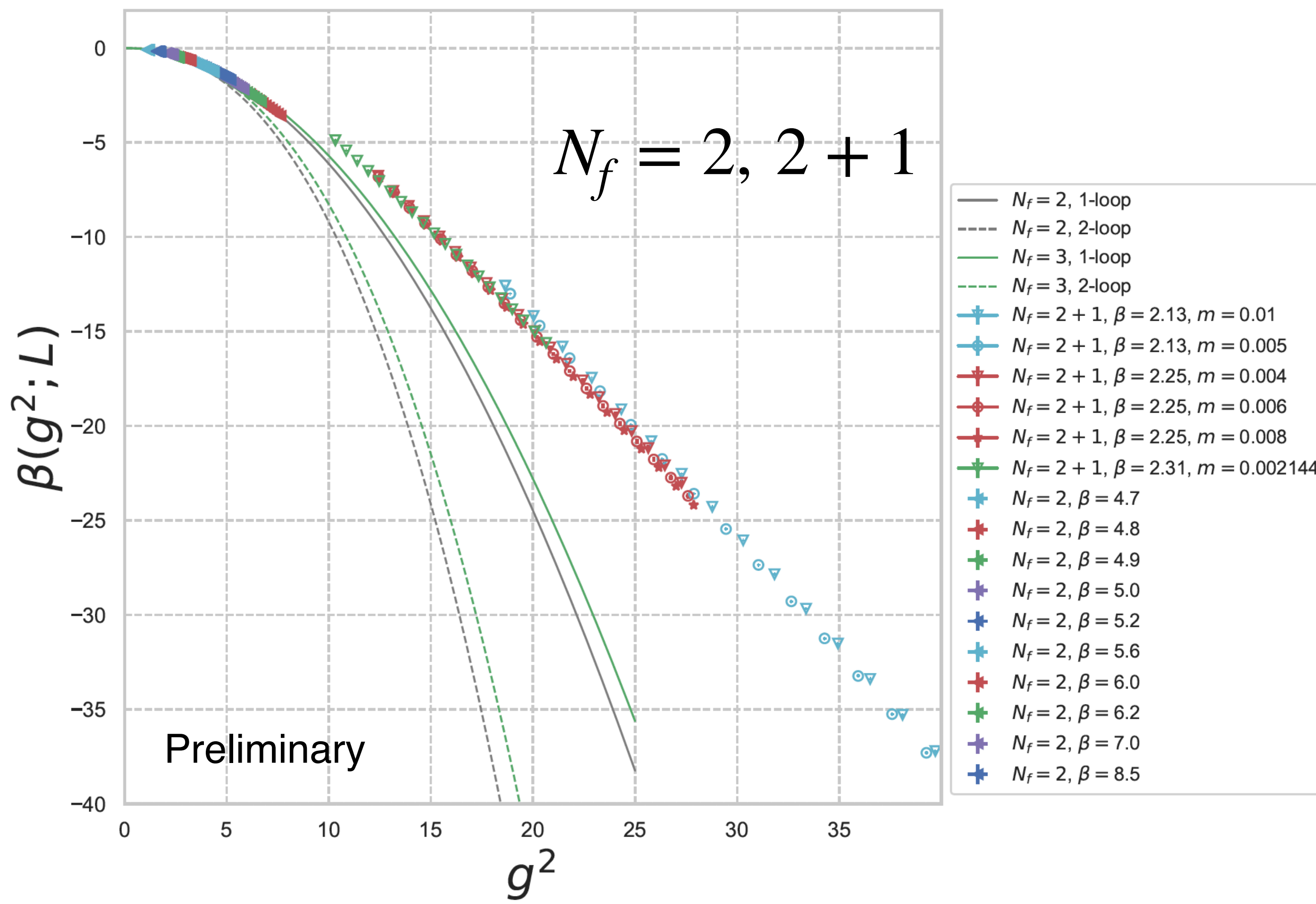
Harlander et al, *JHEP* 06 (2016) 161

The continuous β function, $N_f = 0,\ 2+1$

Continuous β function approach works even in the chirally broken/confining regime where Λ_{QCD} or $\langle \bar{\psi}\psi \rangle$ sets the scale



C. Peterson, talk on Wed 10:30pm
(Symanzik gauge, Zeuthen flow,
Symanzik operator)



C.Monahan, talk on Wed 9:15pm
($N_f = 2+1$: RBC-UKQCD configs,
Iwasaki gauge, Wilson flow, W op)

GF β function consistent with GF 3-loop up to $g^2_{GF} \approx 3$; runs slower than 1-loop

The continuous β function, $N_f = 4$

Preliminary

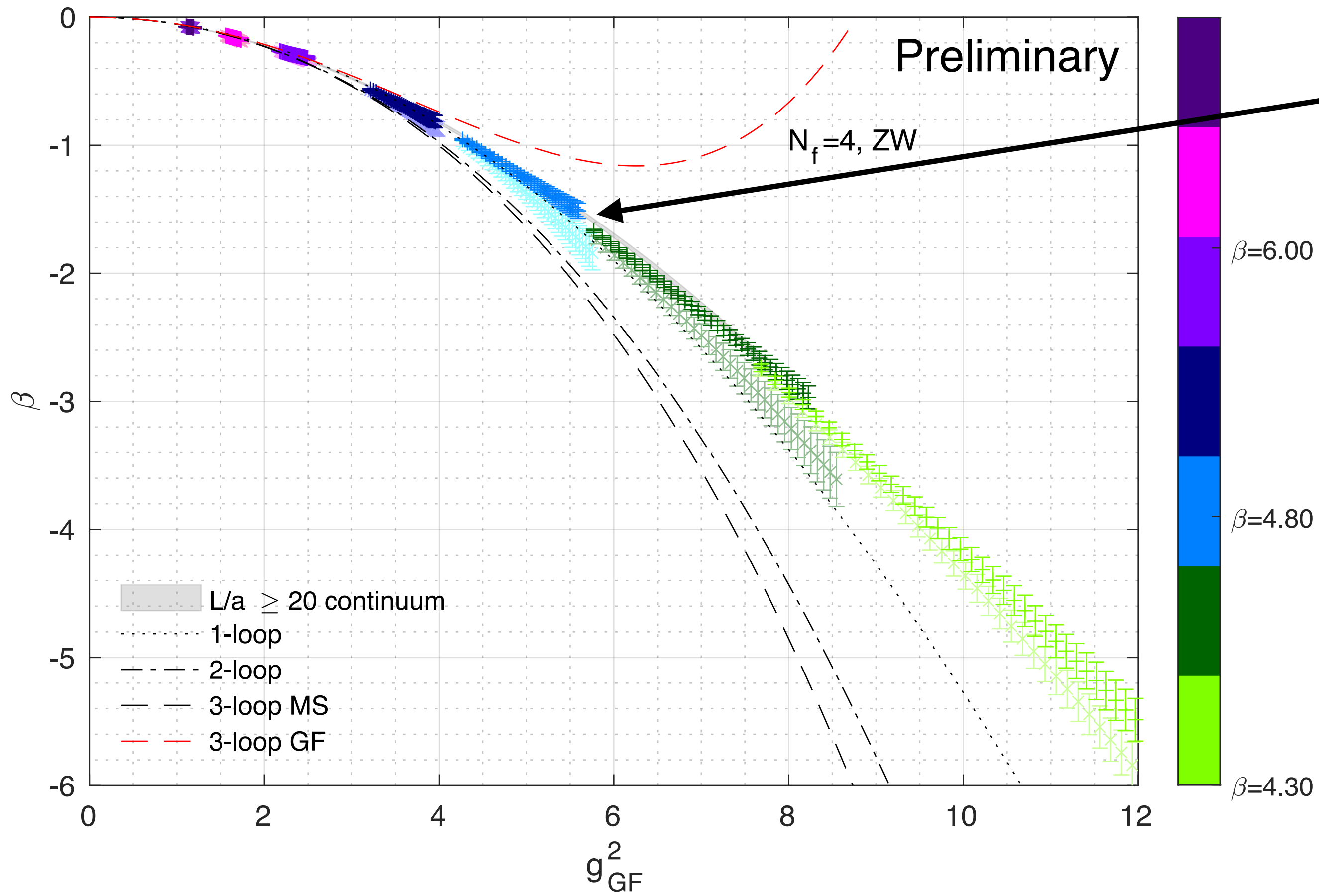
Simulations in the weak coupling chirally symmetric regime, 20^4, 24^4, 32^4 volumes

ZW raw data:

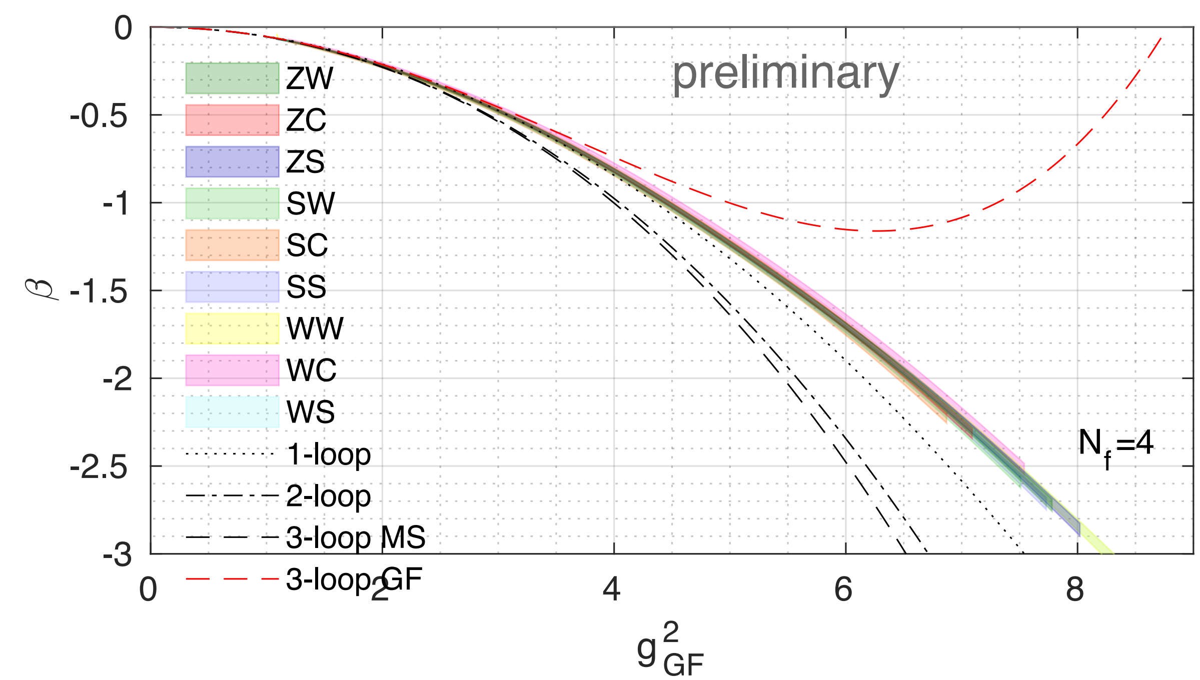
- shows significant overlap and minimal finite volume effects

➡ sits almost on top of continuum extrapolation

- Other flow/operator combos still predict the same continuum limit



continuum limit



GF β function consistent with GF 3-loop up to $g^2_{GF} \approx 3$;
runs between 1 loop and GF 3 loop PT

The continuous β function, $N_f = 6$ Preliminary

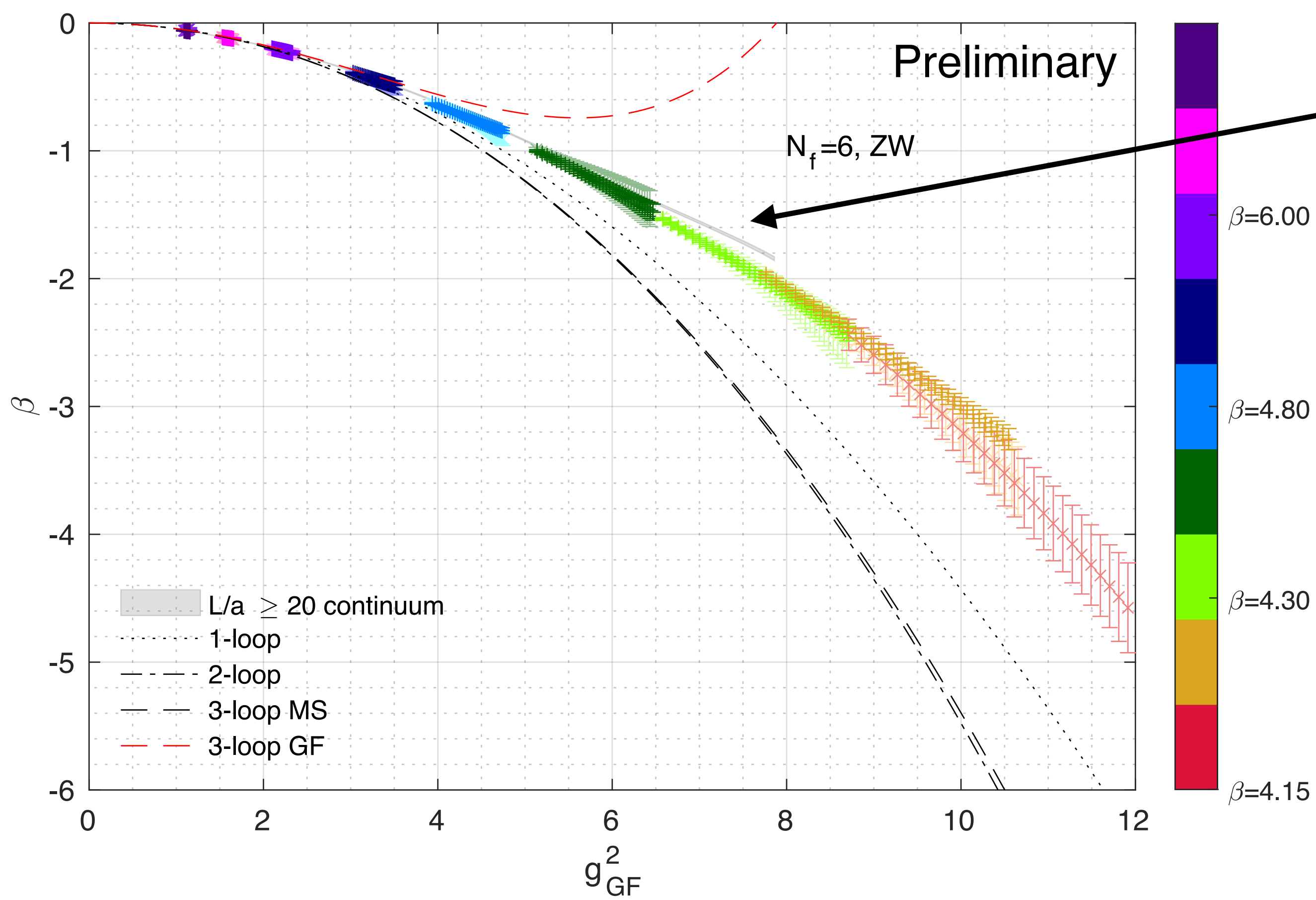
Simulations in the weak coupling chirally symmetric regime, 20^4, 24^4, 32^4, 40^4 volumes

ZW raw data

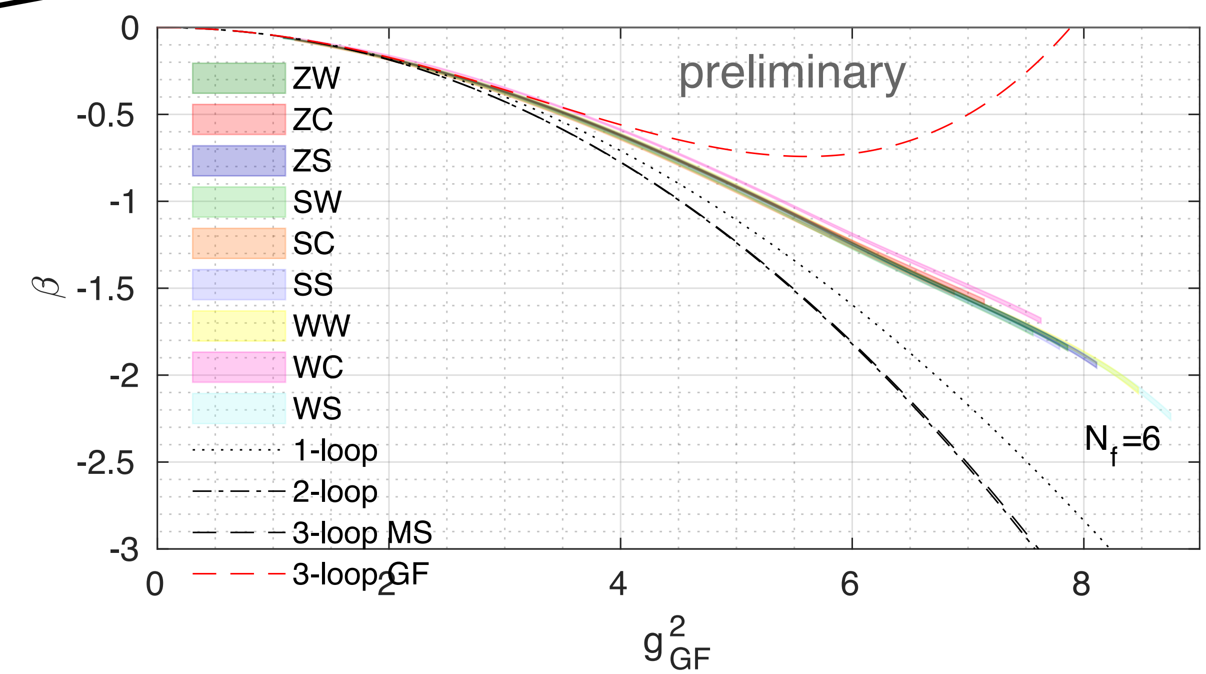
- shows significant overlap and minimal finite volume effects

➡ sits almost on top of continuum extrapolation

- Other flow/operator combos still predict the same continuum limit



continuum limit

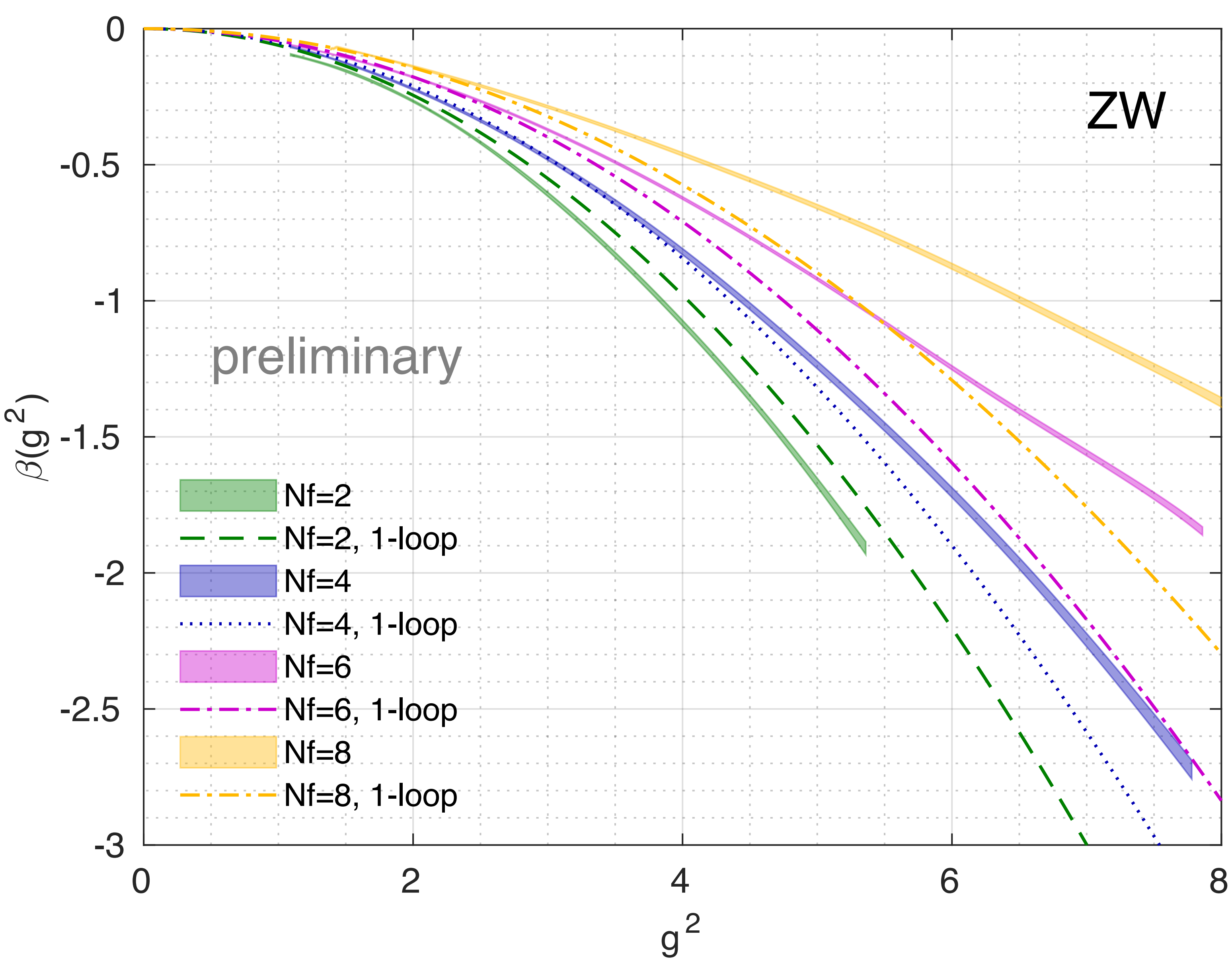


GF β function consistent with GF 3-loop up to $g^2_{GF} \approx 3$;
runs between 1 loop and GF 3 loop PT

The continuous β function, $N_f = 0 - \cancel{6}\ 8$

IN all cases:

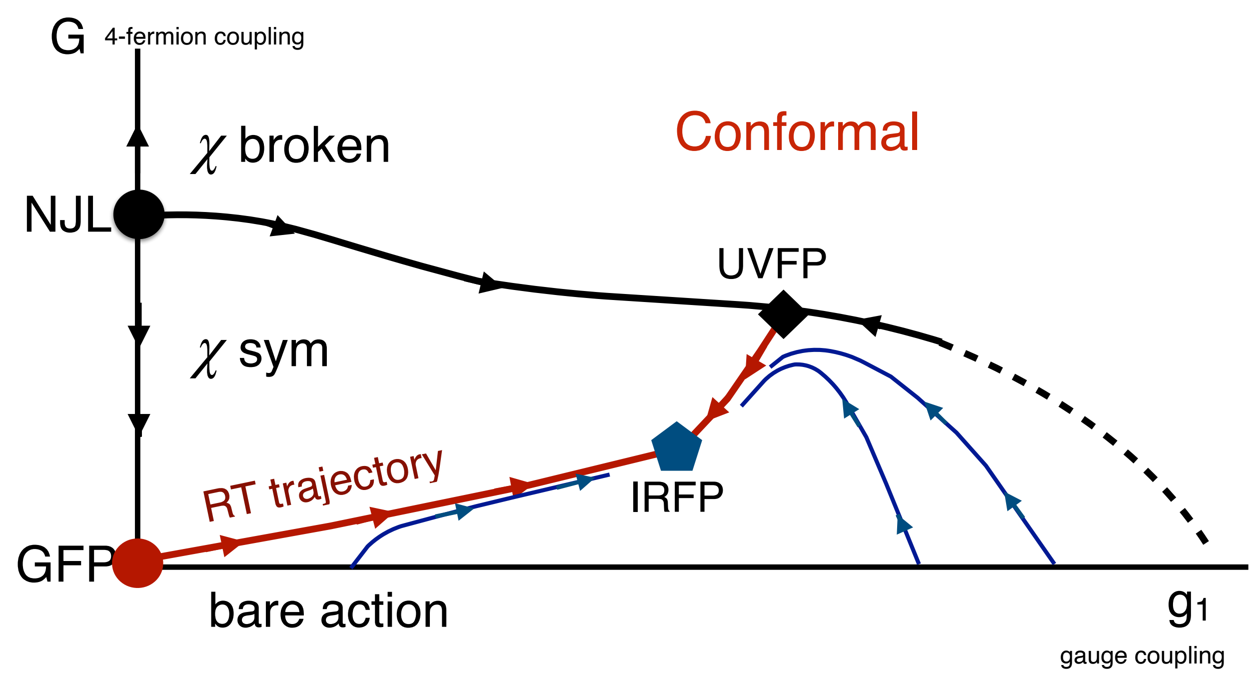
- ZW combination shows small cutoff effects: significant overlap
- Other operators are consistent in the continuum limit
- GF β function consistent with GF 3-loop up to $g^2_{GF} \approx 3$
- The β functions runs between 1-loop and GF 3-loop PT

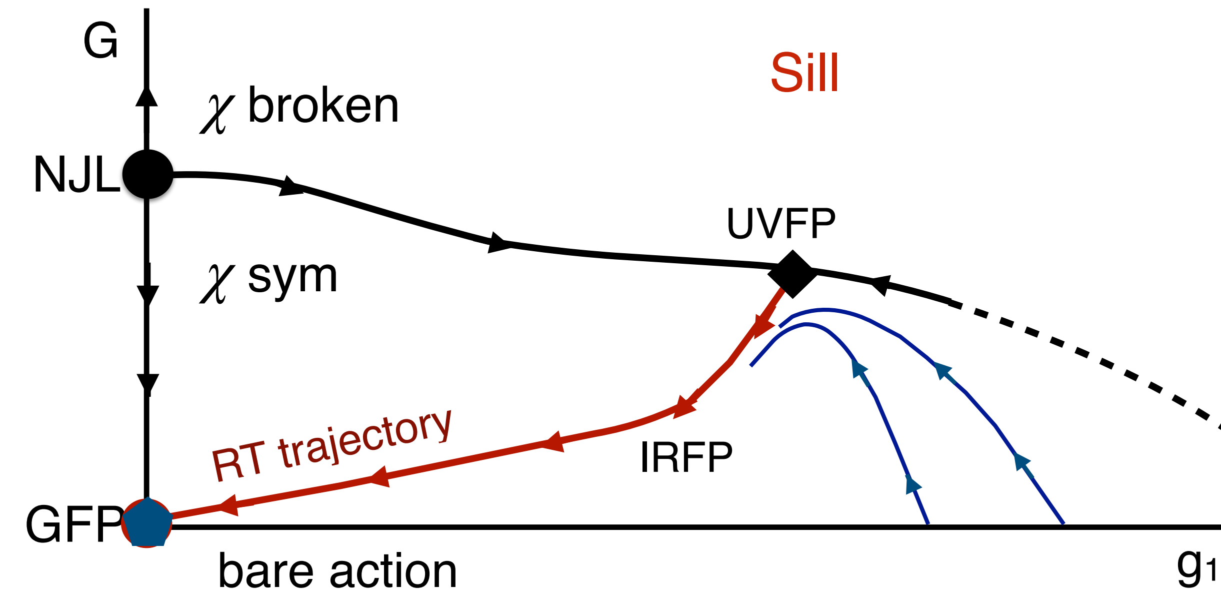


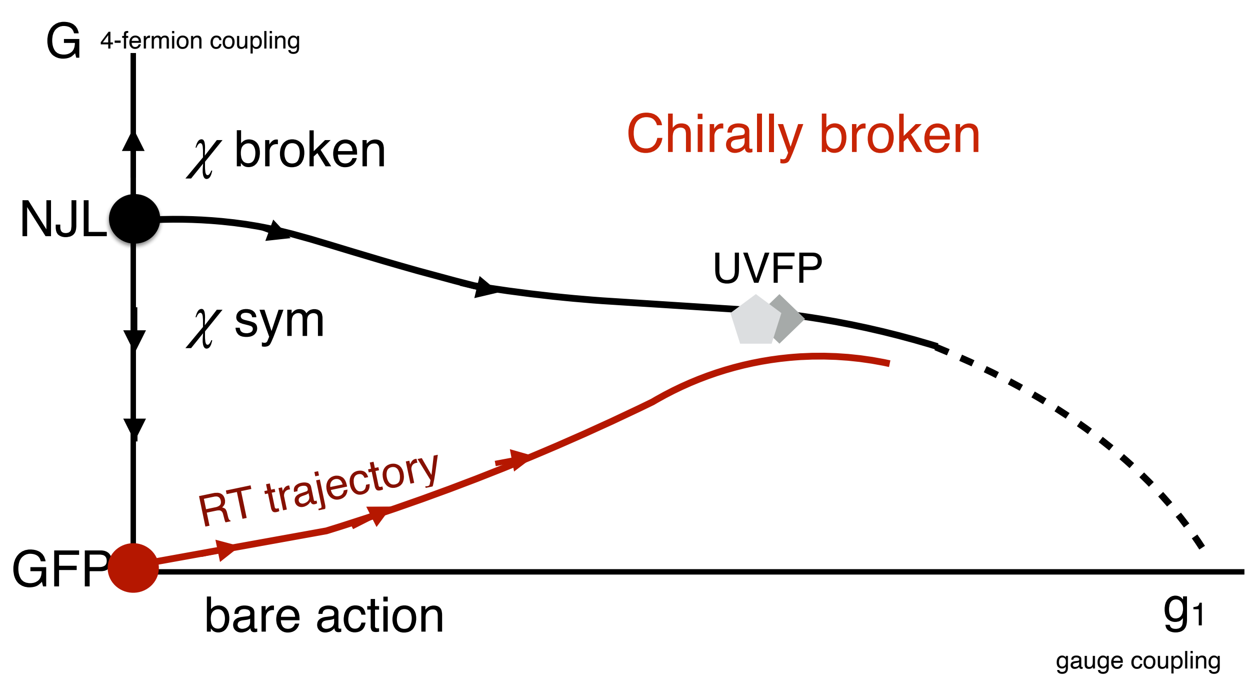
The β function of (near-)conformal systems

Near the conformal sill an UVFP could appear in an extended parameter space (4-fermion interaction)

Kaplan et al, *Phys. Rev. D* 80 (2009) 125005
Gorbenko et al, *JHEP* 10 (2018) 108



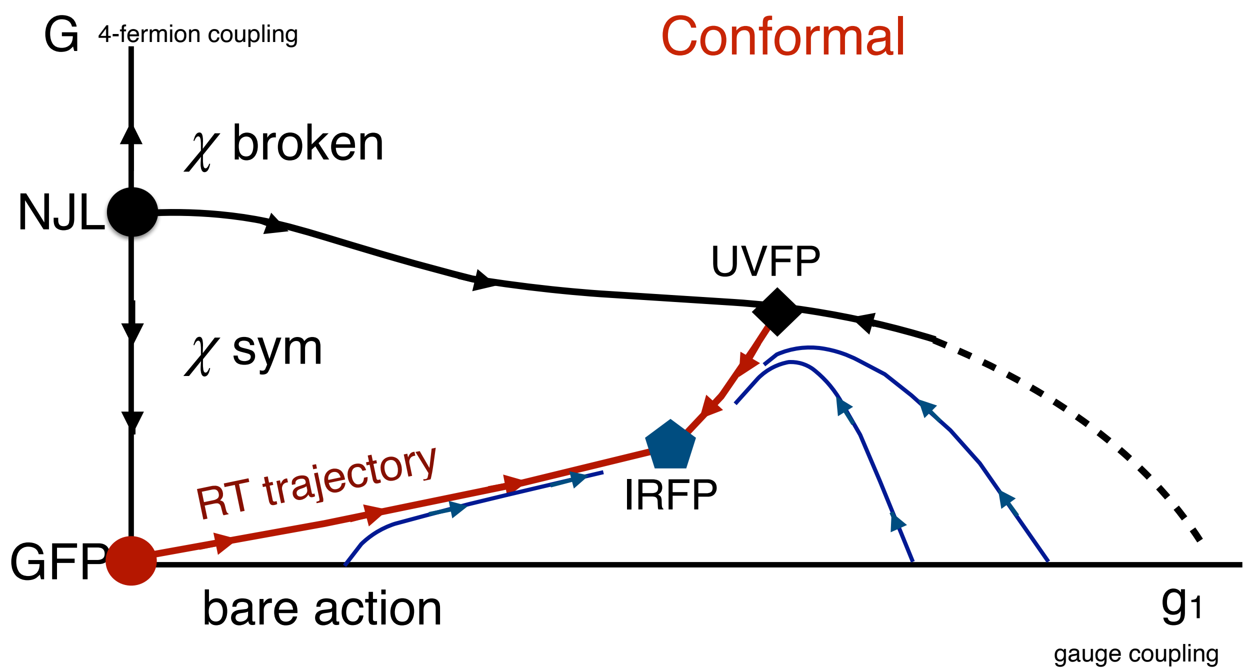




Any calculation based on the perturbative Gaussian FP will cover only (part of) the regime between GFP and IRFP

AH, O. Witzel, *PoS* LATTICE2019 (2019) 094

The β function of (near-)conformal systems



Any calculation based on the perturbative Gaussian FP will cover only (part of) the regime between GFP and IRFP:

- perturbatively justified extrapolation in $(a/L)^2$ or (a^2/t) might need to be replaced with $(a/L)^\alpha$, $(a^2/t)^{\alpha/2}$
- new relevant operator at the UVFP means that a 1-parameter β function might not describe the RG flows
- most simulations are plagued by (bulk) 1st order transition that prevent investigations at strong coupling

To solve these issues will require a new approach

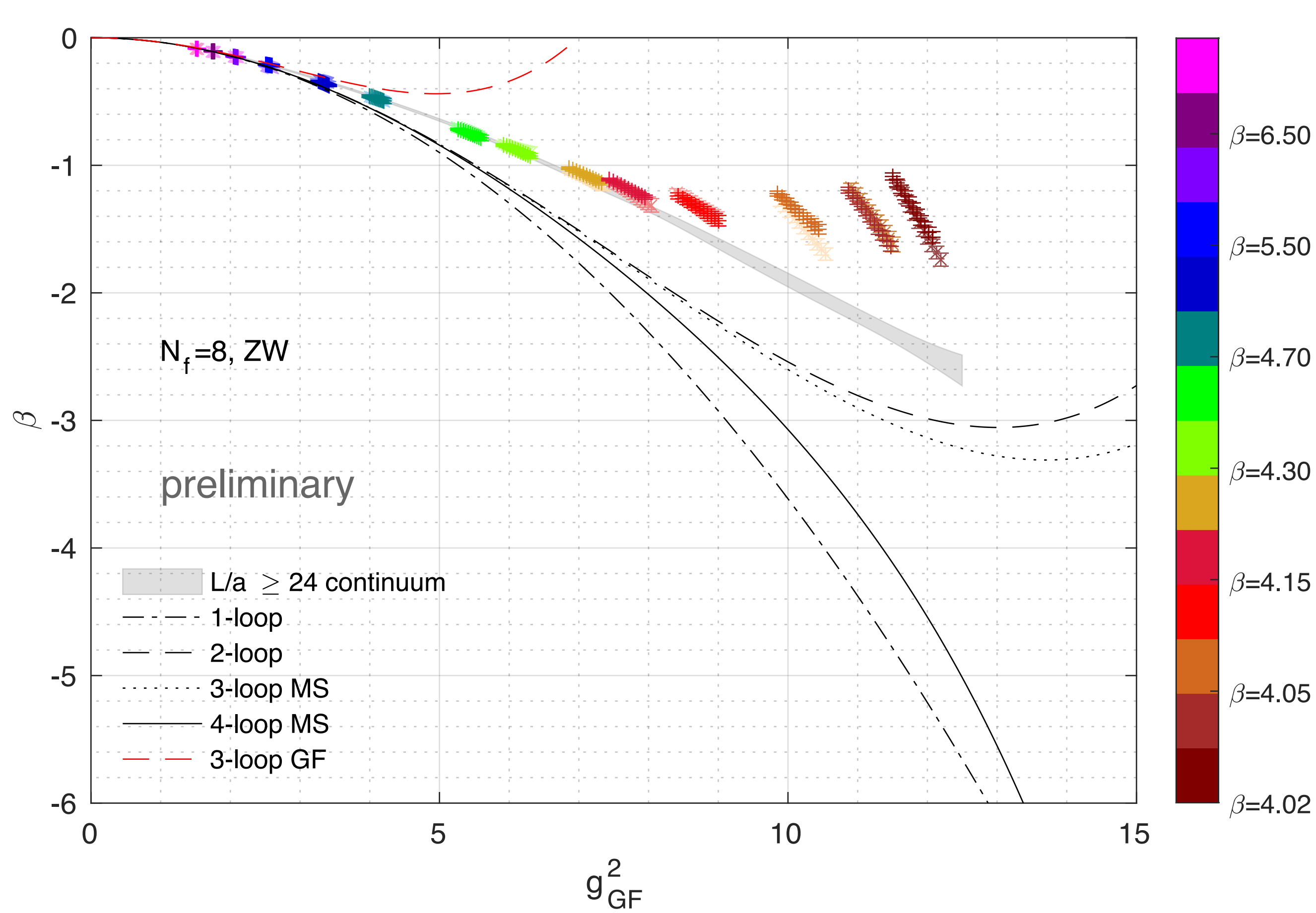
The continuous β function, $N_f = 8$

Simulations in the weak coupling chirally symmetric regime, 16^4, 24^4, 32^4, some 48^4 volumes

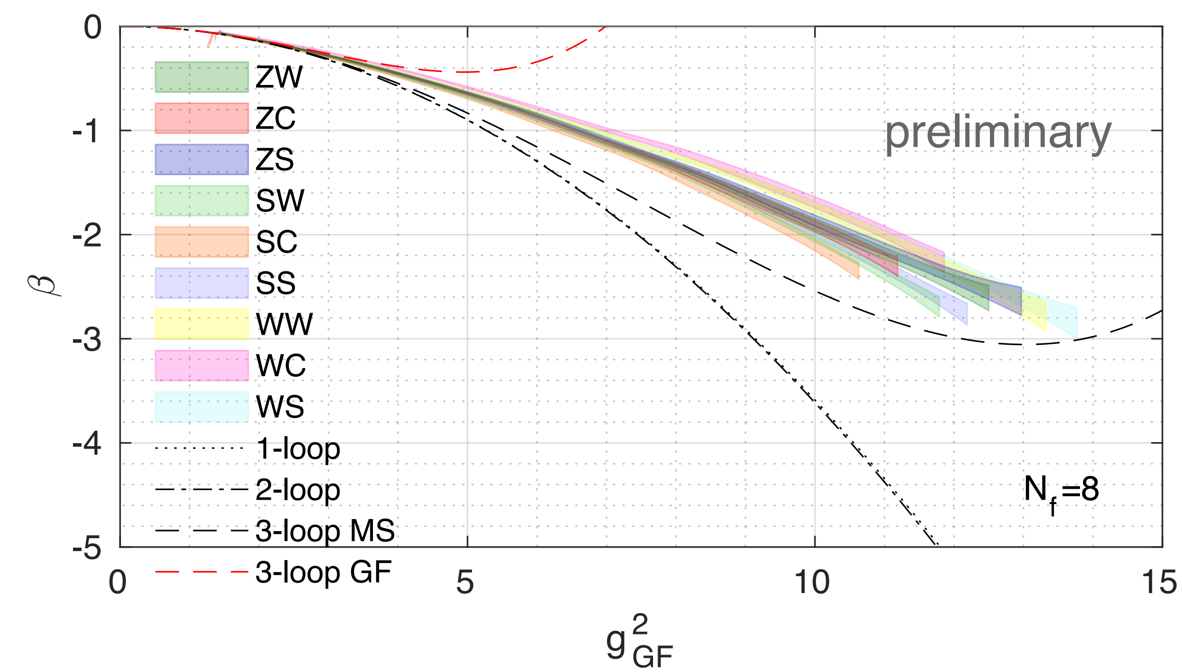
ZW raw data :

➡ sits almost on top of continuum extrapolation up to $g^2_{GF} \approx 8$

at stronger coupling the perturbative guidance breaks down, no overlap, no unique curve traced out by raw data

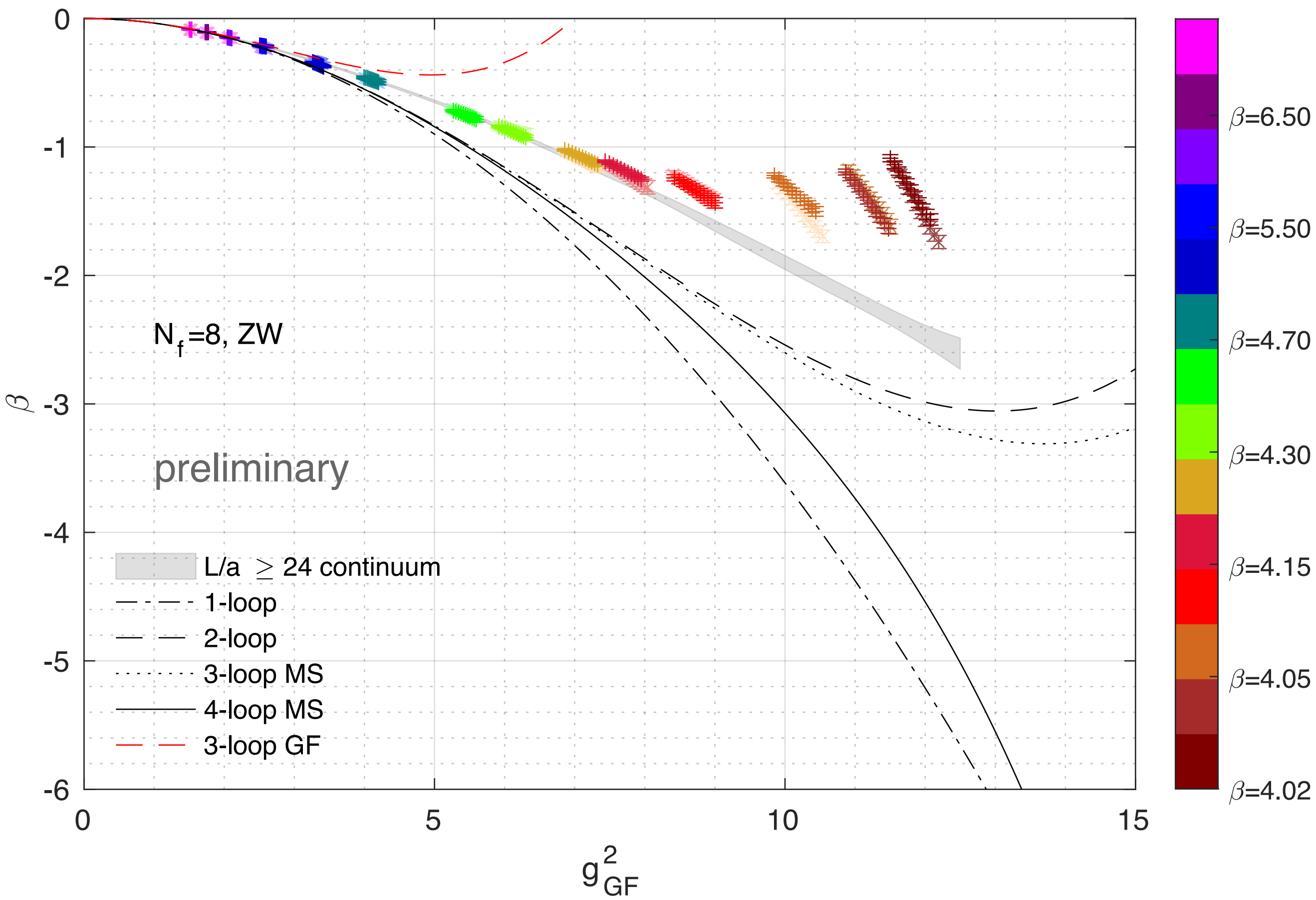


continuum limit

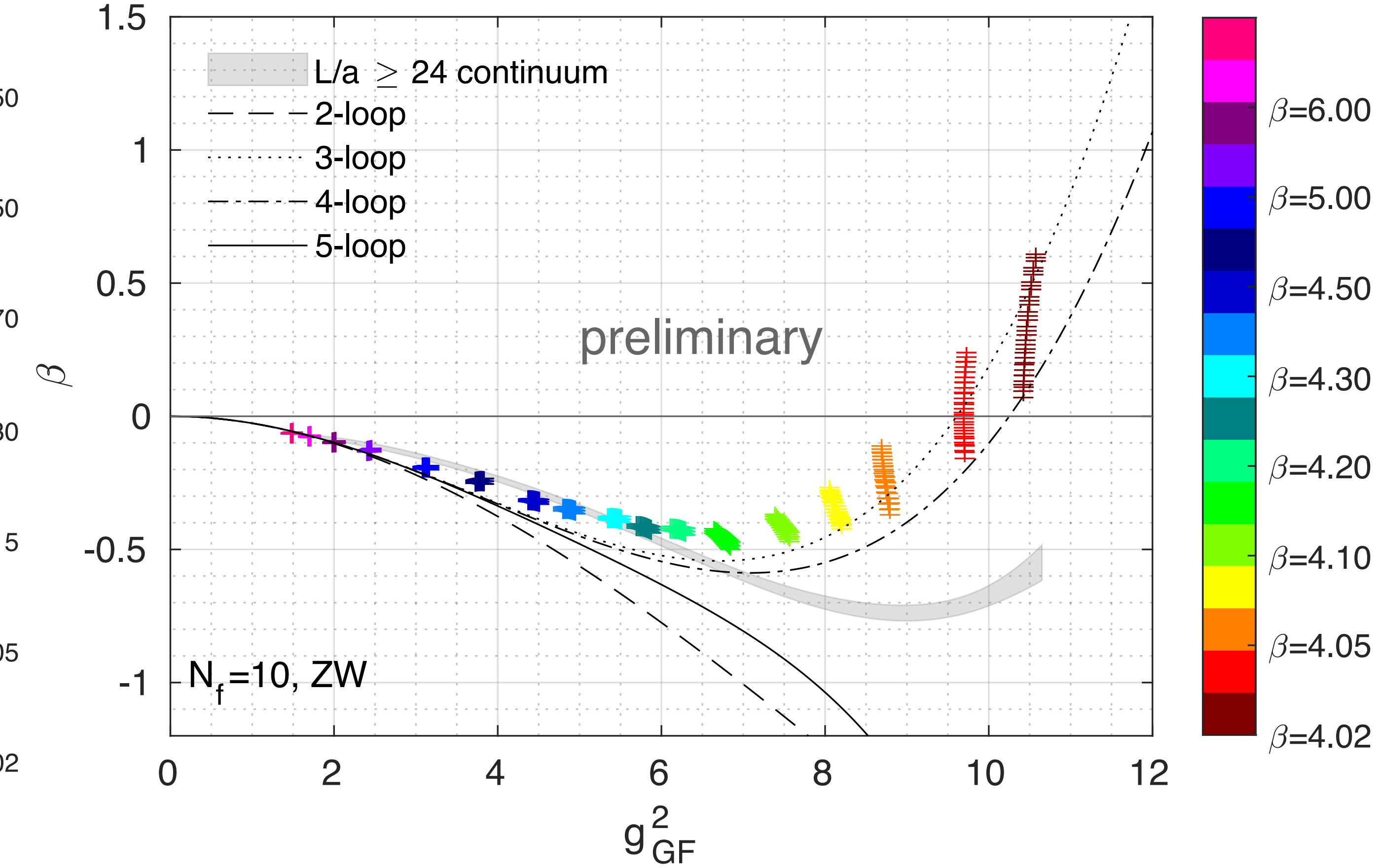


Is this a signal of nearby IRFP/UVFP ? (Somewhere at $N_f \gtrsim 8$)

The continuous β function, $N_f = 8$ and 10



Consistent with
AH,O.Witzel, *Phys.Rev.D* 101 (2020) 11, 114508



- At stronger couplings there is no overlap, no unique curve traced out by raw data
- Is this a signal of nearby IRFP/UVFP ? (Could be at $N_f \gtrsim 8$)

Simulations at even stronger couplings are blocked by a 1st order bulk transition
Improving the action opens up the parameter space to $g^2_{GF} > 20$:

➡Topic for Lattice 2022

AH, Y. Shamir, B. Svetitsky,
"The use of Pauli-Villars fields in lattice simulations", in prep.

Summary and Outlook

The equivalence of Wilsonian RG and gradient flow allows a theoretically solid description of the strong coupling regime of lattice models

✦ This is particularly important in near-conformal / conformal systems where new fixed points, new relevant operators appear

The continuous β function

✦In QCD-like chirally broken systems it is well controlled with minimal cutoff effects with improved action/flow/operator in QCD-like chirally broken systems

✦Near the conformal sill strong non-perturbative effects appear :

- different scaling forms might be necessary
- extended actions (4-fermion term?) might be necessary
- larger volumes (brute-force approach) can extend the accessible range, but will not cover the IRFP or UVFP beyond

Improved actions that avoid the bulk 1st order transition and reduce cutoff effects should be considered