Thermal phase structure of dimensionally reduced super-Yang–Mills

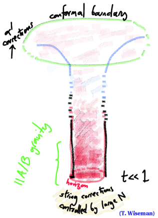
David Schaich (U. Liverpool)

Lattice 2021, July 29

arXiv:2003.01298 and more to come with Raghav G. Jha and Anosh Joseph

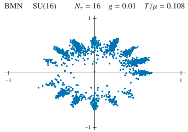
Overview

Dimensional reduction simplifies lattice supersymmetry,
retaining interesting dynamics and holographic dualities

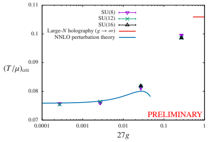


Motivations for lattice susy & dimensional reduction

Berenstein–Maldacena–Nastase (BMN) [hep-th/0202021]
deformation of super-Yang–Mills quantum mechanics



BMN phase diagram results & future plans



These slides: davidschaich.net/talks/2107Lattice.pdf

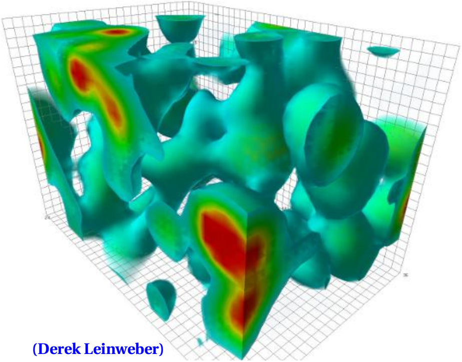
Feel free to follow up on Gather / Slack or directly!

General motivations: Lattice supersymmetry

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

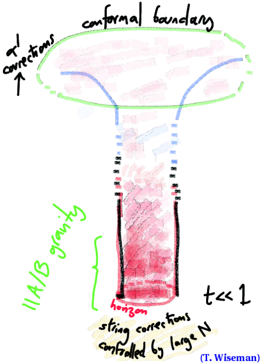
BSM

QFT



(Derek Leinweber)

Holography



(T. Wiseman)

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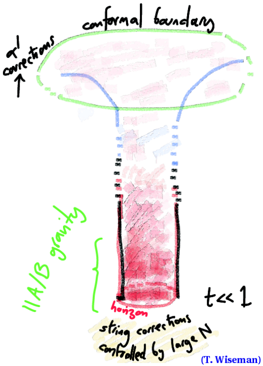


(T. Wiseman)

Today

Focus on 0+1 dimension as testbed for holography

Further motivations: Super-Yang–Mills QM

Supersymmetry is a space-time symmetry, $(\mathrm{I} = 1, \cdots, \mathcal{N})$

adding spinor generators Q_α^I and $\overline{Q}_{\dot\alpha}^\mathrm{I}$ to translations, rotations, boosts

$\left\{Q_\alpha^\mathrm{I}, \overline{Q}_{\dot\alpha}^\mathrm{J}\right\} = 2\delta^\mathrm{IJ}\sigma_{\alpha\dot\alpha}^\mu P_\mu$ broken in discrete space-time

$\longrightarrow$ relevant susy-violating operators

Further motivations: Super-Yang–Mills QM

Supersymmetry broken in discrete space-time $\longrightarrow$ fine-tuning challenges

Simplify via dimensional reduction $\longrightarrow$ significant recent progress

Compactifying all spatial dimensions,

4d $\mathcal{N} = 4$ SU(N) super-Yang–Mills $\longrightarrow$ quantum mechanics of $N \times N$ matrices

[Goldstone, Hoppe, de Wit, Nicolai, $\cdots$, 1980s]

[Holographic duality conjecture by Banks–Fischler–Shenker–Susskind, hep-th/9610043]

$$S_{\text{BFSS}} = \frac{N}{4\lambda} \int d\tau \; \text{Tr}\left[-(D_\tau X_i)^2 + \Psi_\alpha^T \gamma_{\alpha\beta}^\tau D_\tau \Psi_\beta - \frac{1}{2} \sum_{i<j} [X_i, X_j]^2 + \frac{1}{\sqrt{2}} \Psi_\alpha^T \gamma_{\alpha\beta}^i [X_i, \Psi_\beta] \right]$$

16 fermions Ψ_α

9 scalars X_i $\longleftrightarrow$ SO(9) global symmetry

Supersymmetric mass deformation

Berenstein–Maldacena–Nastase, hep-th/0202021

Split the 9 scalars $X_i \longrightarrow 3X_{\text{I}}$ and $6X_A$ with different masses
$\Longrightarrow$ break SO(9) $\longrightarrow$ SO(6) $\times$ SO(3) symmetry

Dimensionful mass parameter $\mu \longrightarrow$ dimensionless coupling $g \equiv \lambda/\mu^3$

$$S_{\text{BMN}} = S_{\text{BFSS}} - \frac{N}{4\lambda}\int d\tau \, \text{Tr}\left[\left(\frac{\mu}{3}X_{\text{I}}\right)^2 + \left(\frac{\mu}{6}X_A\right)^2 + \frac{\mu}{4}\Psi_\alpha^T \gamma_{\alpha\beta}^{123}\Psi_\beta - \frac{\sqrt{2}\mu}{3}\epsilon_{\text{IJK}}X_{\text{I}}X_{\text{J}}X_{\text{K}}\right]$$

Lift flat directions while preserving all 16 supersymmetries

Simple discretization provided in public parallel code

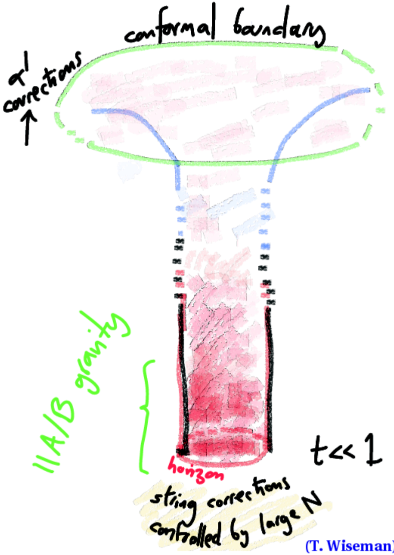
Finite-temperature phase diagram and holography

Holographic duality conjecture

Thermodynamics of maximal SYM
at large N and strong coupling

$\updownarrow$

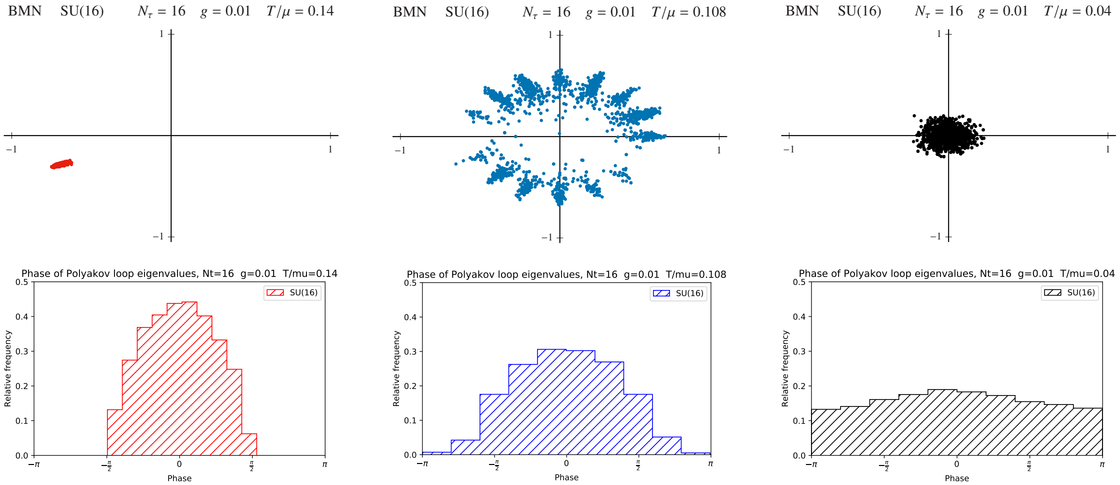
Black holes in string theory



(T. Wiseman)

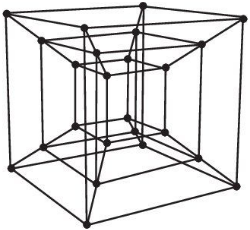
Numerical dual-supergravity [arXiv:1411.5541]
$\longrightarrow$ first-order BMN deconfinement transition

Deconfinement signalled by Polyakov loop



Weak-coupling perturbation theory $\longrightarrow$ first-order BMN deconfinement transition

The lattice phase diagram game



Gauge groups SU(8), SU(12), SU(16)

$\times$

Lattice sizes $N_T = 8,\ 16,\ 24$

$\times$

Dimensionless couplings $g = \lambda/\mu^3 = 0.00001,\ 0.0001,\ 0.001,\ 0.01$

$\times$

Scan dimensionless temperatures T/μ across transition

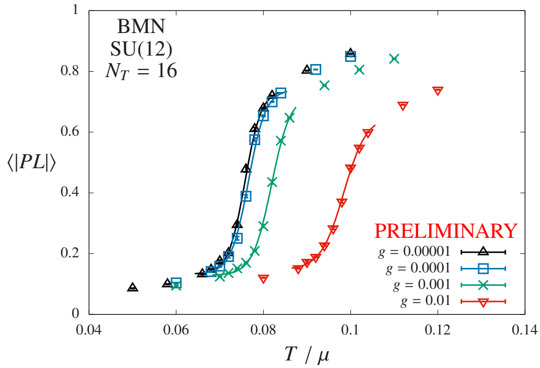
$=$ 333 ensembles

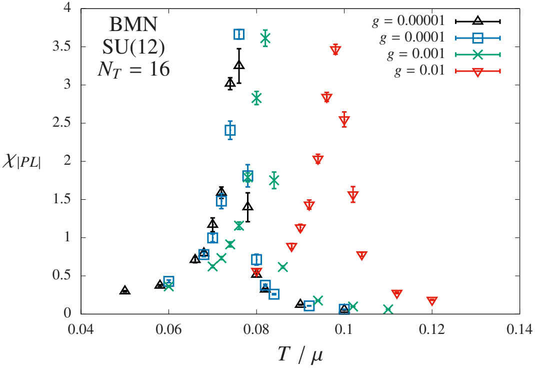
[up to 50,000 MD time units per ensemble]

Polyakov loop and susceptibility

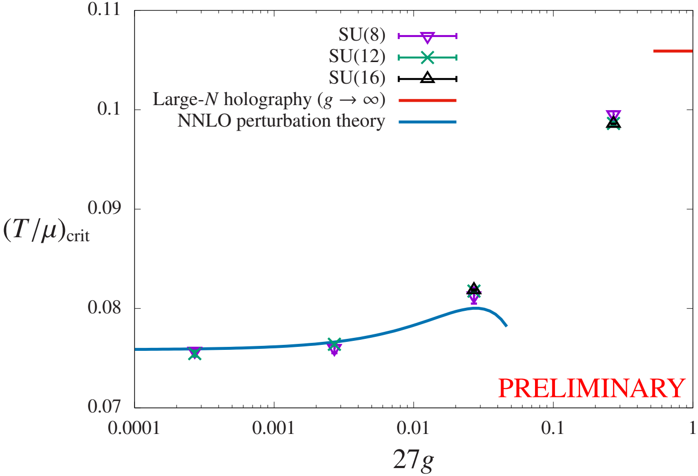
Extract critical T/μ by fitting Polyakov loop to simple sigmoid function

Results match peaks in Polyakov loop susceptibility





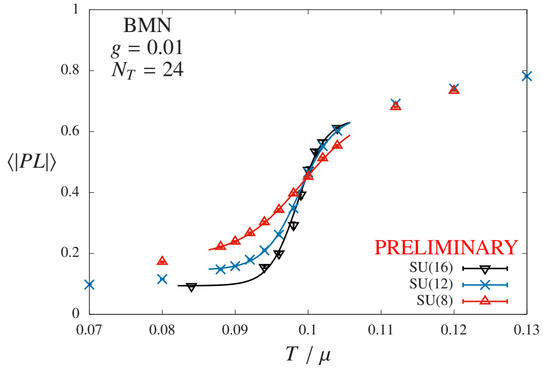
Today's main result: Phase diagram

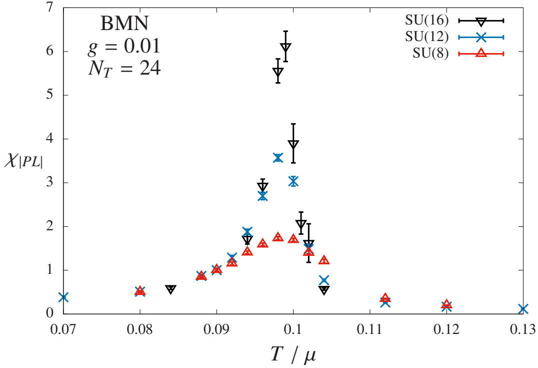


Reproduce weak-coupling pert. theory, approach strong-coupling holography

Dependence on SU(N) gauge group

Larger $N \longrightarrow$ sharper transition at same critical T/μ

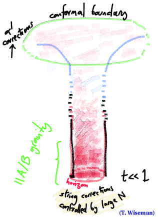




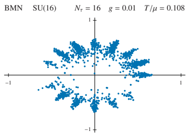
Susceptibility peaks consistent with expected first-order $\sim N^2$ scaling

Recap and outlook

Dimensional reduction simplifies lattice supersymmetry,
retaining interesting dynamics and holographic dualities



BMN quantum mechanics is interesting target
with public code available



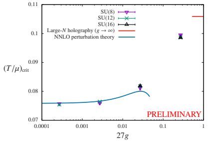
Phase diagram results agree with weak-coupling pert. theory,
approach strong-coupling holography

More investigations underway:

Gauge invariance $\longrightarrow$ global symmetry [1802.02985]

Remove fermions $\longrightarrow$ bosonic BMN

More observables & predictions for holography



Thanks for your attention!

Collaborators

Raghav G. Jha, Anosh Joseph

Funding and computing resources

Backup: Breakdown of Leibniz rule on the lattice

$\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$ is problematic

$\Longrightarrow$ try finite difference $\partial\phi(x) \longrightarrow \Delta\phi(x) = \frac{1}{a}\left[\phi(x+a) - \phi(x)\right]$

Crucial difference between ∂ and Δ

$$\Delta\left[\phi\eta\right] = a^{-1}\left[\phi(x+a)\eta(x+a) - \phi(x)\eta(x)\right]$$

$$= \left[\Delta\phi\right]\eta + \phi\Delta\eta + a\left[\Delta\phi\right]\Delta\eta$$

Full supersymmetry requires Leibniz rule $\partial\left[\phi\eta\right] = \left[\partial\phi\right]\eta + \phi\partial\eta$

only recoverd in $a \to 0$ continuum limit for any local finite difference

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Supersymmetry vs. locality 'no-go' theorems

by Kato–Sakamoto–So [arXiv:0803.3121] and Bergner [arXiv:0909.4791]

Complicated constructions to balance locality vs. supersymmetry

Non-ultralocal product operator $\longrightarrow$ lattice Leibniz rule but not gauge invariance

D'Adda–Kawamoto–Saito, arXiv:1706.02615

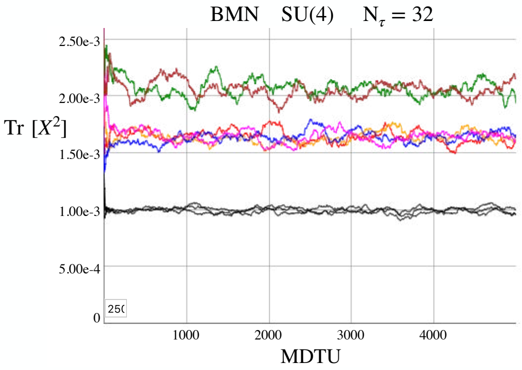
Cyclic Leibniz rule $\longrightarrow$ partial lattice supersymmetry but only (0+1)d QM so far

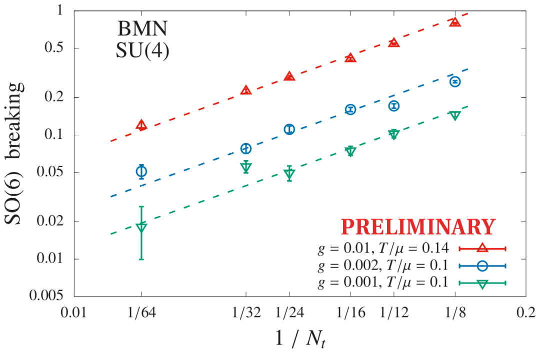
Kadoh–Kamei–So, arXiv:1904.09275

Backup: Discretization artifacts

Discretization artifacts break SO(6) part of BMN $SO(6) \times SO(3)$ symmetry

Breaking increases with g but vanishes in $N_T \to \infty$ continuum limit





Backup: Large-N limit