Large-N limit of two-dimensional Yang-Mills theory with four supercharges

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IN PURSUIT OF KNOWLEDGE

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Work in progress with: Raghav Govind Jha, Anosh Joseph, and David Schaich

Overview

- Motivation
- Lattice construction
- Simulation details
- Results

—Motivation—

Motivation

Why study this **theory** ?

- Existence of holographic dual of its maximally supersymmetric cousin.
- Exploring finite and large-N dependence.
- Absence of sign problem.

  JHEP **01** (2011) 058
  Hanada, Kanamori

  JHEP **01** (2012) 108
  Catterall, Galvez, Joseph, Mehta

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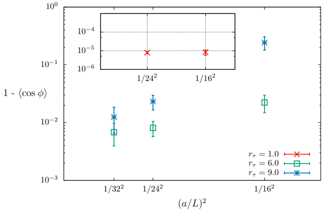

PRD **97**, 054504 (2018)
Catterall, Jha, Joseph

Simple 2d supersymmetric theory to study non-perturbatively

Target

Study distributions of **scalars** with thermal boundary conditions using observable

$$\frac{1}{NN_tN_x}\sum_{i,N_t,N_x} Tr(X_i^2)$$

- To construct bound state in presence of flat directions, we expect that large-N is required

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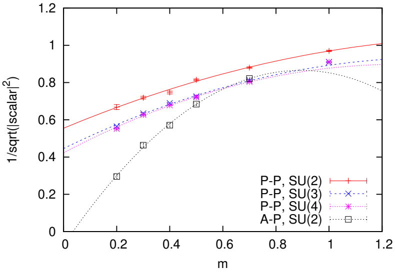
- To construct bound state in presence of flat directions, we expect that large-N is required

- Detailed study with periodic boundary conditions
  PRD **80**, 065014 (2009)
  Hanada, Kanamori



—Lattice construction—

Two-dimensional $\mathcal{N} = (2,2)$ SYM

Formulation of SYM theories on a $2d$ lattice
JHEP **08** (2003) 024 Cohen, Kaplan, Katz, Unsal
JHEP **03** (2004) 067 Sugino
JHEP **11** (2004) 006 Catterall

We use **geometrical discretization** scheme, also used in PRD **97**, 054504 (2018) to probe dynamical SUSY breaking — Catterall, Jha, Joseph

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Dimensional Reduction

$$\mathcal{N} = 1,\ d = 4$$
$$\downarrow$$
$$\mathcal{N} = 2,\ d = 3$$
$$\downarrow$$
$$\mathcal{N} = (2,2),\ d = 2$$

Global symmetry:

Four-dimensional theory

$$SO(4)_E \times U(1)$$

Two-dimensional theory

$$SO(2)_E \times SO(2)_{R_1} \times U(1)_{R_2}$$

Two-dimensional $\mathcal{N} = (2,2)$ SYM

•Two possible twists **A** and **B** as:

• Internal symmetry group contains two $SO(2)$'s

• We work with **B** twist by embedding $SO(2)_E$ with $SO(2)_{R_1}$:

$$SO(2)' = \text{diag}(SO(2)_E \times SO(2)_{R_1})$$

Untwisted theory{ 4 bosonic d.o.f., 4 fermionic d.o.f.
and 4 real supercharges

Fermions and supercharges decomposed into integer spin representation, scalars and gauge fields combine to give complexified field

Twisted theory{ d.o.f. Fermions (η, ψ_a and χ_{ab})
and complexified field $\mathcal{A}_a$

Continuum Action

$$S = \frac{N}{4\lambda}\mathcal{Q}\int d^2x \ \mathrm{Tr}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right] - \frac{1}{2}\eta d\right)$$

where $\quad \mathcal{A}_a = A_a + i\ X_a, \quad \mathcal{D}_a = \partial_a + \mathcal{A}_a \quad$ and $\quad \mathcal{F}_{ab} = [\mathcal{D}_a, \mathcal{D}_b]$

Scalar supercharge $\mathcal{Q}$ acts on twisted fields as:

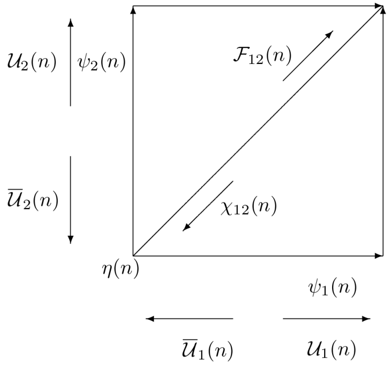
$$\mathcal{Q}\mathcal{A}_a = \psi_a, \qquad \mathcal{Q}\overline{\mathcal{A}}_a = 0, \qquad \mathcal{Q}\psi_a = 0$$

$$\mathcal{Q}\chi_{ab} = -\overline{\mathcal{F}}_{ab}, \qquad \mathcal{Q}\eta = d, \qquad \mathcal{Q}d = 0$$

Performing $\mathcal{Q}$ variation and integrating out field d

$$S = \frac{N}{4\lambda}\int d^2x \ \mathrm{Tr}\left(-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right]^2 - \chi_{ab}\mathcal{D}_{[a}\ \psi_{\ b]} - \eta\overline{\mathcal{D}}_a\psi_a\right)$$

Lattice construction



- Gauge field $\rightarrow$ Wilson links $\mathcal{A}_a(x) \rightarrow \mathcal{U}_a(n)$, on links of square lattice
- To preserve SUSY $\psi_a(n)$ lies on same links as bosonic superpartners
- $\eta(n)$ associated with site
- $\chi_{ab}(n)$ lies on opposite orientation to diagonal

Discretized Action

Covariant difference operators Phys. Lett. B **661**, 52 (2008) Damgaard, Matsuura

$$\overline{\mathcal{D}}_a^{(-)} f_a(n) = f_a(n)\overline{\mathcal{U}}_a(n) - \overline{\mathcal{U}}_a(n-\hat{\mu}_a) f_a(n-\hat{\mu}_a)$$
$$\mathcal{D}_a^{(+)} f_b(n) = \mathcal{U}_a(n) f_b(n+\hat{\mu}_a) - f_b(n)\mathcal{U}_a(n+\hat{\mu}_b)$$

Lattice action :

$$S = \frac{N}{4\lambda}\sum_n \mathrm{Tr}\left(\mathcal{F}_{ab}^{\dagger}(n)\mathcal{F}_{ab}(n) + \frac{1}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n)\right)^2\right)$$
$$+\frac{N}{4\lambda}\sum_n \mathrm{Tr}\left(-\chi_{ab}(n)\mathcal{D}_{[a}^{(+)}\psi_{\ b]}(n) - \eta(n)\overline{\mathcal{D}}_a^{(-)}\psi_a(n)\right)$$

To control flat direction divergences, add scalar potential term:

$$S_\mu = \frac{N}{4\lambda}\mu^2\sum_{n,a}\mathrm{Tr}\left(\overline{\mathcal{U}}_a(n)\mathcal{U}_a(n) - \mathbb{I}_N\right)^2$$

—Simulation details—

Simulation details

Code used publicaly available daschaich/susy

Lattice with physical size $\beta \times L$

Used symmetric lattice $aN_t = \beta = L = aN_x$

Anti-periodic boundary conditions for fermions in temporal direction.

$$r_t = \sqrt{\lambda}\beta$$

$r_t \rightarrowtail$ dimensionless temporal extent, effective coupling

Simulation details

$$r_t = \sqrt{\lambda}\beta, \qquad \mu = \zeta \frac{r_t}{N_t} = \zeta\sqrt{\lambda}a$$

$\mu \rightarrowtail$ mass parameter to control possible flat direction divergences

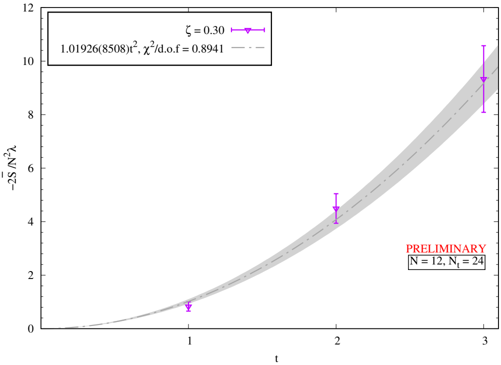
- Worked with :
  - Various ζ values to take $\zeta \to 0$ limit
  - Various combinations of temperature $t = 1/r_t$, N and N_t

Observable in spotlight:

$$\frac{1}{NN_tN_x} \sum_{i,N_t,N_x} Tr(X_i^2)$$

—Results—

Results



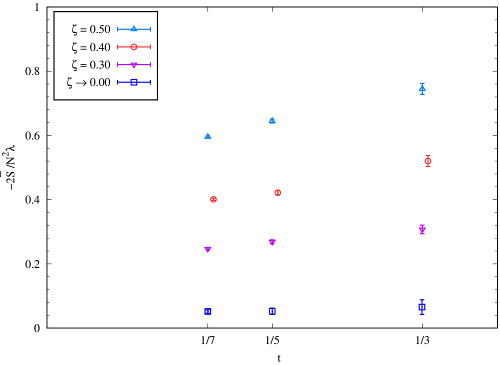
$$\frac{-2\overline{S}}{N^2\lambda} = \frac{3}{\lambda_{lat}}\left(1 - \frac{2}{3N^2}S_B\right)$$

Same t^2 behaviour when studied in Maximal case

PRD **97** 086020 (2018)

Catterall, Jha, Schaich, Wiseman

Results



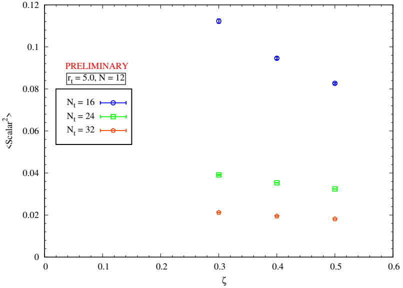
$$\frac{-2\overline{S}}{N^2\lambda} = \frac{3}{\lambda_{lat}}\left(1 - \frac{2}{3N^2}S_B\right)$$

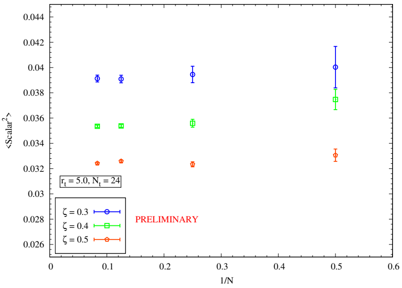
Different behavior to maximal theory dual

JHEP **07** (2013) 101

Wiseman

Results

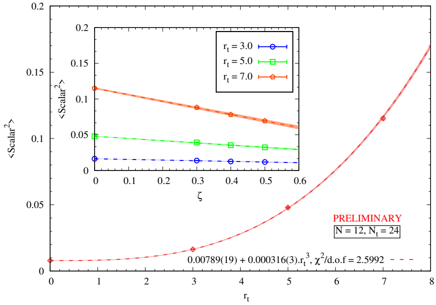




Preliminary analysis: Evident N_t dependence

Preliminary analysis: No strong N dependence

Results



$$<\text{scalar}^2> = \frac{1}{N N_t N_x} \sum_{i,N_t,N_x} Tr(X_i^2)$$

Preliminary analysis: r_t^3 behavior
Different behavior to maximal theory dual
JHEP **07** (2013) 101 Wiseman

Different N, N_t and r_t still in progress

Conclusion and Way Forward

- Preliminary analysis indicates that construction of bound state is possible at finite temperature with even small N values provided we work with larger lattice
- The theory looks to be in different universality class to maximal cousin

  Not discussed in talk: We have not yet seen a finite-volume, large N deconfinement transition like maximal 2d case
- Work in progress with different couplings and N values to make our results more reliable

Thank You

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