University of Cyprus

Department of Physics

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"Perturbative study of the Gluino-Glue operator in SYM"

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General Outline

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1. Introduction to SYM
2. Gluino-Glue Operator, $\mathcal{O}_{Gg}$

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3. Renormalization and Mixing Coefficients of $\mathcal{O}_{Gg}$ using LPT

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1. Introduction to SYM
2. Gluino-Glue Operator, $\mathcal{O}_{Gg}$
3. Renormalization and Mixing Coefficients of $\mathcal{O}_{Gg}$ using LPT
4. Gauge Invariant Renormalization Scheme (GIRS) for $\mathcal{O}_{Gg}$

Supersymmetric Yang-Mills action

The construction of the SYM Lagrangian for $\mathcal{N} = 1$ supersymmetry in 4 dimensions involves only vector superfields V.
The form of a vector superfield $V(x, \theta, \bar{\theta})$ in the Wess-Zumino (WZ) gauge is:

$$V(x; \theta, \bar{\theta}) = -\theta \, \sigma^{\mu} \, \bar{\theta} \, u_{\mu}(x) + i\theta\theta \, \bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta} \, \theta\lambda(x) + \frac{1}{2} \theta\theta \, \bar{\theta}\bar{\theta} \, D(x) \ .$$

where u_{μ} is the gauge field (gluon), λ is a Majorana spinor (gluino) [the superpartner of the gluon] and D is a real auxiliary field.

Supersymmetric gauge action

A Lagrangian, which respects the SUSY trasformations, in terms of superfields is:

$$\mathcal{L} \;=\; \frac{1}{16kg}\mathrm{Tr}\big(W^a\, W_a|_{\theta\theta} + \bar{W}_{\dot{a}}\, \bar{W}^{\dot{a}}|_{\bar{\theta}\bar{\theta}}\big)$$

where $\mathrm{Tr}(T^\alpha T^\beta) = k\,\delta^{\alpha\beta}$, $W_\alpha = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\, e^{-2g\,V}\,\mathcal{D}_\alpha\, e^{2g\,V}$ is the supersymmetric field strength, and the supersymmetric covariant derivative is defined as: $\mathcal{D}_a = \frac{\partial}{\partial\theta^a} \;+\; i\sigma^\mu_{a\dot{a}}\,\bar{\theta}^{\dot{a}}\,\partial_\mu$, $\bar{\mathcal{D}}_{\dot{a}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{a}}} -\; i\theta^a\,(\sigma^\mu)_{a\dot{a}}\,\partial_\mu$.
Taking the corresponding components of the superfields (appropriate powers of θ and $\bar{\theta}$), the continuum Lagrangian in the WZ gauge is:

$$\mathcal{L}_{\mathrm{SYM}} = -\frac{1}{4}u^\alpha_{\mu\nu}u^{\mu\nu\alpha} + \frac{1}{2}D^\alpha D^\alpha - i\bar{\lambda}^\alpha\bar{\sigma}^\mu\mathcal{D}_\mu\lambda^\alpha$$

which is invariant, up to a total derivative, under the supersymmetric transformations (ξ is a Majorana spinor parameter):

$$\delta_\xi u^\alpha_\mu \;=\; -i\bar{\lambda}^\alpha\bar{\sigma}^\mu\xi + i\bar{\xi}\bar{\sigma}^\mu\lambda^\alpha\,, \quad \delta_\xi\lambda^\alpha = \sigma^{\mu\nu}\xi u^\alpha_{\mu\nu} + i\xi\; D^\alpha\,,$$
$$\delta_\xi D^\alpha \;=\; -\xi\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^\alpha - \mathcal{D}_\mu\lambda^\alpha\sigma^\mu\bar{\xi}.$$

Supersymmetric gauge action

- The auxiliary fields may now be eliminated, either by applying their equations of motion (classical case), or by functionally integrating over them (quantum case).
- The Lagrangian can be rewritten in 4 dimensions in Dirac notation and in the Weyl basis.
- The construction of the Euclidean action can be done after a Wick rotation.
- Introduction of a gauge-fixing term in the Lagrangian, along with a compensating Faddeev-Popov ghost term to avoid additional infinities which would appear upon functionally integrating over gauge orbits.

Study of Composite Operators in SYM

In studying the properties of light bound states, the main observables in SYM would be (based on supersymmetric effective Lagrangians):

- Gluino-Glue operator:

$$\mathcal{O}_{Gg} = \sigma_{\mu\nu}\,\mathrm{tr}_c(\,u_{\mu\nu}\lambda), \quad \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$$

- Gluinoball operators:

$$\mathrm{tr}_c(\bar{\lambda}\lambda)$$

$$\mathrm{tr}_c(\bar{\lambda}\gamma_5\lambda)$$

$$\ldots$$

- Glueball operators:

$$\mathrm{tr}_c(u_{\mu\nu}u_{\mu\nu})$$

$$\mathrm{tr}_c(u_{\mu\nu}\tilde{u}_{\mu\nu})$$

$$\ldots$$

Mixing of the Gluino-Glue Operator, $\mathcal{O}_{Gg}$

$$\mathcal{O}_{Gg} = \sigma_{\mu\nu}\,\mathrm{tr}_c(\,u_{\mu\nu}\lambda), \quad \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$$

The operators which mix with the Gluino-Glue operator, $\mathcal{O}_{Gg}$ must necessarily have the same index structure as $\mathcal{O}_{Gg}$:

- one free spinor index
- no free color index
- no free Lorenz index

Additionally, they must have:

- dimensions $\leq 7/2$
- zero ghost number

Supersymmetric gauge lattice action

- We use *Standard discretization*: Gluinos live on the lattice sites, and gluons live on the links of the lattice: $U_\mu(x) = e^{igaT^\alpha u^\alpha_\mu(x+a\hat{\mu}/2)}$; α is a color index in the adjoint representation of the gauge group.
- Our computations are performed to one loop in perturbation theory, employing the standard Wilson action for gluons and the clover improved action for gluinos (for ghost fields we use naive discretization).
- In our ongoing investigation we plan to address also other improved actions (e.g., overlap action, Wilson action using stout-smearing in the fermionic action) and other operators (e.g., Three-gluino operator, Noether Supercurrent operator[1]), so that we can check to what extent some of the SUSY breaking effects can be alleviated.

[1]For detailed information on perturbative and non-perturbative results for this operator, see the poster presentation entitled "Supercurrent renormalization in $N = 1$ Supersymmetric Yang-Mills Theory", by A. Skouroupathis and I. Soler Calero in this conference.

Supersymmetric gauge lattice action

For clover-improved gluinos (λ), the Euclidean action $\mathcal{S}^L_{\rm SYM}$ on the lattice becomes:

$$\mathcal{S}^L_{\rm SYM} = a^4 \sum_x \Bigg[\frac{N_c}{g^2} \sum_{\mu,\nu} \left(1 - \frac{1}{N_c} Tr U_{\mu\nu}\right) + \sum_\mu \left(Tr\left(\bar\lambda \gamma_\mu D_\mu \lambda\right) - \frac{ar}{2} Tr\left(\bar\lambda D^2 \lambda\right)\right) - \sum_{\mu,\nu}\left(\frac{c_{\rm SW}\, a}{4} \bar\lambda^\alpha \sigma_{\mu\nu} \hat{\hat{F}}^{\alpha\beta}_{\mu\nu} \lambda^\beta\right)\Bigg]$$

where $U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat\mu) U^\dagger_\mu(x + a\hat\nu) U^\dagger_\nu(x)$ and $\hat{\hat{F}}^{\alpha\beta}_{\mu\nu}$ in the adjoint representation is defined as:

$$\hat{\hat{F}}^{\alpha\beta}_{\mu\nu} = \frac{1}{8}(\tilde Q^{\alpha\beta}_{\mu\nu} - \tilde Q^{\alpha\beta}_{\nu\mu})$$

$$\begin{aligned}\tilde Q^{\alpha\beta}_{\mu\nu} = 2{\rm tr}_c \Big(& T^\alpha\, U_{x,x+\mu} U_{x+\mu,x+\mu+\nu} U_{x+\mu+\nu,x+\nu} U_{x+\nu,x} T^\beta\, U_{x,x+\nu} U_{x+\nu,x+\mu+\nu} U_{x+\mu+\nu,x+\mu} U_{x+\mu,x} \\ & + T^\alpha\, U_{x,x+\nu} U_{x+\nu,x+\nu-\mu} U_{x+\nu-\mu,x-\mu} U_{x-\mu,x} T^\beta\, U_{x,x+\mu} U_{x+\mu,x+\mu-\nu} U_{x+\mu-\nu,x-\mu} U_{x-\nu,x} \\ & + T^\alpha\, U_{x,x-\mu} U_{x-\mu,x-\mu-\nu} U_{x-\mu-\nu,x-\nu} U_{x-\nu,x} T^\beta\, U_{x,x-\nu} U_{x-\nu,x-\mu-\nu} U_{x-\mu-\nu,x-\mu} U_{x-\mu,x} \\ & + T^\alpha\, U_{x,x-\nu} U_{x-\nu,x-\nu+\mu} U_{x-\nu+\mu,x+\mu} U_{x+\mu,x} T^\beta\, U_{x,x-\mu} U_{x-\mu,x-\mu+\nu} U_{x-\mu+\nu,x+\nu} U_{x+\nu,x} \Big)\end{aligned}$$

The definitions of the covariant derivatives are as follows:

$$\mathcal{D}_\mu \lambda(x) \equiv \frac{1}{2a}\Big[U_\mu(x)\lambda(x+a\hat\mu)U^\dagger_\mu(x) - U^\dagger_\mu(x-a\hat\mu)\lambda(x-a\hat\mu)U_\mu(x-a\hat\mu)\Big]$$

$$\mathcal{D}^2 \lambda(x) \equiv \frac{1}{a^2}\sum_\mu \Big[U_\mu(x)\lambda(x+a\hat\mu)U^\dagger_\mu(x) - 2\lambda(x) + U^\dagger_\mu(x-a\hat\mu)\lambda(x-a\hat\mu)U_\mu(x-a\hat\mu)\Big]$$

Mixing of composite operators

The Gluino-Glue operator, being composite, could in principle mix with four classes of operators having the same quantum numbers.
The four classes are as follows:

- Class G are gauge invariant operators: only $\mathcal{O}_{Gg}$.
- Class A are operators which are not gauge invariant but are the BRST variation of some other operators: $\mathcal{O}_A$.
- Class B operators vanish by the equations of motion: $\mathcal{O}_B$.
- Class C are operators which are not linear combinations of class G, A and B: $\mathcal{O}_C$.

Mixing of Gluino-Glue operator under Renormalization

We present all candidate operators which can mix with $\mathcal{O}_{Gg}$ ($\beta = 1-\alpha$: gauge fixing parameter, c: ghost field):

$$
\begin{aligned}
\mathcal{O}_{A1} &= \frac{1}{1-\beta}\mathrm{tr}_c(\lambda\,\partial_\mu u_\mu) - ig\,\mathrm{tr}_c(\lambda[c,\bar{c}]) \\
\mathcal{O}_{B1} &= \mathrm{tr}_c(\not{u}\not{D}\lambda) \\
\mathcal{O}_{C1} &= \mathrm{tr}_c(\partial_\mu\lambda\, u^\mu) \\
\mathcal{O}_{C2} &= \mathrm{tr}_c(\not{u}\lambda) \\
\mathcal{O}_{C3} &= ig\,\sigma_{\mu\nu}\mathrm{tr}_c(\,\lambda[u_\mu, u_\nu]) \\
\mathcal{O}_{C4} &= ig\,\mathrm{tr}_c(\lambda[c,\bar{c}])
\end{aligned}
$$

Renormalized fields and operators are related to bare ones through:

$$
\begin{aligned}
u_\mu^R &= \sqrt{Z_u}\, u_\mu^B, \quad \lambda^R = \sqrt{Z_\lambda}\,\lambda^B, \quad c^R = \sqrt{Z_c}\, c^B, \\
\mathcal{O}_{Gg}^R &= Z_{Gg}\mathcal{O}_{Gg}^B + z_{A1}\mathcal{O}_{A1}^B + z_{B1}\mathcal{O}_{B1}^B + \sum_{i=1}^{4} z_{Ci}\mathcal{O}_{Ci}^B
\end{aligned}
$$

Renormalization conditions are imposed on amputated renormalized Green's functions. Let us relate the latter to the bare ones. To 1-loop order:

For the gluino-gluon Green's function:

$$\begin{aligned}\langle u_\nu^R\, \mathcal{O}_{Gg}^R\, \bar{\lambda}^R\rangle_{amp} &= Z_\lambda^{-1/2}\, Z_u^{-1/2} Z_{Gg} \langle u_\nu^B\, \mathcal{O}_{Gg}^B\, \bar{\lambda}^B\rangle_{amp} \\ &+ z_{A1}\langle u_\nu^B\, \mathcal{O}_{A1}^B\, \bar{\lambda}^B\rangle_{amp}^{tree} + z_{B1}\langle u_\nu^B\, \mathcal{O}_{B1}^B\, \bar{\lambda}^B\rangle_{amp}^{tree} \\ &+ z_{C1}\langle u_\nu^B\, \mathcal{O}_{C1}^B\, \bar{\lambda}^B\rangle_{amp}^{tree} + z_{C2}\langle u_\nu^B\, \mathcal{O}_{C2}^B\, \bar{\lambda}^B\rangle_{amp}^{tree}\end{aligned}$$

Similarly for the gluino-gluon-gluon Green's function:

$$\begin{aligned}\langle u_\nu^R\, u_\mu^R\, \mathcal{O}_{Gg}^R\, \bar{\lambda}^R\rangle_{amp} &= Z_\lambda^{-1/2}\, Z_u^{-1} Z_{Gg} \langle u_\nu^B u_\mu^B\, \mathcal{O}_{Gg}^B\, \bar{\lambda}^B\rangle_{amp} \\ &+ z_{B1}\langle u_\nu^B u_\mu^B\, \mathcal{O}_{B1}^B\, \bar{\lambda}^B\rangle_{amp}^{tree} + z_{C3}\langle u_\nu^B u_\mu^B\, \mathcal{O}_{C3}^B\, \bar{\lambda}^B\rangle_{amp}^{tree}\end{aligned}$$

Lastly, for the gluino-ghost-antighost Green's function:

$$\begin{aligned}\langle c^R\, \mathcal{O}_{Gg}^R\, \bar{c}^R\, \bar{\lambda}^R\rangle_{amp} &= Z_c^{-1}\, Z_\lambda^{-1/2} Z_{Gg} \langle c^B\, \mathcal{O}_{Gg}^B\, \bar{c}^b \bar{\lambda}^B\rangle_{amp} \\ &+ z_{A1}\langle c^B\, \mathcal{O}_{A1}^B\, \bar{c}^B \bar{\lambda}^B\rangle_{amp}^{tree} + z_{C4}\langle c^B\, \mathcal{O}_{C4}^B\, \bar{c}^B \bar{\lambda}^B\rangle_{amp}^{tree}\end{aligned}$$

Tasks for extracting renormalization and mixing coefficients on the lattice

- Decision of the choice of the external momenta for Green's functions.
- Calculations of these Green's functions using dimensional regularization.
- *Lattice computations of the same Green's functions.*
- Take the difference between the renormalized Green's functions ($\overline{\text{MS}}$) and lattice bare Green's functions. The resulting expressions have a polynomial form on external momentum.

The tree-level Green's functions of the operators naturally show up in these subtractions thus allowing us to deduce the corresponding mixing coefficients.

Tree-level Green's functions

The nonvanishing two-point amputated tree-level Green's functions, with an operator insertion at point x, are:

$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{Gg}(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle^{tree}_{amp} = \frac{1}{2}\delta^{\alpha_1\alpha_2}\,ie^{i(q_1+q_2)x}\sigma_{\mu\rho}(q_{1\mu}\delta_{\nu\rho} - q_{1\rho}\delta_{\mu\nu})$$
$$= -\delta^{\alpha_1\alpha_2}\,ie^{i(q_1+q_2)x}(\gamma_\nu \not{q}_1 - q_{1\nu})$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{A1}(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle^{tree}_{amp} = \frac{1}{2}\delta^{\alpha_1\alpha_2}\,ie^{i(q_1+q_2)x}q_{1\nu}$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{B1}(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle^{tree}_{amp} = \frac{1}{2}\delta^{\alpha_1\alpha_2}\,ie^{i(q_1+q_2)x}(\gamma_\nu\gamma_\rho)q_{2\rho}$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{C1}(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle^{tree}_{amp} = \frac{1}{2}\delta^{\alpha_1\alpha_2}\,ie^{i(q_1+q_2)x}q_{2\nu}$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{C2}(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle^{tree}_{amp} = \frac{1}{2}\delta^{\alpha_1\alpha_2}\,e^{i(q_1+q_2)x}\gamma_\nu$$

and the three-point amputated tree-level Green's functions of $\mathcal{O}_{Gg}$, $\mathcal{O}_{B1}$, $\mathcal{O}_{C3}$ and $\mathcal{O}_{C4}$:

$$\langle u_\nu^{\alpha_1}(-q_1)\,u_\mu^{\alpha_2}(-q_2)\,\mathcal{O}_{Gg}(x)\,\bar\lambda^{\alpha_3}(q_3)\rangle^{tree}_{amp} = -g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1+q_2+q_3)x}\sigma_{\nu\mu}$$
$$= -g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1+q_2+q_3)x}(\gamma_\nu\gamma_\mu - \delta_{\mu\nu})$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,u_\mu^{\alpha_2}(-q_2)\,\mathcal{O}_{B1}(x)\,\bar\lambda^{\alpha_3}(q_3)\rangle^{tree}_{amp} = -g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1+q_2+q_3)x}\sigma_{\nu\mu}$$
$$\langle u_\nu^{\alpha_1}(-q_1)\,u_\mu^{\alpha_2}(-q_2)\,\mathcal{O}_{C3}(x)\,\bar\lambda^{\alpha_3}(q_3)\rangle^{tree}_{amp} = -g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1+q_2+q_3)x}\sigma_{\nu\mu}$$
$$\langle c^{\alpha_3}(q_3)\,\mathcal{O}_{A1}(x)\,\bar c^{\alpha_2}(q_2)\,\bar\lambda^{\alpha_1}(q_1)\rangle^{tree}_{amp} = \frac{1}{2}g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1-q_2+q_3)x}$$
$$\langle c^{\alpha_3}(q_3)\,\mathcal{O}_{C4}(x)\,\bar c^{\alpha_2}(q_2)\,\bar\lambda^{\alpha_1}(q_1)\rangle^{tree}_{amp} = -\frac{1}{2}g\,f^{\alpha_1\alpha_2\alpha_3}\,e^{i(q_1-q_2+q_3)x}$$

One-loop Green's functions

More specifically, we calculate the two-point Green's function $\langle u_\nu^{\alpha_1}(-q_1)\mathcal{O}_{Gg}(x)\bar{\lambda}^{\alpha_2}(q_2)\rangle$, for three choices of the external momenta q_1 and q_2. The one-loop Feynman diagrams (one-particle irreducible (1PI)) contributing to these Green's functions are shown below.

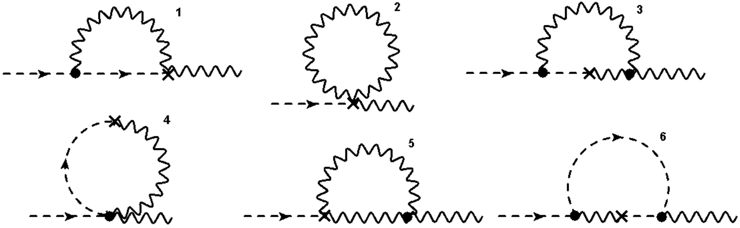

Figure: One-loop Feynman diagrams contributing to the two point Green's function of the Gluino-Glue operator, $\langle u_\nu \mathcal{O}_{Gg} \bar{\lambda}\rangle$. A wavy (dashed) line represents gluons (gluinos). A cross denotes the insertion of the Gluino-Glue operator. Diagrams 2, 4 do not appear in dimensional regularization; they do however show up in the lattice formulation.

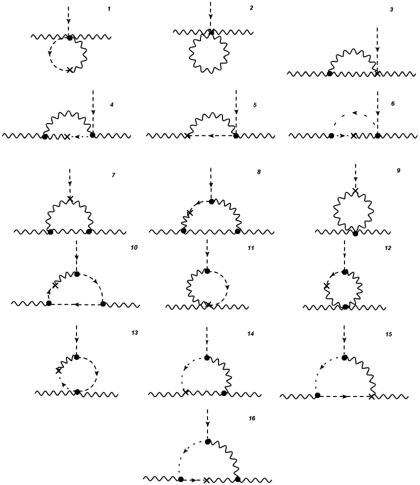

Figure: One-loop Feynman diagrams contributing to the three point Green's function of the Gluino-Glue operator, $\langle u_\nu u_\mu \mathcal{O}_{Gg} \bar{\lambda} \rangle$. A wavy (dashed) line represents gluons (gluinos). Diagrams 1, 2, 3, 5, 6, 11, and 13 do not appear in dimensional regularization but they contribute in the lattice regularization. A cross denotes the insertion of the operator. A mirror version (under exchange of the two external gluons) of diagrams 3, 4, 5, 6, 8, 10, 14, 15 and 16 must also be included.

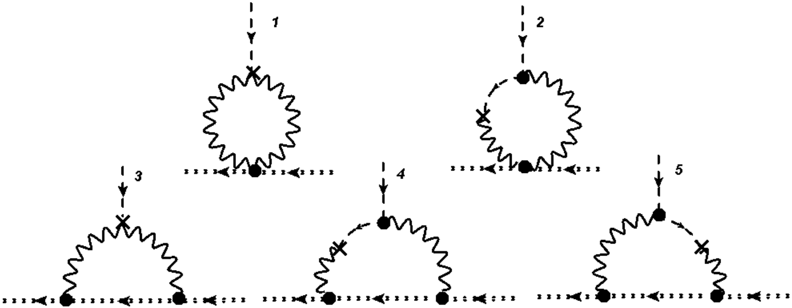

Figure: One-loop Feynman diagrams contributing to the three point Green's function of the Gluino-glue operator, $\langle c\, \mathcal{O}_{Gg}\, \bar{c}\, \bar{\lambda} \rangle$. A wavy (dashed) line represents gluons (gluinos). A cross denotes the insertion of the operator. The "double dashed" line is the ghost field. Diagrams 1 and 2 do not appear in dimensional regularization; they do however show up in the lattice formulation.

Results of Green's functions for the lattice regularization

The first two-point Green's function for $q_2 = 0$ is:

$$\langle u^{\alpha_1}(-q_1)\,\mathcal{O}_{Gg}\,\bar{\lambda}_\nu^{\alpha_2}(q_2)\rangle_{amp}\Big|^{L}_{q_2=0} = -\delta^{\alpha_1\alpha_2}\,ie^{iq_1x}(\gamma_\nu \not{q}_1 - q_{1\nu})$$
$$+ \quad \frac{g^2 N_c}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2}\,e^{iq_1x}\,i\,(\gamma_\nu\not{q}_1 - q_{1\nu})\Big(\frac{-39.4784}{N_c^2} + 27.5552 + 4.1783\beta$$
$$+ \quad \frac{1}{2}\beta^2 - 4.6002\,{c_{\rm SW}}^2 - 12.8568\,c_{\rm SW}\,r + 6\log(a^2q_1^2) - \frac{3\beta}{2}\log(a^2q_1^2)\Big)$$

- It is proportional to its tree-level value!
- It provides us with the renormalization of the Gluino-Glue operator: $Z_{Gg}^{L,\overline{\rm MS}}$.

Results of Green's functions for the lattice regularization

The same Green's function, evaluated at $q_1 = 0$ is:

$$\langle u^{a_1}(-q_1)\, \mathcal{O}_{Gg}\, \bar{\lambda}^{a_2}_{\nu}(q_2)\rangle_{amp}\Big|^{L}_{q_1=0} = \frac{g^2 N_c}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2}\, e^{iq_2 x}\Big[-i\, q_{2\nu}$$
$$-i\gamma_\nu \not{q}_2\left(1.42407 - \frac{3}{2}\log(a^2 q_2^2)\right)\Big]$$

• It has two tensor structures.
• $q_{2\nu}$ appears also in the continuum Green's function. By taking the difference between the lattice and the $\overline{\text{MS}}$ Green's functions, it cancels out.
• $\gamma_\nu \not{q}_2$ determines the mixing coefficient with $\mathcal{O}_{B1}$: $z^{L,\overline{\text{MS}}}_{B1}$.

Results of Green's functions for the lattice regularization

An immediate check of our results is the extraction of the $\overline{\rm MS}$-renormalized Green's function at $q_2 = -q_1$. The bare lattice Green's function at $q_2 = -q_1$ is:

$$\langle u_\nu^{\alpha_1}(-q_1)\,\mathcal{O}_{Gg}\,\bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|^{L}_{q_2=-q_1} = \frac{g^2 N_c}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2}\Bigg[$$

$$i(\gamma_\nu \not{q}_1 - q_{1\nu})\Bigg(\frac{-39.4784}{N_c^2} + 26.5552 + 5.1783\beta$$

$$+\quad \frac{1}{2}\beta^2 - 4.6002\,c_{\rm SW}{}^2 - 12.8568\,c_{\rm SW}\;r + 6\log(a^2 q_1^2) - \frac{3\beta}{2}\log(a^2 q_1^2)\Bigg)$$

$$+\quad i\gamma_\nu \not{q}_1\Bigg(2.4241 - \frac{3}{2}\log(a^2 q_1^2)\Bigg)\Bigg]$$

By applying $Z_{Gg}^{L,\overline{\rm MS}}$ and $z_{B1}^{L,\overline{\rm MS}}$ to the above, we indeed recover the $\overline{\rm MS}$-renormalized Green's function.

Results of Green's functions for the lattice regularization

We now turn to the three-point Green's functions. The lattice three-point Green's function, $\langle c^{\alpha_3}(q_3)\,\mathcal{O}_{Gg}\,\bar{c}^{\alpha_2}(q_2)\bar{\lambda}^{\alpha_1}(q_1)\rangle_{amp}\Big|^{L}_{q_1=q_2,\,q_3=0}$ coincides with the one in the continuum:

$$\langle c^{\alpha_3}(q_3)\,\mathcal{O}_{Gg}\,\bar{c}^{\alpha_2}(q_2)\bar{\lambda}^{\alpha_1}(q_1)\rangle_{amp}\Big|^{DR}_{q_1=q_2,\,q_3=0} = \frac{g^2 N_c}{16\pi^2}\left(\frac{3}{4}(1-\beta)g\, f^{\alpha_1\alpha_2\alpha_3}\right)$$

It follows that $z_{C4}^{L,\overline{\mathrm{MS}}}$ vanishes.

Results of Green's functions for the lattice regularization

For the lattice Green's function with one external gluino and two external gluons, we find:

$$\langle u_\nu^{\alpha_1}(-q_1) u_\mu^{\alpha_2}(-q_2)\, \mathcal{O}_{Gg}\, \bar{\lambda}^{\alpha_3}(q_3)\rangle_{amp}\Big|^{L}_{q_2=0,q_3=-q_1} =$$

$$-g\, f^{\alpha_1\alpha_2\alpha_3}\,(\gamma_\nu\gamma_\mu - \delta_{\mu\nu}) Z_g^{-1} - 19.7392\,\frac{g^3}{16\pi^2 N_c}\, f^{\alpha_1\alpha_2\alpha_3}\,(\gamma_\nu\gamma_\mu - \delta_{\mu\nu})$$

$$+\frac{g^3 N_c}{16\pi^2}\, f^{\alpha_1\alpha_2\alpha_3}\Bigg[\delta_{\mu\nu}\Big(-7.83744 - 3.9073\beta - \frac{1}{4}\beta^2 + 6.4284\, c_{\rm SW}\, r + 2.3001\, c_{\rm SW}^2$$

$$-\frac{5}{2}\log\left(a^2 q_1^2\right) + \beta\log\left(a^2 q_1^2\right)\Big)$$

$$+\gamma_\nu\gamma_\mu\Big(9.8999 + 3.5323\beta + \frac{\beta^2}{4} - 6.4284\, c_{\rm SW}\, r - 2.3001\, c_{\rm SW}^2$$

$$-\beta\log\left(a^2 q_1^2\right) + \frac{5}{2}\log\left(a^2 q_1^2\right)\Big)$$

$$+\gamma_\nu\frac{\not{q}_1 q_{1\mu}}{q_1^2}\left(\frac{77}{16} - \frac{13\beta}{8} + \frac{\beta^2}{4}\right) + \gamma_\mu\frac{\not{q}_1 q_{1\nu}}{q_1^2}\left(\frac{1}{16} - \frac{\beta}{4}\right) - \frac{q_{1\nu} q_{1\mu}}{q_1^2}\left(\frac{63}{8} - \frac{9\beta}{4} + \frac{\beta^2}{4}\right)\Bigg]$$

It determines $z_{C3}^{L,\overline{\rm MS}}$.

Results of Renormalization and Mixing Coefficients on the lattice

The mixing coefficients $z_{A1}^{L,\overline{\rm MS}} = z_{C1}^{L,\overline{\rm MS}} = z_{C2}^{L,\overline{\rm MS}} = z_{C4}^{L,\overline{\rm MS}}$ vanish. Below are the results for the renormalization factor and the non vanising mixing coefficients:

$$Z_{Gg}^{L,\overline{\rm MS}} = 1 - \frac{g^2 N_c}{16\pi^2}\Big(\frac{9.8696}{N_c^2} - 1.7626 - 9.9198\, c_{\rm SW}^2 + 4.9765\, c_{\rm SW}\, r - 3\log(a^2\,\bar\mu^2)\Big)$$

$$z_{B1}^{L,\overline{\rm MS}} = \frac{g^2 N_c}{16\pi^2}\Big(0.4241 - \frac{3}{2}\log(a^2\,\bar\mu^2)\Big)$$

$$z_{C3}^{L,\overline{\rm MS}} = -\frac{g^2 N_c}{16\pi^2}0.000114$$

Further details on our calculation can be found in: "Renormalization and mixing of the Gluino-Glue operator on the lattice"

Eur.Phys.J.C 81 (2021) 5, 401 — arXiv:2010.02683.

Renormalization of Gluino-Glue operator in the GIRS scheme

In the Gauge Invariant Renormalization Scheme (GIRS), renormalization factors are defined via correlation functions of gauge invariant operators (in coordinate space): $G(x-y) \equiv \langle \mathcal{O}_{Gg}(x) \overline{\mathcal{O}}_{Gg}(y) \rangle$.

Advantage:

- No mixing with gauge variant operators.
- No need to fix the gauge.
- All necessary correlation functions can be computed nonperturbatively in simulations, without need for gauge fixing or ghost fields.

The downside of this scheme is that the calculations to order g^{2n} requires evaluation of $(n+1)$-loop Feynman diagrams.

In order to achieve renormalization in a more standard scheme, such as $\overline{\mathrm{MS}}$, one must calculate appropriate conversion factors via perturbation theory (e.g. for $\overline{\mathrm{MS}}$ using dimensional regularization).

See our publication: "Gauge-invariant Renormalization of the Gluino-Glue operator" Phys.Lett.B 816 (2021) 136225 — arXiv:2102.02036, for older references on coordinate-space renormalization schemes.

Renormalization of Gluino-Glue operator in the GIRS scheme

Note that:

- In order to compute the conversion factor from GIRS to $\overline{\rm MS}$, it suffices to regularize the theory in d dimensions $(d = 4 - 2\,\epsilon)$.
- No lower dimensional operator mixes with $\mathcal{O}_{Gg}$.
- In the GIRS scheme all non-gauge invariant operators will not contribute to $G(x-y)$, and one will obtain directly the multiplicative renormalization of $\mathcal{O}_{Gg}$, which is the only renormalization factor which is relevant for physical matrix elements.

Green's function of Gluino-Glue operator in the GIRS scheme

The tree-level contribution has the following momentum-integral form:

$$G(x-y)^{\text{tree}} = -4i(N_c^2-1)(d-2)\int \frac{d^d p}{(2\pi)^d}\int \frac{d^d k}{(2\pi)^d}\, e^{-i(x-y)\cdot k}\, \frac{\not{p}\,(p\cdot k)}{p^2(p-k)^2}$$

Integrating over the loop momenta, the resulting expression is:

$$-2\,\frac{(N_c^2-1)\,\Gamma(2-\epsilon)^2}{\pi^{4-2\epsilon}}(-1+\epsilon)(-3+2\epsilon)\,\not{z}\,(z^2)^{-4+2\epsilon}, \quad z^\mu \equiv y^\mu - x^\mu$$

The Feynman diagram for the tree-level value is shown below.

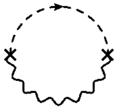

Figure: One-loop ($O(g^0)$) Feynman diagram contributing to the expectation value $\langle \mathcal{O}_{Gg}(x)\mathcal{O}_{Gg}(y)\rangle$. A wavy (dashed) line represents gluons (gluinos). A cross denotes the insertion of the Gluino-Glue operator.

Green's function of Gluino-Glue operator in GIRS scheme

The Feynman diagrams contributing to $G(x-y)$ at $O(g^2)$ are shown below.

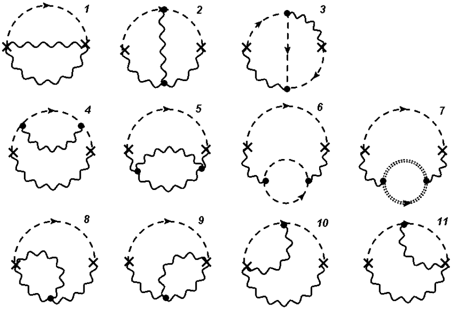

Figure: Two-loop ($O(g^2)$) Feynman diagrams contributing to the expectation value $\langle \mathcal{O}_{Gg}(x)\mathcal{O}_{Gg}(y)\rangle$. A wavy (dashed) line represents gluons (gluinos). The double dashed line is the ghost field. A cross denotes the insertion of the operator.

Green's function of Gluino-Glue operator in GIRS scheme

By adding $O(g^0)$ and $O(g^2)$ contributions, the bare Green's function takes the following form:

$$G(x-y)^{\rm bare} = G(x-y)^{\rm tree} \times \left\{1 - \frac{g^2\ N_c}{16\pi^2} \left(\bar\mu^2 z^2\right)^{\epsilon} \frac{e^{\epsilon\gamma_E}\Gamma(-\epsilon)}{4^{\epsilon}\ \epsilon\ (1-\epsilon)^3\ (3-2\epsilon)} \times \left[(1-\epsilon)\ (12-48\epsilon+70\epsilon^2-39\epsilon^3+\epsilon^4) + \frac{(1-3\epsilon+2\epsilon^2+\epsilon^3)\ \Gamma(-\epsilon)\ \Gamma(\epsilon)^2\ \Gamma(4-3\epsilon)}{4\ (1-2\epsilon)\ \Gamma(-2\epsilon)^2\ \Gamma(2\epsilon)}\right]\right.$$

where $\bar\mu$ is the $\overline{\rm MS}$ renormalization scale relating the dimensionful coupling constant g_L in the d-dimensional Lagrangian to the dimensionless "bare" coupling constant g^B: $g_L = \mu^\epsilon g^B$ $(\mu = \bar\mu\sqrt{e^{\gamma_E}/4\pi})$. To this perturbative order, the distinction between bare and renormalized coupling constants is inessential; we thus denote both by g.

Green's function of Gluino-Glue operator in GIRS scheme

The renormalization factor $Z_{Gg}^{B,R}$ relating the bare Gluino-Glue operator in the "B" regularization to the renormalized operator in the "R" renomalization scheme is defined by:

$$\mathcal{O}_{Gg}^{R} = Z_{Gg}^{B,R}\mathcal{O}_{Gg}^{B} + \text{other operators which will not contribute in } G(x-y).$$

In (DR, $\overline{\text{MS}}$), the renormalization factor $Z_{Gg}^{\text{DR},\overline{\text{MS}}}$ is defined to have only negative integer powers of ϵ and the following condition is imposed:

$$\left[\left(Z_{Gg}^{\text{DR},\overline{\text{MS}}}\right)^2 \langle \mathcal{O}_{Gg}(x)\mathcal{O}_{Gg}(y)\rangle^{\text{bare}}\right]\Big|_{\epsilon^{-n}} = 0, \quad n \in \mathbb{Z}^+.$$

Results in the GIRS scheme

The renormalization factor in GIRS can be obtained by imposing the following condition on the renormalized Green's function:

$$\begin{aligned} & \mathrm{Tr}\Big[(\not{y} - \not{x})\,\langle \mathcal{O}^{\rm GIRS}_{Gg}(x)\mathcal{O}^{\rm GIRS}_{Gg}(y)\rangle\Big]\big|_{y-x=\bar{z}} \equiv \\ & (Z^{B,{\rm GIRS}}_{Gg})^2\ \mathrm{Tr}\Big[(\not{y} - \not{x})\,\langle \mathcal{O}^{B}_{Gg}(x)\mathcal{O}^{B}_{Gg}(y)\rangle\Big]\big|_{y-x=\bar{z}} \\ = \ & \mathrm{Tr}\Big[(\not{y} - \not{x})\,\langle \mathcal{O}^{B}_{Gg}(x)\mathcal{O}^{B}_{Gg}(y)\rangle^{\rm tree}\Big]\big|_{y-x=\bar{z}}, \end{aligned}$$

where the 4-vector $\bar{z}$ is the GIRS renormalization scale ($\bar{z} \neq 0$).

Results in the GIRS scheme - Conversion factor to $\overline{\rm MS}$

The ratio between the $\overline{\rm MS}$ and GIRS renormalization factors gives the corresponding conversion factor:

$$C_{Gg}^{\overline{\rm MS},{\rm GIRS}} = Z_{Gg}^{DR,\overline{\rm MS}} / Z_{Gg}^{DR,{\rm GIRS}}.$$

Being regularization independent, the same conversion factor can then be also used in the lattice regularization (L):

$$C_{Gg}^{\overline{\rm MS},{\rm GIRS}} = Z_{Gg}^{L,\overline{\rm MS}} / Z_{Gg}^{L,{\rm GIRS}}.$$

Combining the *ipso facto* perturbative evaluation of $C_{Gg}^{\overline{\rm MS},GIRS}$ with the nonperturbative evaluation of $Z_{Gg}^{L,GIRS}$, one is thus led to the desired renormalization factor $Z_{Gg}^{L,\overline{\rm MS}}$. Using the relation:

$$\mathrm{Tr}\left[(\not{y}-\not{x})\,\langle \mathcal{O}_{Gg}^{\overline{\rm MS}}(x)\mathcal{O}_{Gg}^{\overline{\rm MS}}(y)\rangle\right]|_{y-x=\bar{z}} \equiv (C_{Gg}^{\overline{\rm MS},{\rm GIRS}})^2\ \mathrm{Tr}\left[(\not{y}-\not{x})\,\langle \mathcal{O}_{Gg}^{\rm GIRS}(x)\mathcal{O}_{Gg}^{\rm GIRS}(y)\rangle\right]|_{y-x=\bar{z}},$$

we obtain the result for the conversion factor:

$$C_{Gg}^{\overline{\rm MS},{\rm GIRS}} = 1 + \frac{g_{\overline{\rm MS}}^2 N_c}{16\pi^2}\left(\frac{5}{3} + 6\gamma_E - 3\ln(4) + 3\ln(\bar{\mu}^2\bar{z}^2)\right) + \mathcal{O}(g^4).$$

The END

Thank you!