$N_f = 1$ QCD as an analog computer for SUSY

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$N_f = 1$ QCD

Physics

- study BSM physics and SUSY
- update on hadronic spectrum

  see earlier results by Farchioni et al.

History

Corrigan & Ramond, 1979

Sannino et al. hep-th/0309252

Armoni et al. hep-th/0403071

Veneziano et al. hep-th/0603045

Creutz, hep-th/0609187

Farchioni et al. 0810.0161[hep-lat]

Numerics

- simulate SUSY without need to simulate SUSY
- single flavour of Wilson fermions $\rightarrow$ sign problem

Studying SUSY inspired by Corrigan and Ramond

(effective) theories of **larks**
$\equiv$ fermions in the two-index anti-symmetric representation of SU(N_c)

Corrigan & Ramond, Phys. Lett. B **87**, 1979

Sannino & Shifman, hep-th/0309252, 2003

$$\mathcal{L} = \frac{1}{2g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}(m_0 + \gamma_\mu D_\mu)\psi$$

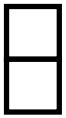
$$\psi^{ij} = \psi^b \, (t^b)^{ij} \,, \quad i,j = 1,..,N_c \,,$$

$$a = 1,..,N_c^2 - 1 \,, \quad b = 1,..,\frac{N_c^2 - N_c}{2}$$

N_c dependence of the lark theory

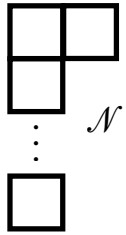
Source: The Guardian, Photograph: Mike Lane/Alamy

theory of a single lark



$N_f = 1$ **QCD**

$\cdots$



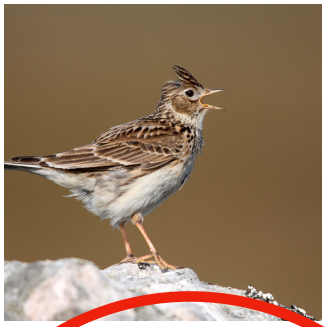
$\mathcal{N} = 1$ **SYM**

N_c

3 4 5 $\cdots$ ∞

N_c dependence of the lark theory

Source: The Guardian, Photograph: Mike Lane/Alamy



theory of a single lark

good proxy for SUSY?

$N_f = 1$ **QCD**

$\mathcal{N} = 1$ **SYM**

N_c

3 4 5 ⋯ ∞

The One Flavour Plan

Probe lark theory for $N_c \to \infty$

Lattice

compute hadron spectrum

study the sign problem by computing sgn (det(D))

low-energy effective Lagrangians

$$\frac{M_{PS}}{M_S} = 1 - \frac{22}{9N_c} - \frac{4}{9}\beta + O(1/N_c^2)$$

Sannino & Shifman, hep-th/0309252

planar equivalence

$$\frac{M_{PS}}{M_S} = 1 - \frac{2}{N_c} + \ldots$$

Armoni & Imeroni, hep-th/0508107

Check deviation from degeneracy of even and odd parity mesons!

Lattice simulation of $N_f = 1$ QCD

Setup

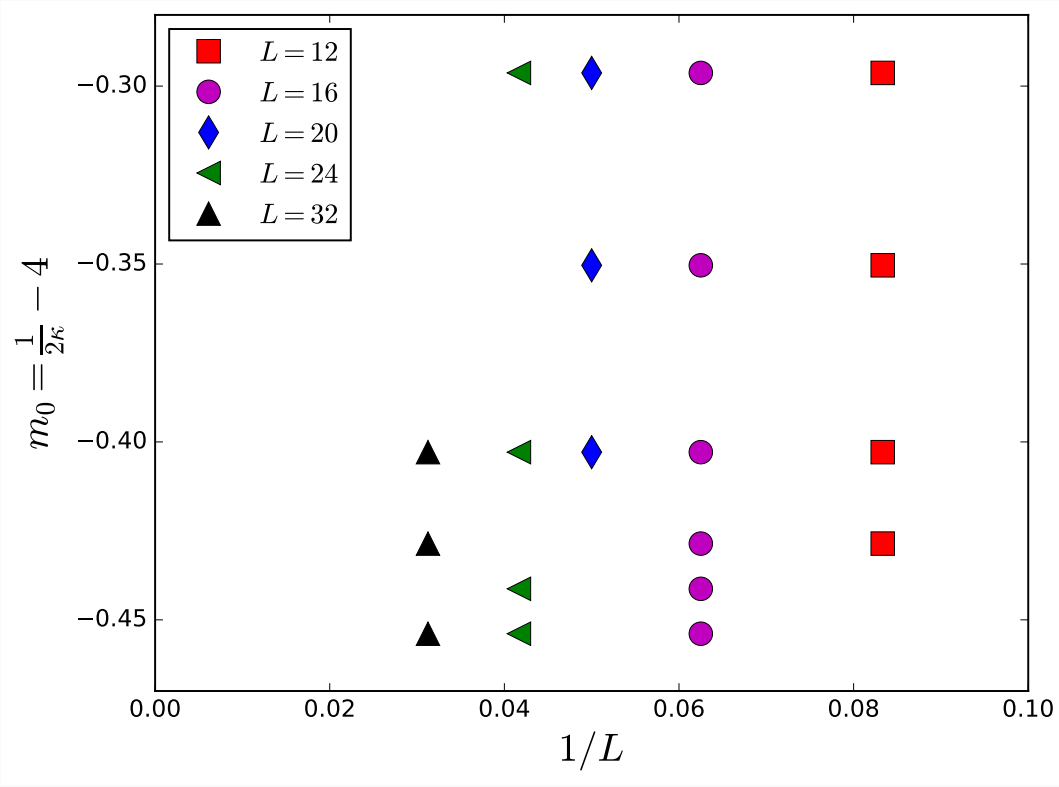
- Symanzik improved gauge action, $O(a)$ improved Wilson fermions, $c_{SW} = 1$
- RHMC algorithm
- single gauge coupling $\beta = 4.5$, $\Rightarrow a \approx 0.06\mathrm{fm}$ (gradient flow)

Peculiarities of $N_f = 1$ QCD

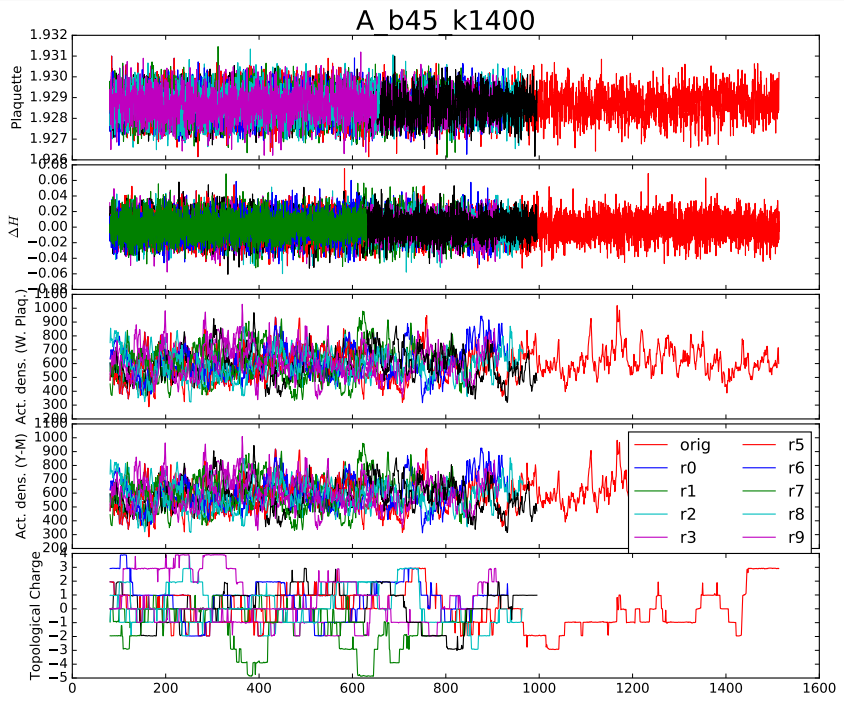
- lack of chiral symmetry
- unknown physical scale

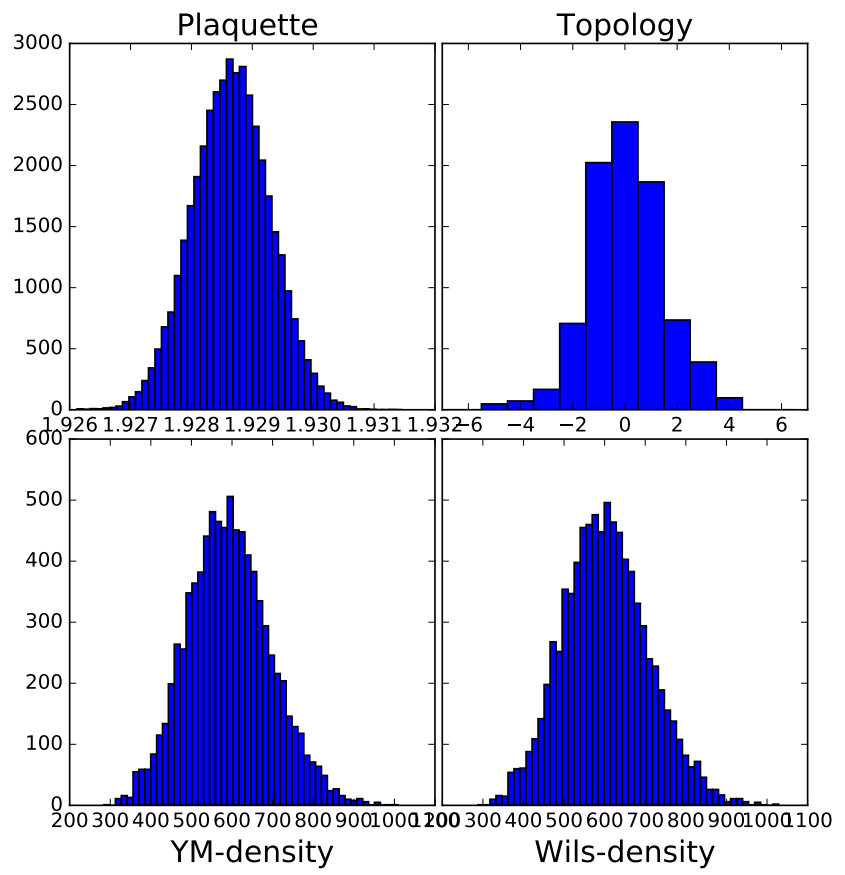
Challenge

choose suitable physical volume and parameters of the action



Configuration generation





Finite volume spectrum

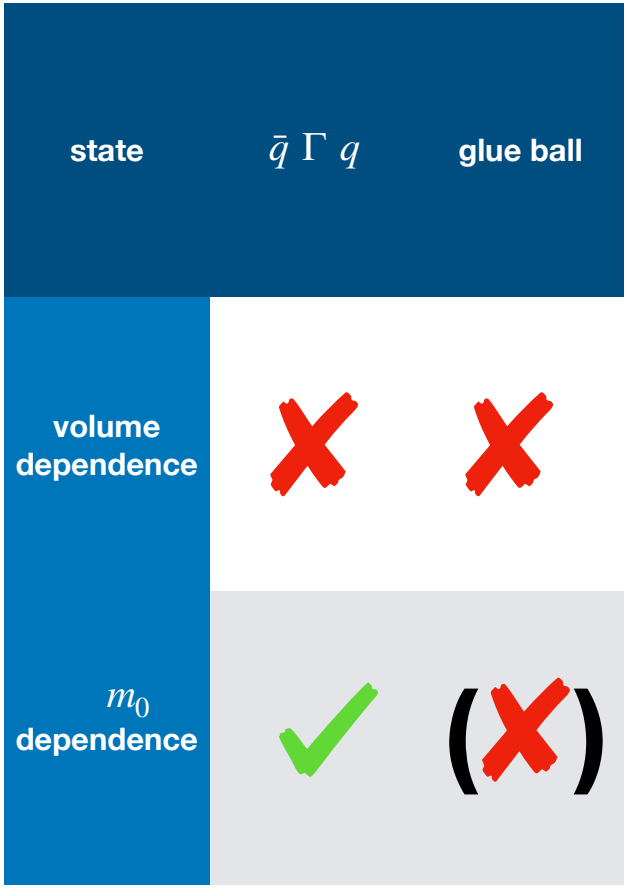
- Hadron masses + **excited states**
- Constraint fits

$$C_{ij}(t) = \sum_n \langle 0|\hat{O}_i|n\rangle\langle n|\hat{O}_j^\dagger|0\rangle e^{-t\, m_n}$$

- scalar glueball, $\bar{q}\ \Gamma q$ - scalar, vector, pseudo-scalar, $i, j \in \{PS, S, V, G\}$

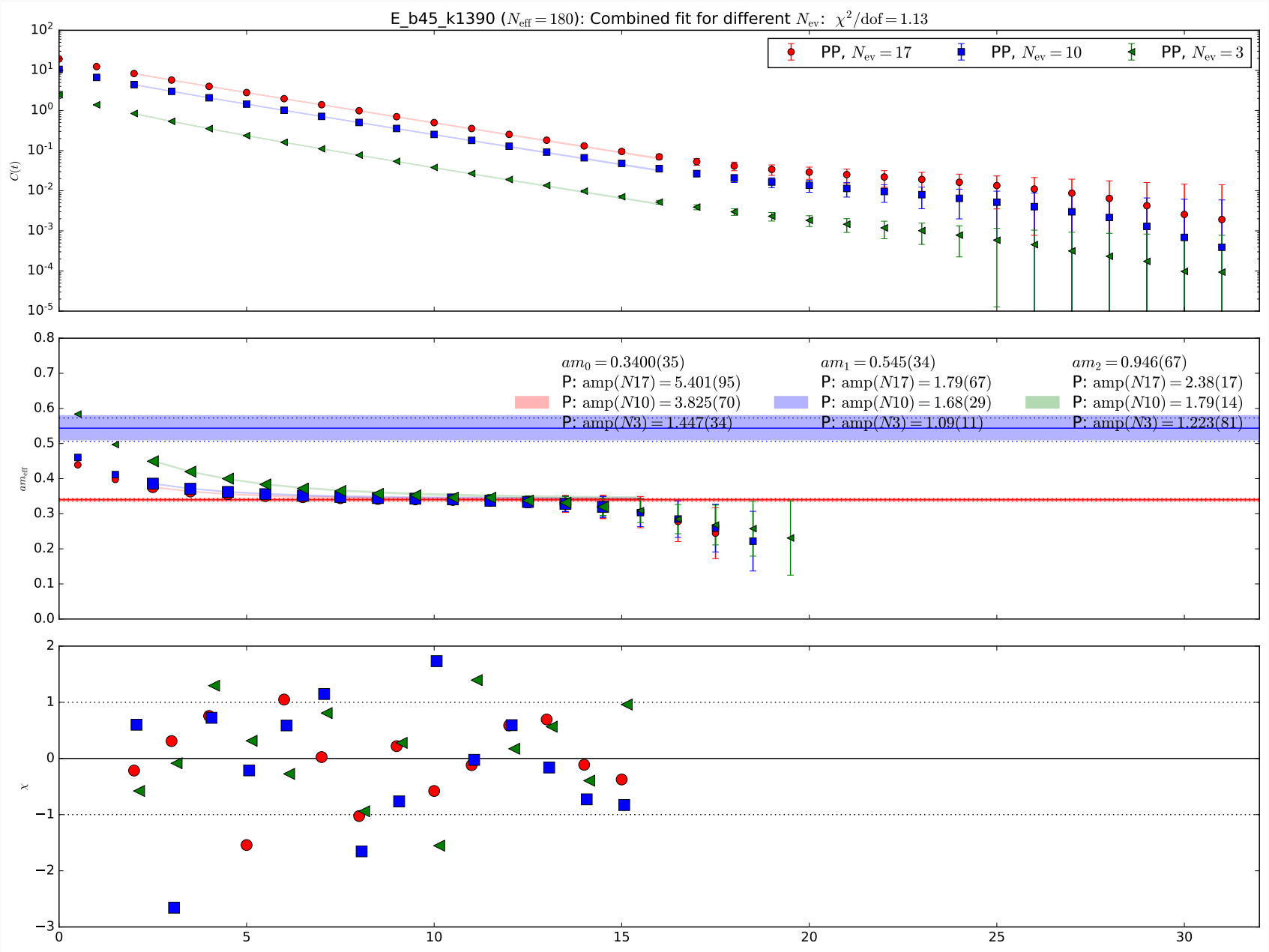
LapH method

- disconnected diagrams
- smearing



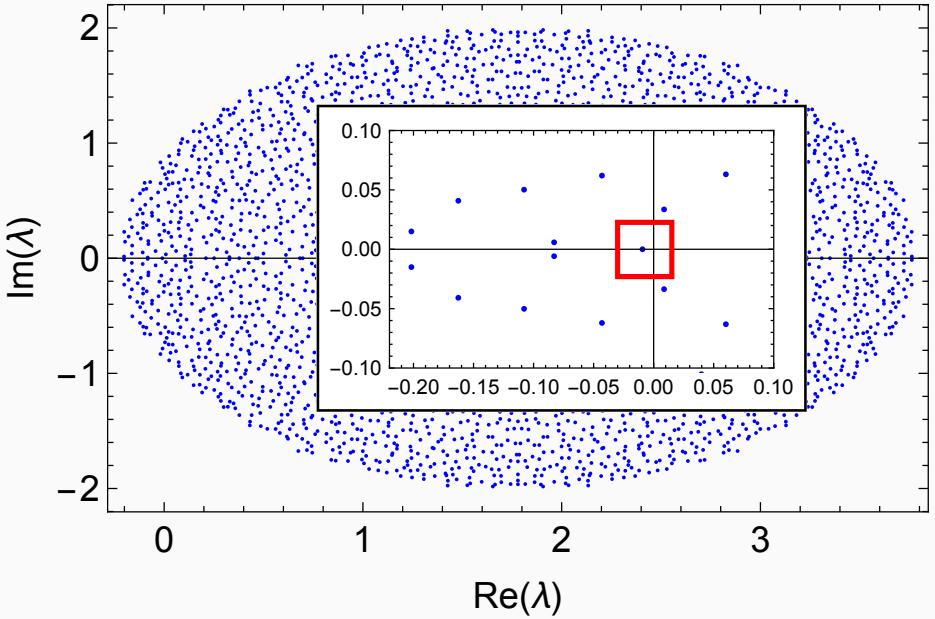
naive expectation

Example for a correlator fit



The sign problem in $N_f = 1$ QCD

- **Wilson fermions**
  $\Rightarrow \det(D) < 0$ possible
- **Need to monitor sign of the fermion determinant**
- Look at **real eigenvalues** of D where they change sign.



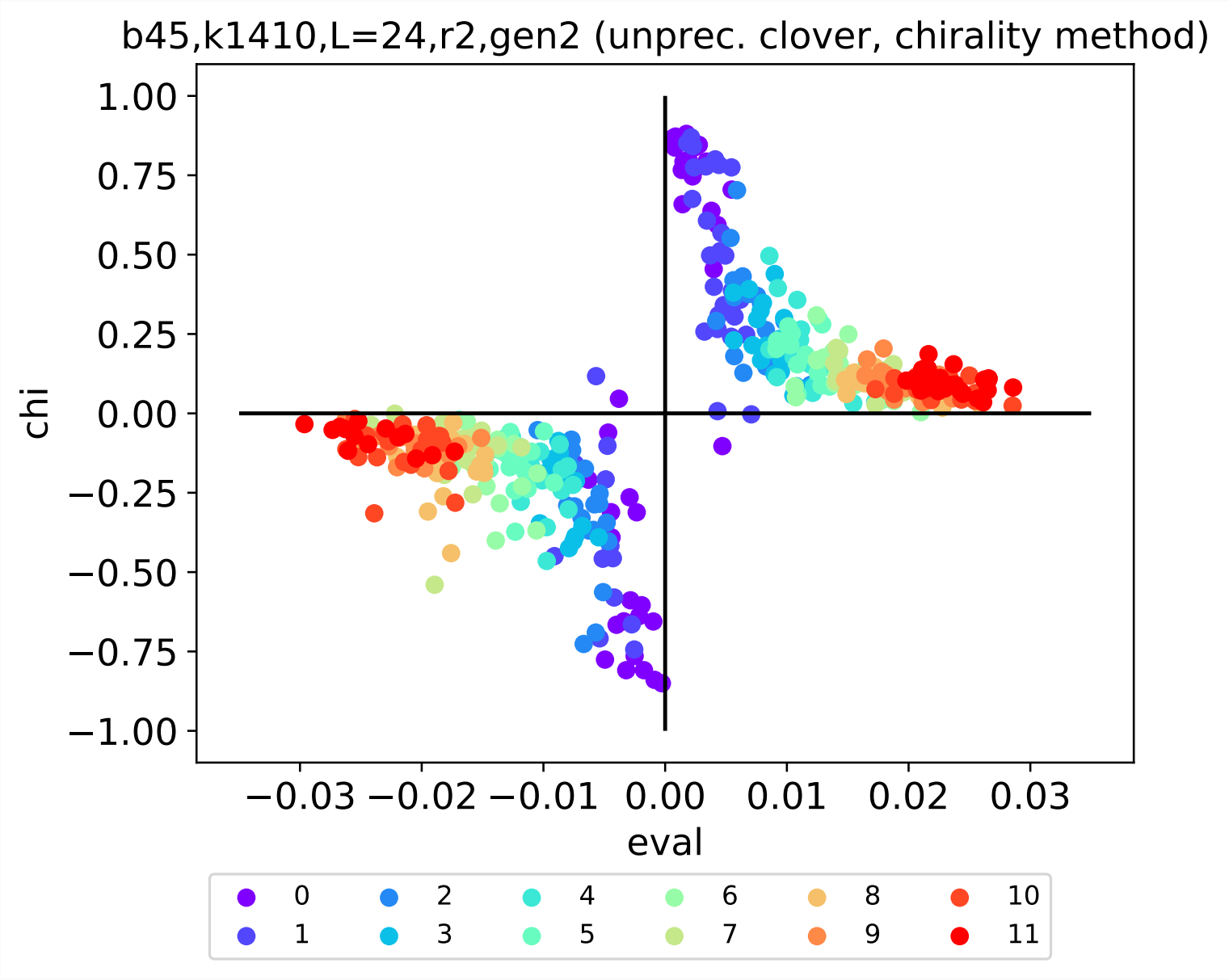
In practice

- analyze zeros of eigenvalues of $Q = \gamma_5 D$

$$Q\psi_i = \lambda_i \psi_i$$

- Quantities: eigenvalue function of $\lambda_i(m_0)$, chirality $\chi_i(m_0) = (\psi_i, \gamma_5 \psi_i)(m_0)$ (slope)

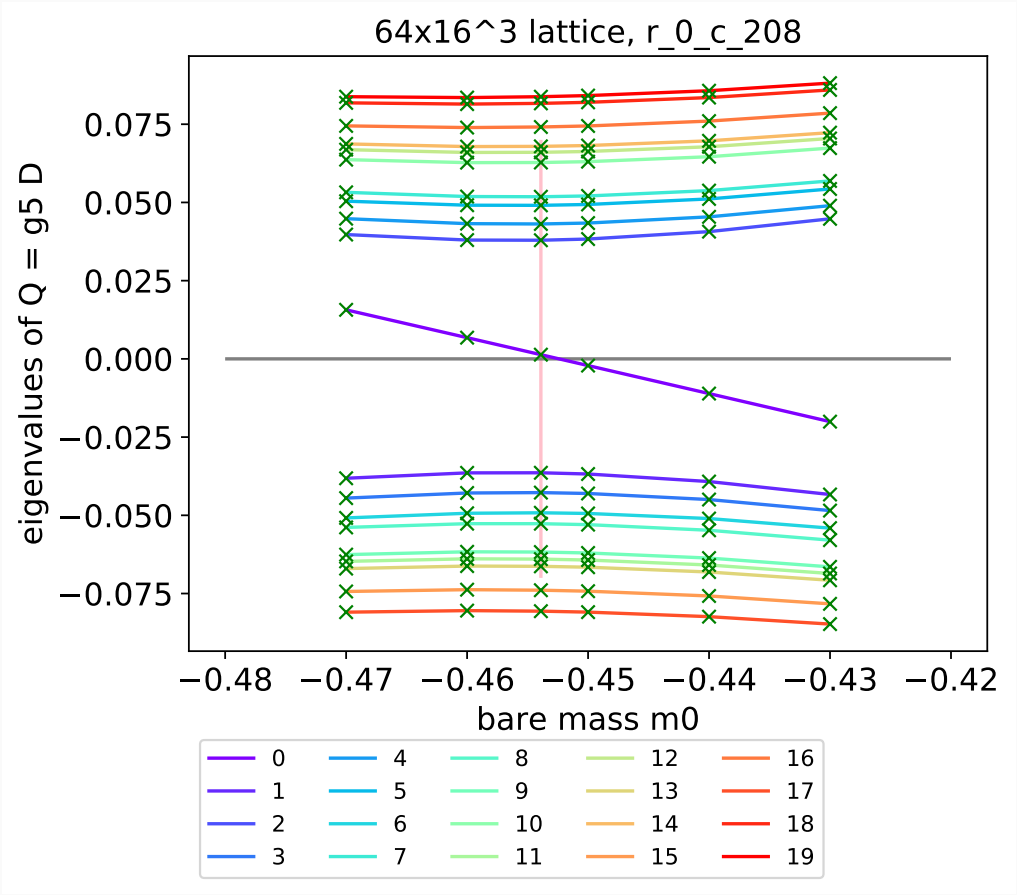
Sign problem analysis - stage 1



Sign problem analysis - stage 2

Tracking method

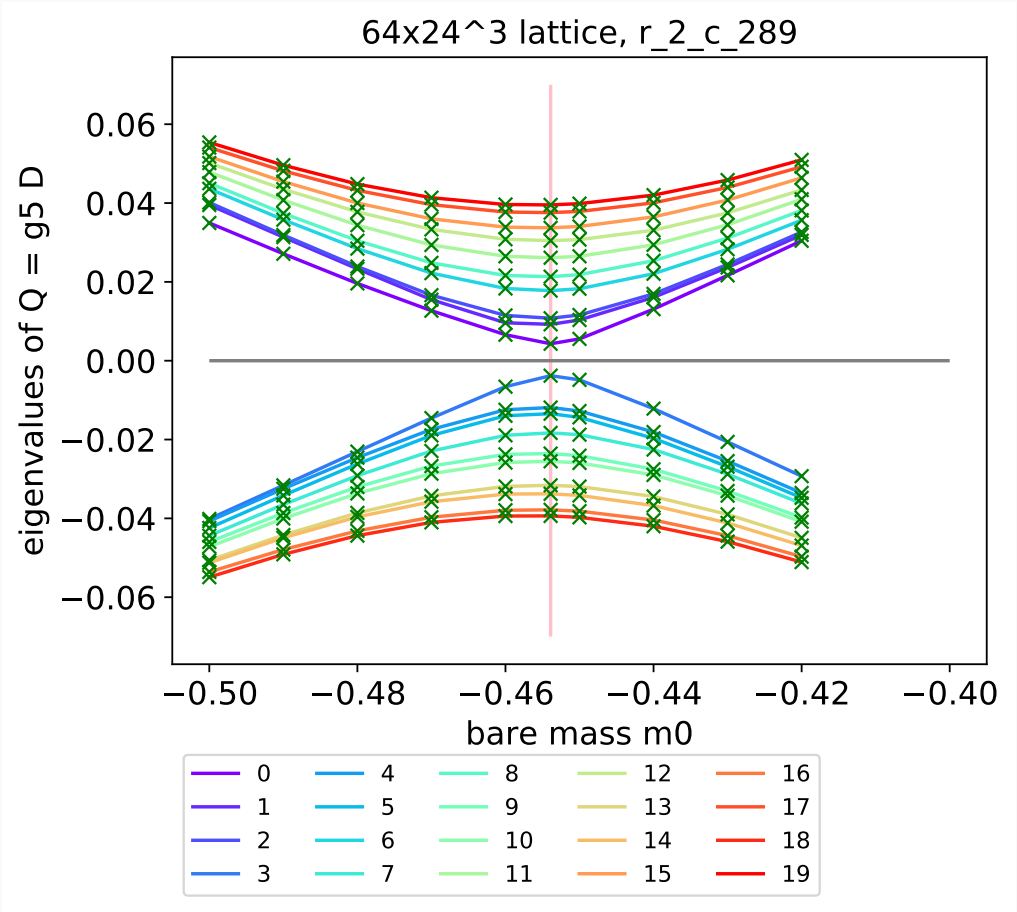
- see Mohler & Schaefer, 2003.13359[hep-lat], 2020
- monitor eigenvalue behaviour as a function of m_0
  - assume basis $\{\psi_i\}$ varies slowly and continuously
  - overlap of $\psi_i(m_0)$ and $\psi_j(m_0 + \Delta m_0)$



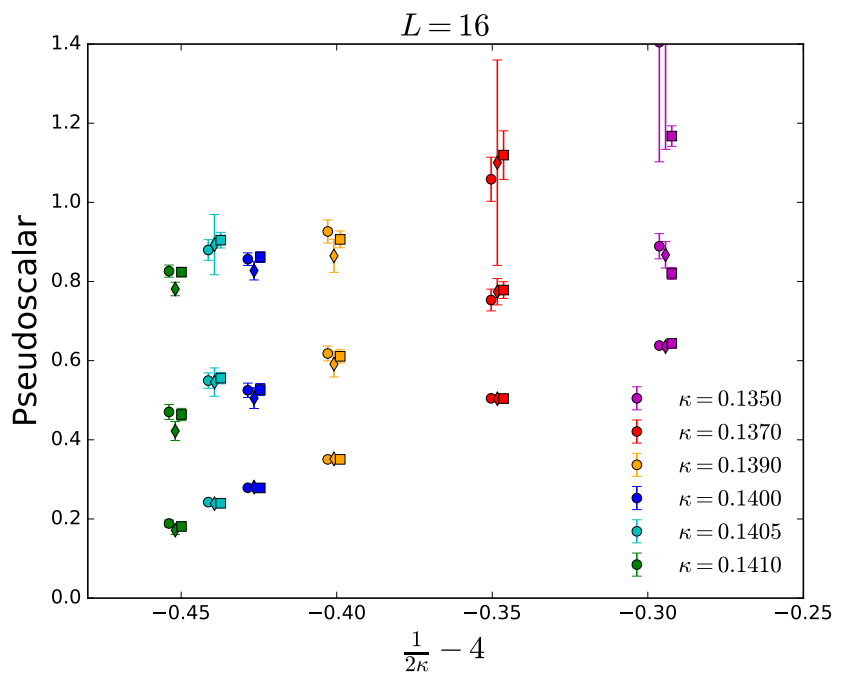
PRELIMINARY - Sign problem analysis - summary

Results at **lowest** bare mass ($\kappa = 0.1410$)

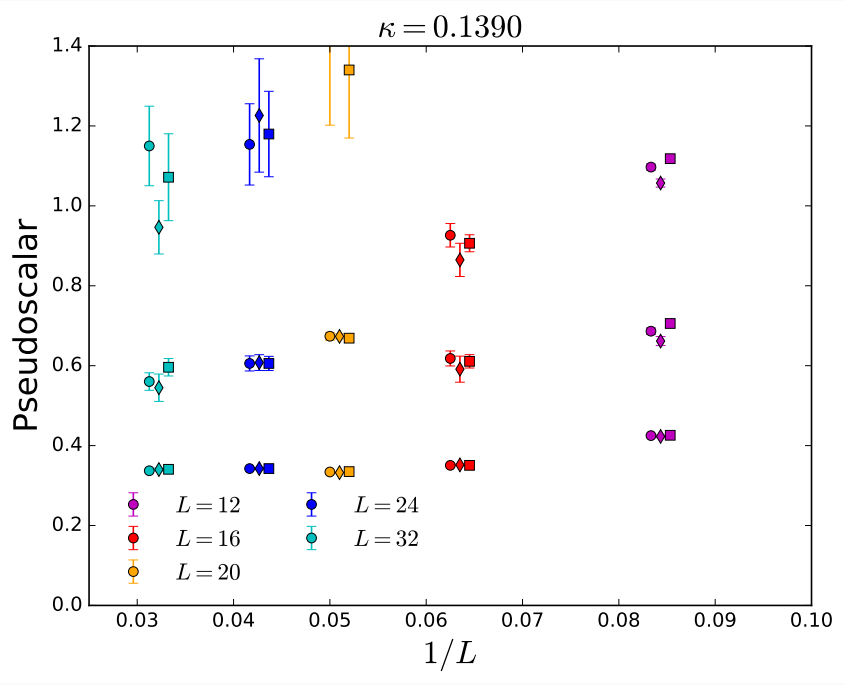
- $L/a = 24$: $\det D > 0$ for all configs
- $L/a = 16$: $\det D < 0$ for $\lesssim 1\%$ of the configs
- **sign problem mild**



Pseudo-scalar state 0^{-+}

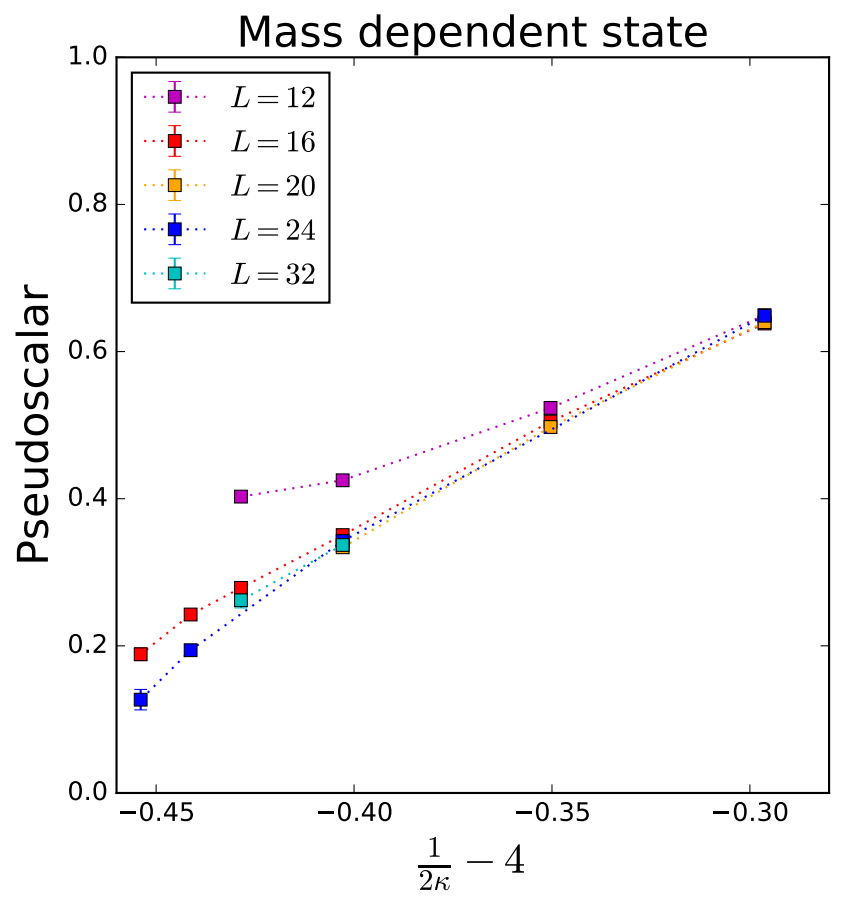


PRELIMINARY

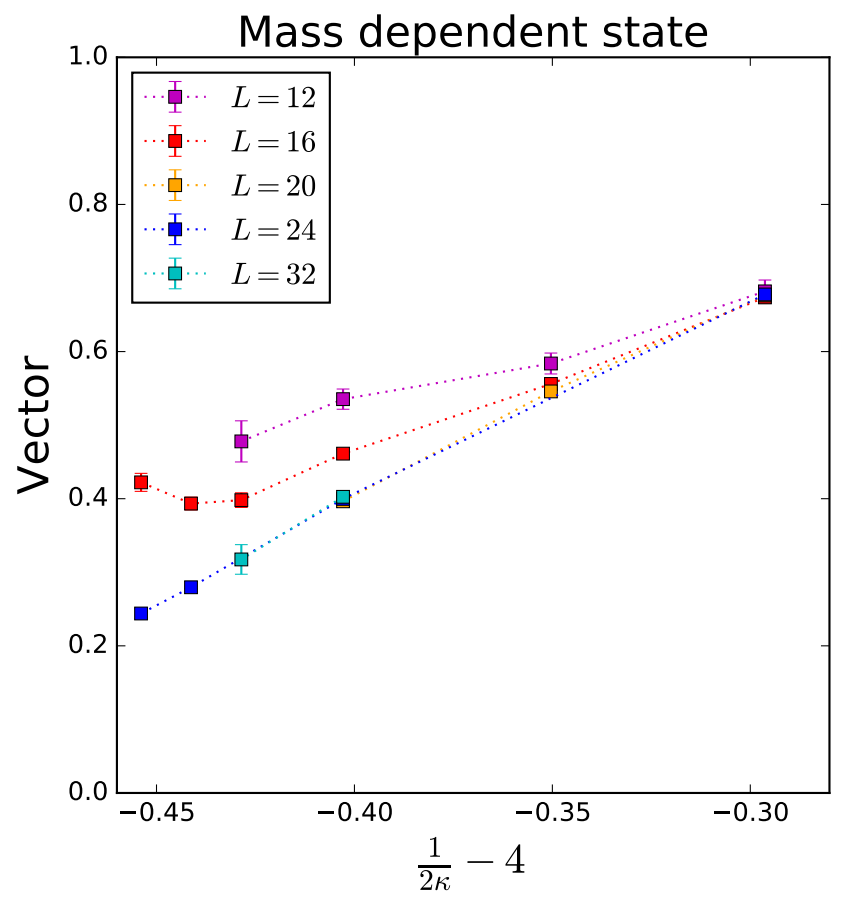


PRELIMINARY

Mass dependence - pseudo-scalar and vector

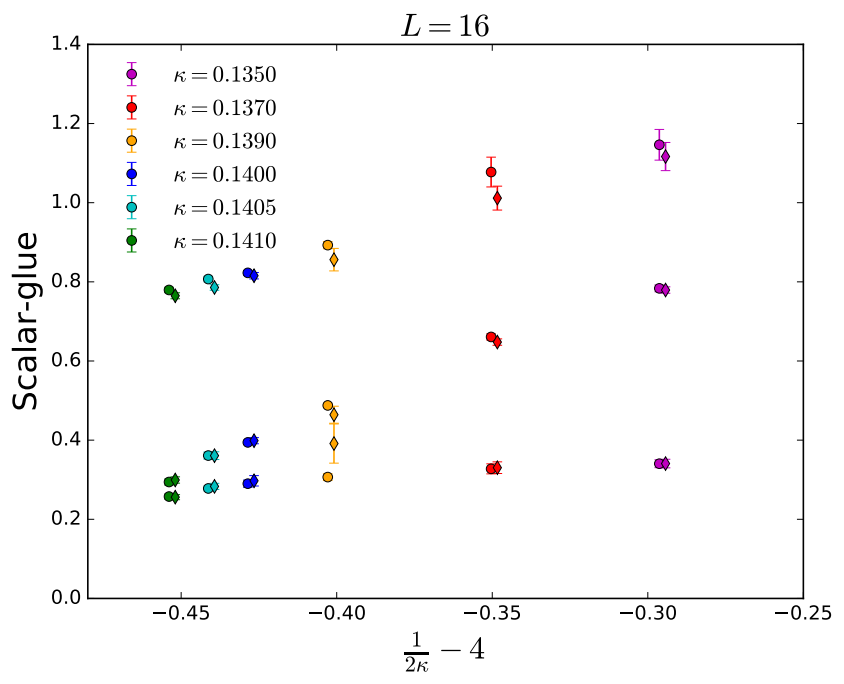


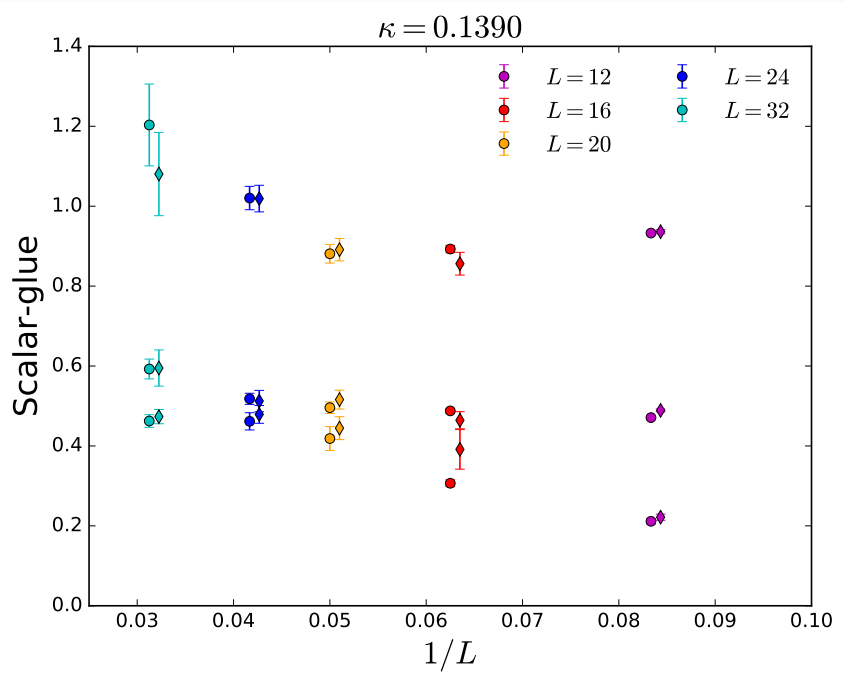
PRELIMINARY



PRELIMINARY

PRELIMINARY - scalar state 0^{++}





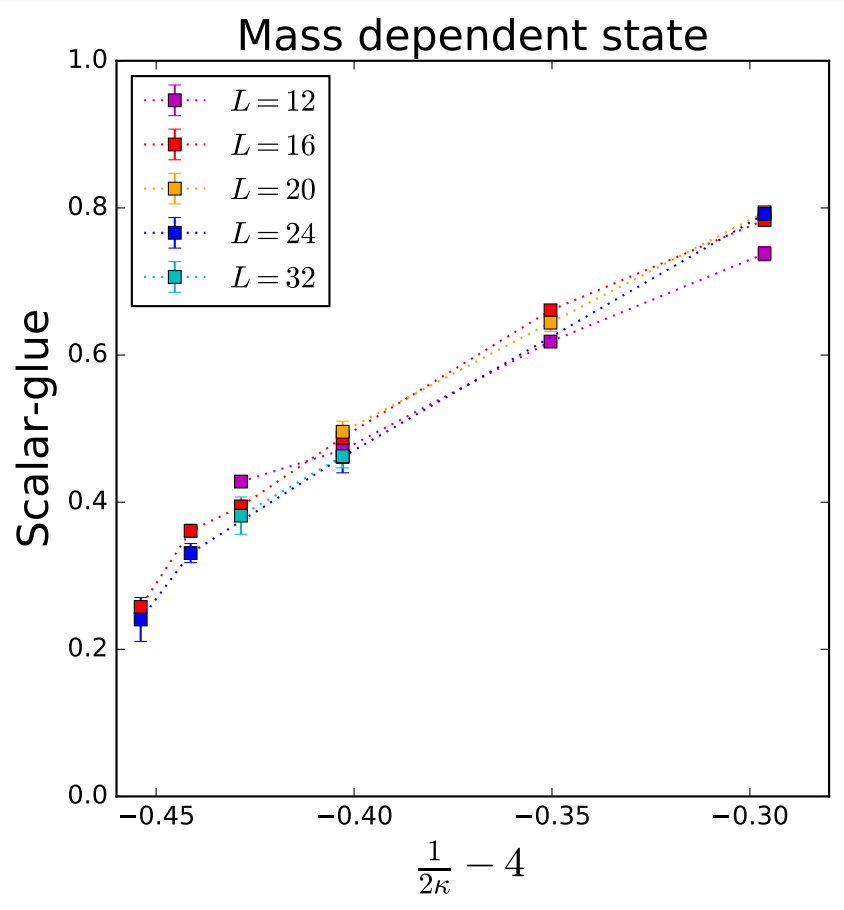
PRELIMINARY

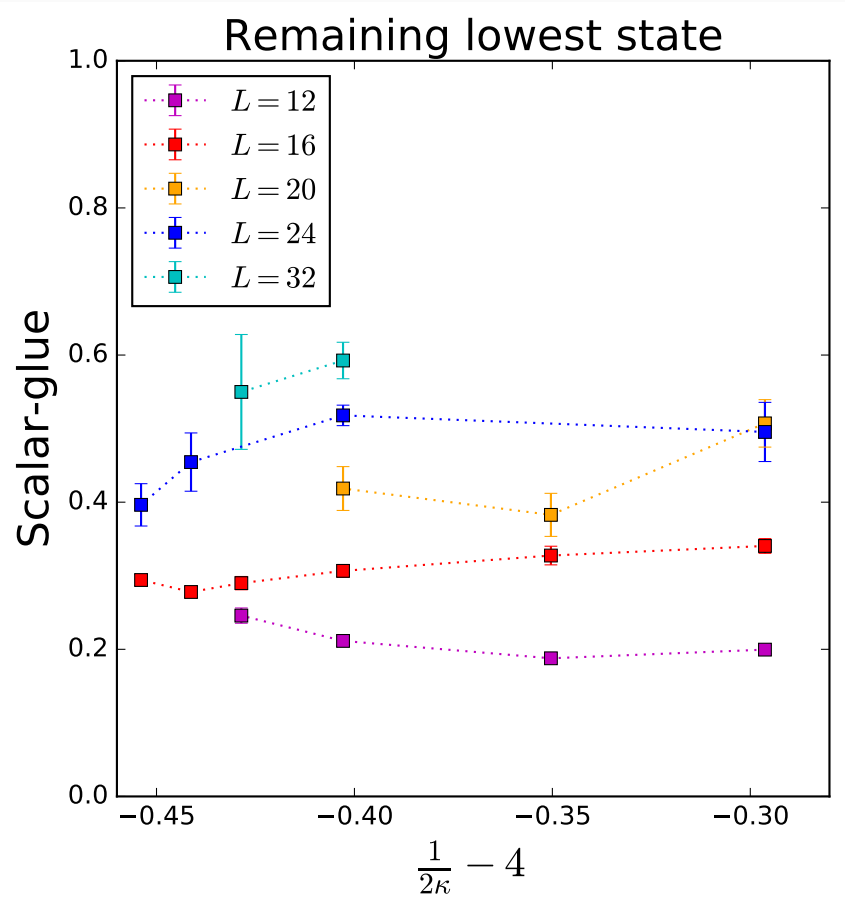
- 1 mass dependent state
- 1 mass independent state

PRELIMINARY

- 1 volume dependent state
- 1 volume independent state

Categorizing the scalar state 0^{++}





PRELIMINARY

- volume independent

PRELIMINARY

- volume dependent

From remaining lowest state: inconsistent with glueball $\rightarrow$ torelon

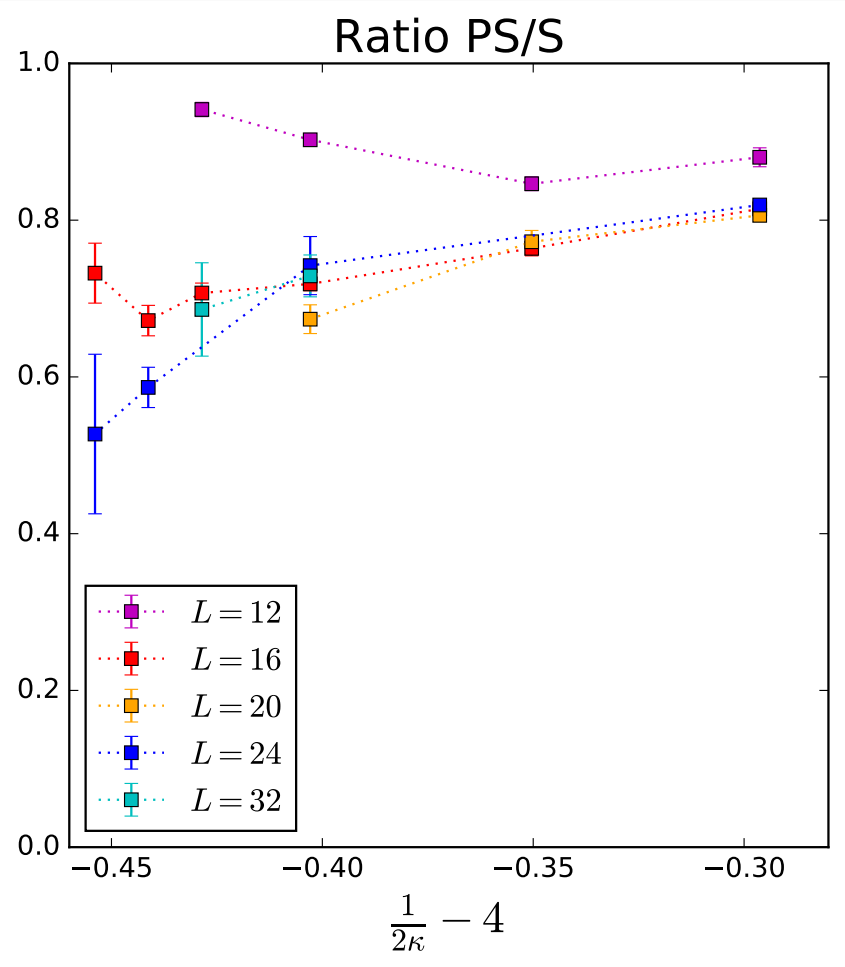
PRELIMINARY - Lark matter predictions

Our result

- large and intermediate masses show agreement
- ToDo: determine zero-quark mass limit and perform fit to extrapolate ratio

Sannino & Shifman

$$\frac{M_{PS}}{M_S} = 1 - \frac{22}{9N_c} + \ldots \lesssim 0.19$$



PRELIMINARY

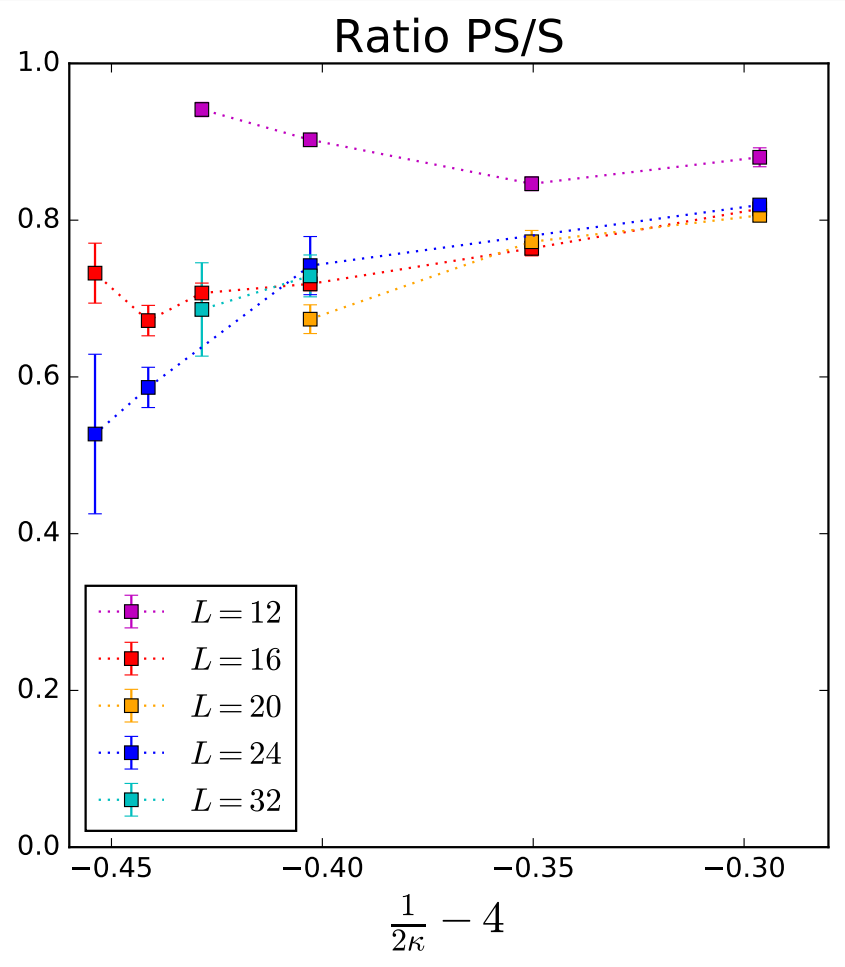
PRELIMINARY - Lark matter predictions

Our result

- large and intermediate masses show agreement
- ToDo: determine zero-quark mass limit and perform fit to extrapolate ratio

Result by Farchioni et al

$$\frac{M_{PS}}{M_S} = 0.410(32)(15)$$



PRELIMINARY

Conclusions on $N_f = 1$ QCD ... for now

Summary

- update on hadron spectrum including **excited states**
  $\Rightarrow$ mass dependence of scalar state
- ToDo: extrapolation to zero quark mass and fit of the data for low-energy effective theory comparison
- sign problem mild **BUT** must be monitored $\rightarrow$ multi-flavour QCD

Perspectives

- larger N_c
- code development $\rightarrow$ moving to GPUs

Thank you very much and stay tuned!

Back-up slides

Orientifold-A Theory - Theory of Larks

- larks are fermions in the two-indexed anti-symmetric representation of SU(N_c), $q_{ij} = -q_{ji}$
- dimension: $N_c(N_c - 1)/2$
- (theory of a single lark) $\to \mathcal{N} = 1$ SYM as $N_c \to \infty$
- $N_c = 3$: lark is an anti-quark and anti-lark is a quark $\Rightarrow$ lark theory equivalent to $N_f = 1$ QCD

Some representation theory

	gauge fields	fermion fields	repr. dimension (fermions)	spinor
lark theory	adjoint, 8	two-index anti-symmetric	$\frac{N_c(N_c-1)}{2}$	Dirac (2 x 4)
$N_f = 1$ **QCD**	adjoint, 8	fundamental	N_c	Dirac (2 x 4)
$\mathcal{N} = 1$ **SYM**	adjoint, 8	adjoint	$N_c^2 - 1$	Majorana (2 x 2)

The LapH method

Use here

- disconnected diagrams (all-to-all correlators)
  $\Rightarrow$ improve correlator signal
- reduce excited state contamination

How does it work

- project quark sinks into subspace spanned by lowest N_{ev} eigenmodes of the covariant Laplacian
  Morningstar, "Hadron Spectroscopy in Lattice QCD", 2016
- tune approach to plateau varying N_{ev}