Universität Bielefeld

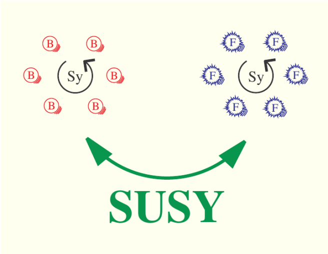
CRC-TR 211 Strong-interaction matter under extreme conditions

Estimates for the lightest baryon masses in $\mathscr{N} = 1$ supersymmetric Yang-Mills theory

Speaker: Sajid Ali (Bielefeld University)

Collaborators: G. Bergner, I. Montvay, G. Münster, C. Lopes

July 29, 2021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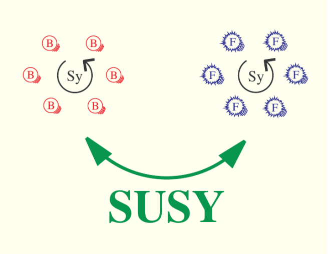
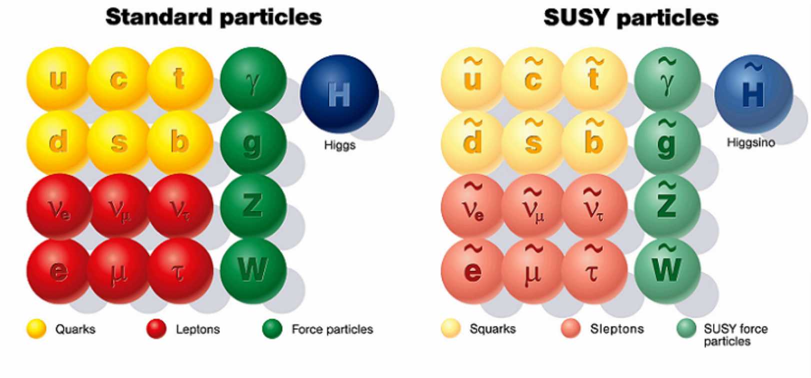


- Natural solution to the hierarchy problem
- Unifies three couplings at 2×10^{16} GeV
- Provides dark matter candidate: LSP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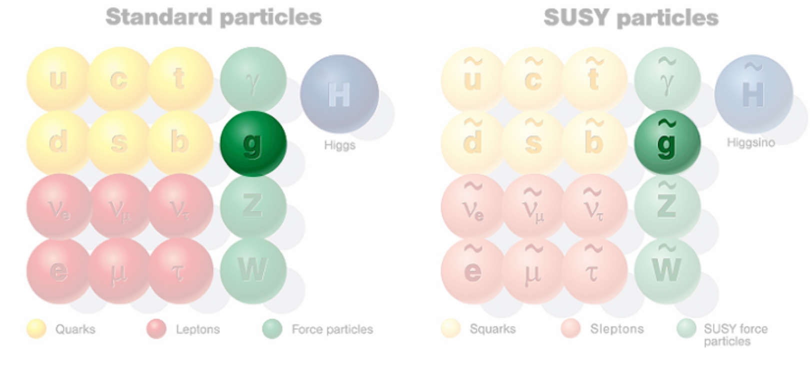
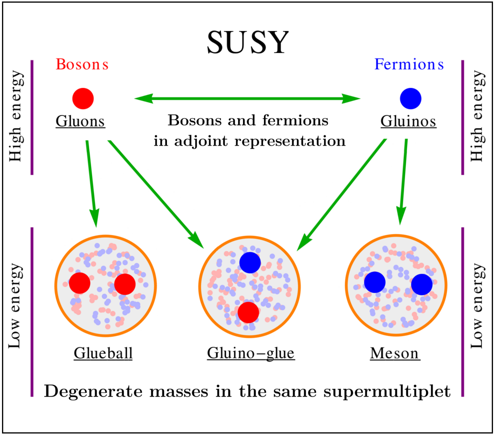
SUSY force particles



$\mathcal{N}$=1 supersymmetric Yang-Mills theory

Spectrum



Formation of bound states at low energy

Supersymmetric Yang-Mills theory

$$S_{SYM} = \int d^4x \left\{ -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \frac{\mathrm{i}}{2} \bar{\lambda}_a \gamma_\mu (\mathscr{D}_\mu \lambda)^a - \frac{1}{2} m_{\tilde{g}} \bar{\lambda}_a \lambda^a \right\}$$

- Field strength tensor
  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \mathrm{i}\, g\, f_{bc}^a A_\mu^b A_\nu^c$, Gauge field A_μ^a (gluon)

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  $(\mathscr{D}_\mu \lambda)^a = \partial_\mu \lambda^a + g\, f^a_{bc} A^b_\mu \lambda^c$, $a = 1, \ldots, N_c^2 - 1$
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  fermion field λ^a (gluino)
- Gluino mass term $\boxed{\frac{1}{2} m_{\tilde{g}} \bar{\lambda}_a \lambda^a}$ breaks SUSY softly
- In contrast to QCD:
  1) adjoint representation of SU(N_c)
  2) Majorana condition, $\boxed{\bar{\lambda}_a = \lambda_a^T C}$
  3) λ^a is Majorana spinor field, "$N_f = \frac{1}{2}$"

Motivations

- SYM: simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD
- Similar to QCD:
  1) Asymptotic freedom
  2) Confinement
  3) Numerical lattice simulation of bound states

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  3) Numerical lattice simulation of bound states
- Guide towards realistic models i. e. super-QCD
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)
- Spectrum of bound states $\rightarrow$ Chiral supermultiplet
- SUSY restoration on the lattice

Supersymmetric Yang-Mills theory on the lattice

$$S_{CV}^{lat} = \beta \sum_{\mu\nu} \left(1 - \frac{1}{N_c} \mathrm{Re} \left[\mathrm{tr} \left[U_{\mu\nu} \right] \right] \right)$$

$$+ \frac{1}{2} \sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat{\mu}} V_{ab,x\mu} (1+\gamma_\mu) \lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu} (1-\gamma_\mu) \lambda^b_{x+\hat{\mu}} \right] \right\}$$

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$$\beta = \frac{2N_c}{g^2} \text{(inverse gauge coupling)},$$

$$\kappa = \frac{1}{2m_{\tilde{g}}+8} \text{(hopping parameter)},$$

$$V_{ab,x\mu} = 2\,\mathrm{tr}[U_{x\mu}^\dagger T_a U_{x\mu} T_b] \text{(adjoint link variables)}$$

Supersymmetric Yang-Mills theory on the lattice

Basic principles:

- Cont. space-time $\rightarrow$ lattice
- Imaginary time $t = \mathrm{i}\tau$
- Functional integral approach

allow Monte Carlo simulations

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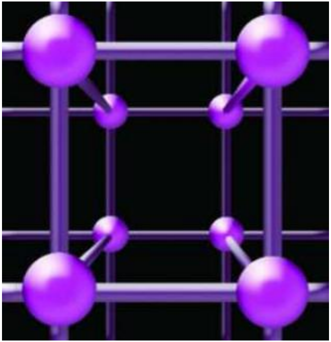

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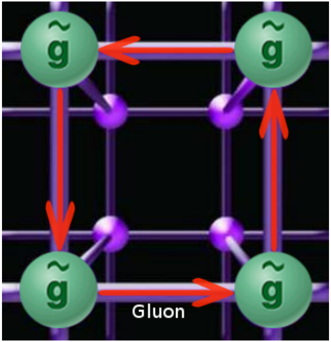

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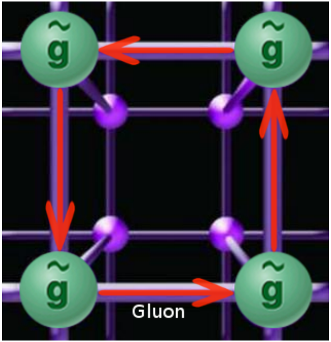

$$\textit{Link}: U_{x\mu} = e^{\mathrm{i}ag A_\mu(x)}$$

$$\textit{Plaquette}: U_{x\mu\nu} = e^{\mathrm{i}a^2 F_{\mu\nu}(x)}$$

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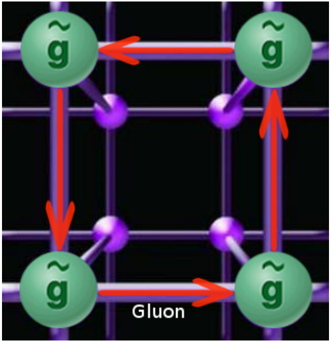
allow Monte Carlo simulations

Challenges:

- Finite volume effects
- Chiral symmetry
- Back to cont. space-time



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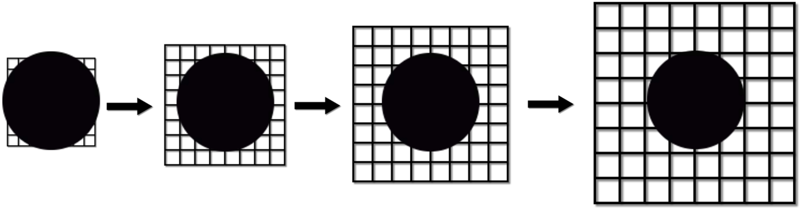
Finite volume effects



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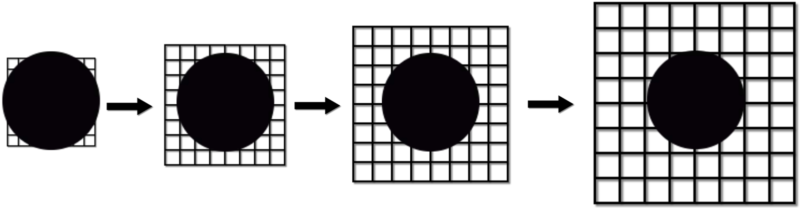
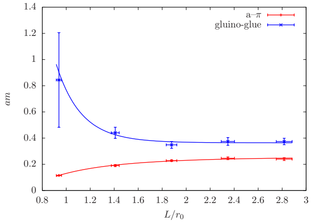
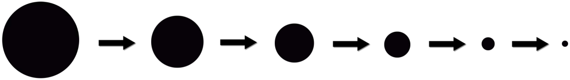




Lattice volume dependence of the gluino-glue (blue) and a–π (red) masses. The numerical simulations are performed at $\beta = 5.6$ and $\kappa = 0.1660$.

Non-zero gluino mass

Gluino mass term $\frac{1}{2}m_{\tilde{g}}\bar{\lambda}^a\lambda^a$ breaks SUSY softly $\rightarrow$ Chiral limit



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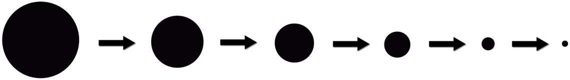

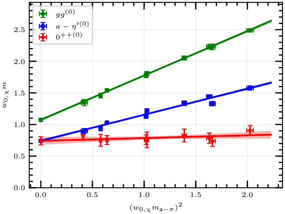

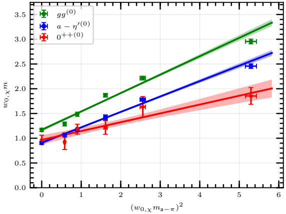

Continuum limit

- Lattice breaks SUSY
- Can SUSY be restored in the continuum limit?
- Search for continuum limit $a \to 0$ with restored SUSY

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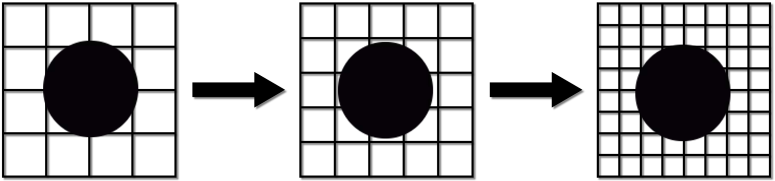
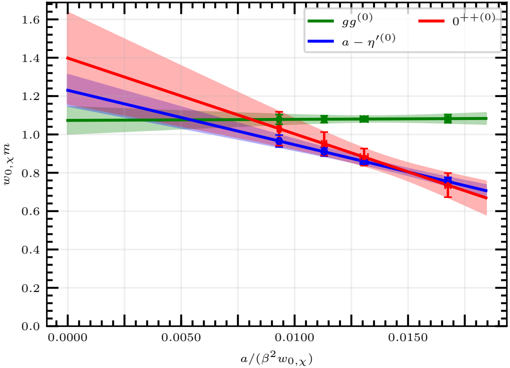


Continuum limit



Extrapolations of ground state masses towards the continuum limit at performed at 4 different lattice spacings and formation of the chiral supermultiplet.

Baryons in $\mathcal{N}$=1 SUSY Yang-Mills theory on the lattice

Baryons in SYM

Baryon interpolator:

$$W(x) = t_{abc}\lambda_a\left(\lambda_b^T \Gamma \lambda_c\right)$$

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Baryon correlator:

$$\begin{aligned} B(x,y) &= \langle W(x)\overline{W}(y)\rangle \\ &= t_{abc}t_{a'b'c'}\Gamma^{\beta\gamma}\Gamma^{\beta'\gamma'}C^{\delta\alpha'}\langle\lambda_a^\alpha(x)\lambda_b^\beta(x)\lambda_c^\gamma(x)\lambda_{a'}^{\alpha'}(y)\lambda_{b'}^{\beta'}(y)\lambda_{c'}^{\gamma'}(y)\rangle \end{aligned}$$

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Wick's theorem gives:

- Sunset
- Spectacle

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Wick's theorem gives:

- Sunset
- Spectacle

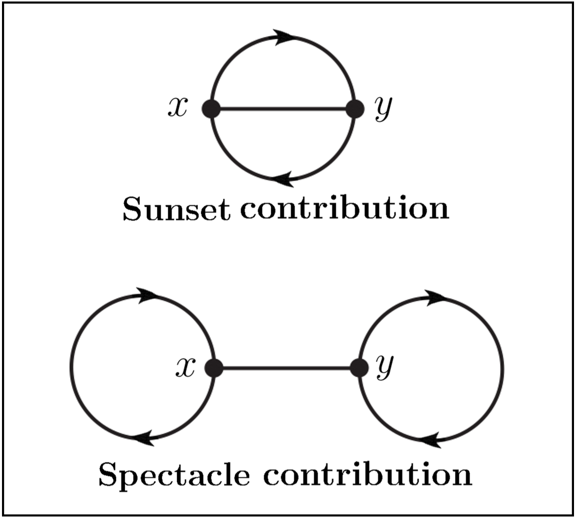
$$\langle\lambda_a^\alpha(x)\lambda_b^\beta(y)\rangle = -(\triangle(x,y)C)_{ab}^{\alpha\beta},$$

Baryons in SYM

$$B_{Sset}(x,y) = t_{a'b'c'}t_{abc}\Gamma^{\beta\gamma}\Gamma^{\beta'\gamma'}P_{\pm}^{\alpha\alpha'} \times \{ +2\triangle_{aa'}^{\alpha\alpha'}(x,y)\triangle_{bb'}^{\beta\beta'}(x,y)\triangle_{cc'}^{\gamma\gamma'}(x,y) +4\triangle_{ab'}^{\alpha\beta'}(x,y)\triangle_{bc'}^{\beta\gamma'}(x,y)\triangle_{ca'}^{\gamma\alpha'}(x,y)\} \quad (1)$$

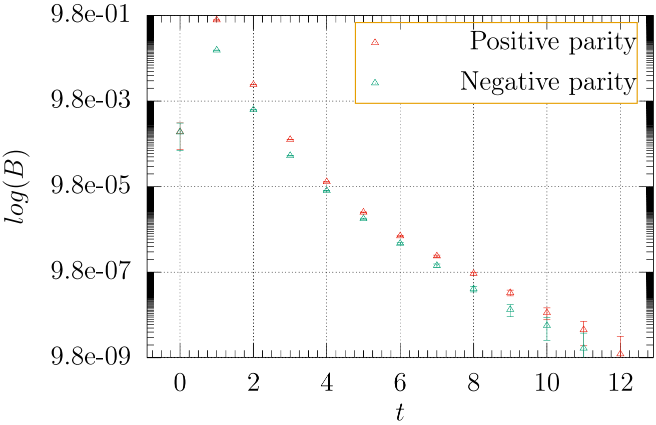
$$B_{Spec}(x,y) = t_{a'b'c'}t_{abc}\Gamma^{\beta\gamma}\Gamma^{\beta'\gamma'}P_{\pm}^{\alpha\alpha'} \times \{ +2\triangle_{ab}^{\alpha\beta}(x,x)\triangle_{ca'}^{\delta\alpha'}(x,y)\triangle_{c'b'}^{\delta'\beta'}(y,y)C^{\gamma\delta}C^{\delta'\gamma'} +4\triangle_{ab}^{\alpha\beta}(x,x)\triangle_{cb'}^{\delta\delta'}(x,y)\triangle_{c'a'}^{\gamma'\alpha'}(y,y)C^{\gamma\delta}C^{\beta'\delta'} +1\triangle_{bc}^{\beta\delta}(x,x)\triangle_{aa'}^{\alpha\alpha'}(x,y)\triangle_{c'b'}^{\delta'\beta'}(y,y)C^{\gamma\delta}C^{\delta'\gamma'} +2\triangle_{bc}^{\beta\delta}(x,x)\triangle_{ab'}^{\alpha\beta'}(x,y)\triangle_{c'a'}^{\delta'\alpha'}(y,y)C^{\gamma\delta}C^{\gamma'\delta'}\} \quad (2)$$

Baryons in SYM



Baryon correlation function

Correlator (preliminary)



Baryon correlator (Sunset) in $\mathcal{N}$=1 SUSY Yang-Mills theory, $\beta = 1.75$, $\kappa = 0.14925$

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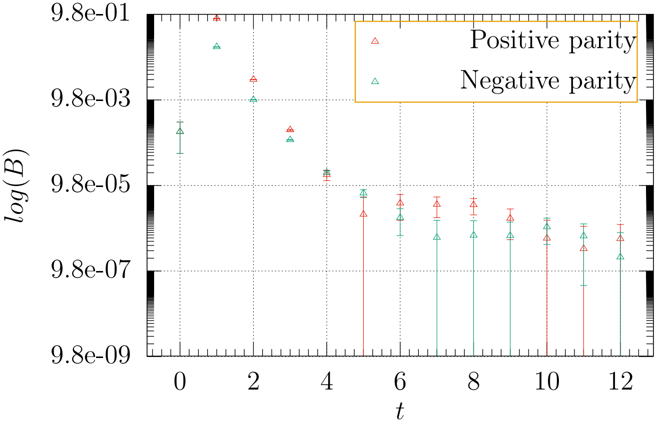
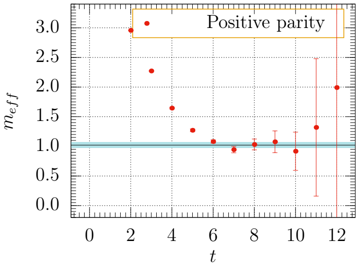
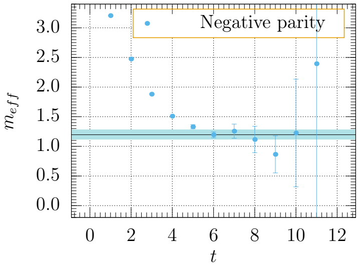


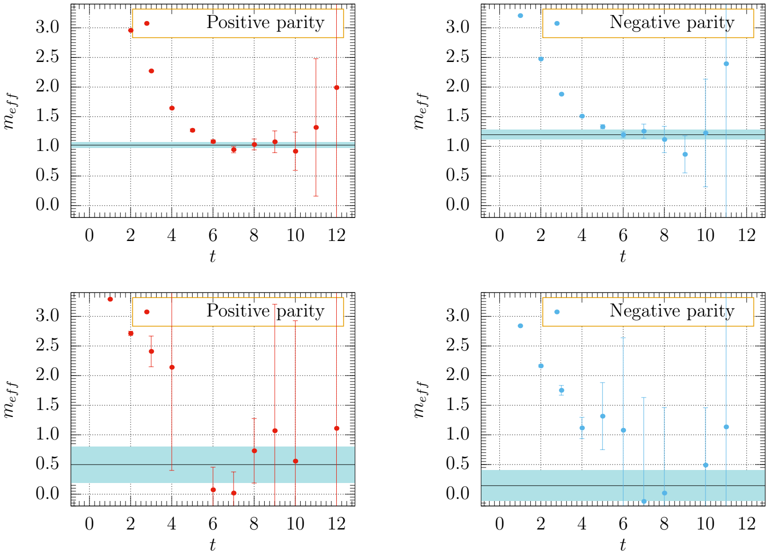
Baryon correlator (Sunset+Spectacle) in $\mathcal{N}=1$ SUSY Yang-Mills theory, $\beta = 1.75$, $\kappa = 0.14925$

Effective mass (preliminary)





Effective mass (preliminary)



Effective masses of two pieces in $\mathcal{N}$=1 SUSY Yang-Mills theory β

Summary

- $\mathcal{N}=1$ supersymetric Yang-Mills theory
  1) Introduction
  2) SYM theory on the lattice
  3) Spectrum

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  2) SYM theory on the lattice
  3) Spectrum
- Baryonic states in $\mathcal{N}=1$ SUSY Yang-Mills theory
  1) Baryon correlation function
  2) Effective mass of Baryons

Thank You!

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