Anomalies and symmetric mass generation with staggered fermions

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Fermion mass

Typically fermions acquire mass by breaking symmetries:

- Explicitly eg Dirac mass breaks axial symmetry.
- Spontaneously eg. chiral condensate $< \overline{q}q > \neq 0$ in QCD.
- Via anomalies eg η'

Does this exhaust the possibilities ?

No!

Fermion masses can arise without breaking global symmetries provided **all** 't Hooft anomalies vanish

Symmetric Mass Generation (SMG)

't Hooft anomalies

Imagine gauging global symmetry $G \rightarrow$ non-zero anomaly coeff **A**

Anomaly is RG invariant $\rightarrow$
requires massless particles in I.R with same **A**

Options:

- Massless composite fermions
- G breaks spontaneously - massless Goldstone bosons

't Hooft anomaly matching gives non-perturbative information on I.R dynamcs

An example: Georgi-Glashow

Consider $SU(5)$ gauge theory with global $G = U(1)$

L Weyl fields: $\chi_{\alpha\beta}[10]^{(1)}$ and $\psi^{\alpha}[\bar{5}]^{(-3)}$

chiral gauge theory

No invariant mass term but gauge anomaly cancels $A(\bar{\mathbf{5}}) = -A(\mathbf{10})$

Imagine weakly gauging $G \to$ 't Hooft anomaly:
$A(G) = \sum_a Q_a^3 = 5 \times (-3)^3 + 10 \times (1)^3 = -125$

One obvious color singlet composite fermion in I.R
$\bar{\xi} = \chi_{ab}\psi^a\psi^b$ – $U(1)$ charge -5

Precisely what is needed for the anomaly $(-5)^3 = -125$!

Symmetric mass generation for GG model

Turning this around ...

To make all states **massive** in I.R must cancel 't Hooft anomaly in U.V

Just add a singlet $\xi^{(+5)}$!

Can now couple $\xi, \bar{\xi}$ with Dirac mass

$\lambda\bar{\xi}\xi = \lambda\chi_{ab}\psi^a\psi^b\xi \leftarrow$ four fermion term - preserves G

Notice: naively irrelevant 4 fermion operator yields a relevant mass term in I.R as a result of confinement.

Take home point

Four fermion terms can gap fermions without breaking symmetries if all 't Hooft anomalies vanish

Another observation

Notice sixteen Weyl fermions needed...

Discrete anomalies

Recent work focused on discrete global anomalies ...

Cancelling 't Hooft anomalies for these symmetries gives new constraints on fermion content of consistent QFTs

D=1	Time reversal	8 Majorana
D=2	Chiral fermion parity	8 Majorana/Weyl
D=3	Time reversal	16 Majorana
D=4	Spin-Z_4 symmetry	16 Majorana/Weyl

eg. Spin-Z_4 symmetry

$$\psi_L \to -i\psi_L \quad \psi_R \to +i\psi_R$$

n_L, n_R number of L/R Weyl fermions

anomaly cancellation: $n_L - n_R = 0 \mod 16$

How are these anomalies manifested ?

Consider $Z(A) = \int D\psi\, e^{iS(\psi,A)}$
$A \equiv$ gauge or scalar field

Classical symmetry G: $S(A,\psi) = S(A^G,\psi^G)$
Imagine closed path P: $A \overset{\text{path in A space}}{\rightarrow} A^G$

Anomaly: when $Z(A^G) = e^{2\pi i\eta} Z(A)$

- P infinitessimal: local anomaly eg ABJ
- P finite: global anomaly eg Witten's global $SU(2)$ anomaly for single Weyl corresponds to $\eta = \frac{1}{2}$. Implies: $Z = \int DA\, Z(A) = 0$

Hard to compute η in general

Dai-Freed method: calculate $\delta_G Z$ for free fermions in curved background geometry - **gravitational** anomaly

Example of lattice model capable of SMG

Reduced staggered fermions

$$S = \sum_{x,\mu} \chi^a(x)\eta_\mu(x)D^S_\mu \chi^a(x) - \frac{G^2}{8}\sum_x \epsilon_{abcd}\chi^a(x)\chi^b(x)\chi^c(x)\chi^d(x)$$

$\chi^a(x)$: 4 single component Grassmanns

$\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu-1} x_i}$ and $\xi_\mu(x) = (-1)^{\sum_{i=\mu+1}^{d} x_i}$

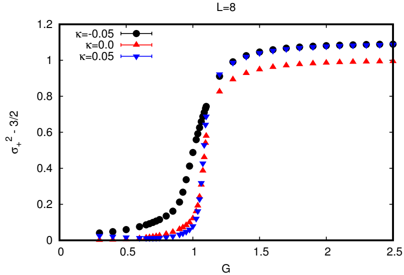
Symmetries

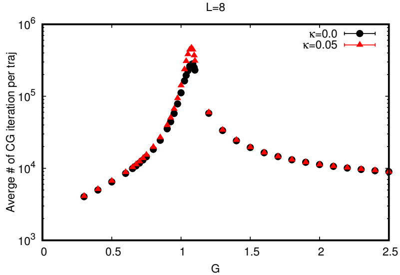
- $SU(4)$
- shift: $\chi(x) \to \xi_\mu(x)\chi(x+\mu)$
- Z_4: $\chi^a(x) \to i\epsilon(x)\chi^a(x)$

Symmetries prohibit all bilinear terms in $\Gamma(\chi)$

Phase diagram $D = 4$

see Nouman Butt's talk in this meeting...





Evidence for direct, continuous phase transition between massless and massive phases with no symmetry breaking (S.C et al. PRD98 (2018) 114514)

Notice: continuum limit describes $4 \times 4 = 16$ Majorana fermions !

Lattice anomalies ?

Simulations indicate that fermions gapped in continuum - 16 Majorana fermions as required by Dai-Freed arguments
Is there any anomaly argument based directly in lattice for this ?
Yes !

A gravitational anomaly for staggered fermions

- Staggered fermions may be generalized to random lattices approximating curved space by replacing them by Kähler–Dirac fermions
- Massless free Kähler–Dirac fermions have an exact $U(1)$ symmetry on **any** random lattice – analog of $U_\epsilon(1) = e^{i\alpha\epsilon(x)}$ for staggered.
- This symmetry is **anomalous even for finite lattices**: $Z \rightarrow e^{2i\alpha\chi} Z \quad \text{where } \chi = \text{Euler character} = \sum_{i=0}^{d} N_i \, (-1)^i$

Example of anomaly for finite number dof !
S.C et al JHEP 2010 (2018) 013.

Cancelling the anomaly

- Consider the sphere in even dims: $\chi(S^{2n}) = 2 \rightarrow$ phase=$e^{4i\alpha}$. Lattice gravitational anomaly breaks $U(1) \rightarrow Z_4$.
- Cancel anomaly if we have $4n$ flavors of Kähler–Dirac (staggered) field. Equal to $16n$ Dirac in flat space limit.
- SMG is then possible using four fermion terms built from 4 flavors of Kähler–Dirac /staggered fermion.

Can impose a Majorana-like condition $\rightarrow$ **reduced** Kähler–Dirac or **reduced** staggered fermions (1/2 dof)
$\rightarrow \rightarrow$ anomaly cancelled for sixteen Majorana fermions
Agrees with Dai-Freed arguments (Weyl fermions) and explains observed lattice phases

Summary

- SMG allows fermions to acquire masses without breaking symmetries and leads to new phase transitions for strongly interacting fermions. Employs 4 fermion interactions.
- Necessary condition for SMG is cancellation of all 't Hooft anomalies. Leads to magic numbers of fermions (8 Weyl in 2d, 16 Majorana in 3d, 16 Weyl 4d). These numbers can be gotten from exact **gravitational** anomalies of staggered fermions.
- $D = 3, 4$ there are well studied lattice examples which use staggered/Kähler–Dirac fermions
- SMG may offer new mechanism to gap out mirrors in efforts to construct chiral lattice gauge theories

Backups