Newtonian Binding from Lattice Quantum Gravity

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Overview

Asymtotic Safety with EDT

It was pointed out by Weinberg that if garvity is asymptotically safe, it would be renormalizable non-perturbatively.[1]

Euclidean dynamical triangulations (EDT) is an approach to lattice quantum gravity. Geometry is constructed by gluing 4-simplices together. In [arXiv:1604.02745], it was shown that when a non-trivial measure term is added and associated coupling is fine-tuned, EDT gives the correct Hausdorff dimension and spectral dimension (≈ 4).

This talk summarizes the work done in arXiv:2102.04492.

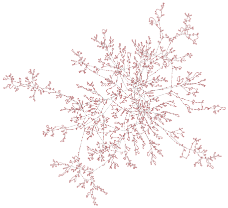

Figure 1: Visualization of one configuration. Dots represent 4-simplices and the lines show connection between the nearest neighbors.

The path integral of 4-d Euclidean Einstein gravity in the **continuum** is:

$$Z_E = \int \mathcal{D}[g]\mathcal{D}[\phi]\, e^{-S_{EH}[g]-S_M[\phi]}, \tag{1}$$

where S_{EH} is the Euclidean Einstein-Hilbert action:

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{g}(R-2\Lambda), \tag{2}$$

and S_M is the matter sector (only scalar for now):

$$S_M = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m_0^2\phi^2\right), \tag{3}$$

Working in"quenched approximation"[1], for **lattice** QG, the path integral becomes:

$$Z_E=\sum_T \frac{1}{C_T}\left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^{\beta}\right] e^{-S_{ER}} \tag{4}$$

where C_T divides out equivalant ways of labeling the vertices in a given geometry, N_2 is the total number of triangles in our geometry, β is a free parameter, and S_{ER} is the Einstein-Regge action:[3]

$$S_{ER}=-\kappa \sum_{j=1}^{N_2} V_2 \delta_j+\lambda \sum_{j=1}^{N_4} V_4=-\kappa_2 N_2+\kappa_4 N_4, \tag{5}$$

where $\kappa = (8\pi G)^{-1}$, $\lambda = \kappa\Lambda$, $\delta_j = 2\pi - \mathcal{O}(t_j)\arccos(1/4)$ is the deficit angle around a triangular hinge t_j, and where the volume of a d-simplex of equilateral edge length a is given by

$$V_d=\frac{\sqrt{d+1}}{d!\sqrt{2^d}}\, a^d. \tag{6}$$

The action of the matter sector becomes:

$$S_M^{\text{lat}}=\frac{1}{2} \sum_{\langle xy\rangle} (\phi_x-\phi_y)^2+\frac{m_0^2}{2} \sum_x \phi_x^2 \tag{7}$$

where x represents the 4-simplex the scalar field lives on.

[1] 'Quenched' means that the matter field does not influence the geometry.

Overview

We can write the action of matter sector as

$$S_M^{\text{lat}} = \sum_{x,y} \phi_x L_{xy} \phi_y \tag{8}$$

where $L_{xy} = (D_x + m_0^2) - A_{xy}$, D_x is the number of neighbors each simplex has and

$$A_{xy} = \begin{cases} 1 & \text{if } x \text{ and } y \text{ share a dual edge} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Then the one-particle and two-particle correlators can be calculated as

$$G(r) = \left\langle \frac{\sum_{x,y} L_{xy}^{-1} \delta_{|x-y|,r}}{\sum_{x,y} \delta_{|x-y|,r}} \right\rangle \text{ and } G^{(2)}(r) = \left\langle \frac{\sum_{x,y} (L_{xy}^{-1})^2 \delta_{|x-y|,r}}{\sum_{x,y} \delta_{|x-y|,r}} \right\rangle \tag{10}$$

We can compute the renormalized mass and the binding energy by fitting G and $G^{(2)}$ to their asymptotic form:

$$G(r) \propto \frac{e^{-mr}}{r^p}, \quad G^{(2)} \propto \frac{e^{-Mr}}{r^q}, \tag{11}$$

The binding energy is defined to be

$$E_b \equiv 2m - M, \tag{12}$$

If the mass of the scalar field is much lighter than the Planck mass, we can find the binding energy by solving the non-relativistic Schrödinger equation with Newtonian gravitational potential:[4]

$$-\nabla^2 \psi(r,\theta,\phi) + 2\mu \left(-\frac{Gm^2}{r} - E \right) \psi(r,\theta,\phi) = 0, \tag{13}$$

where μ is the reduced mass (in this case $m/2$), we will get the energy levels to be:

$$E_n = \frac{G^2 m^5}{4n^2} \tag{14}$$

Therefore, the relation between the ground state binding energy and particle mass is as follows:

$$E_1 = \frac{G^2 m^5}{4} \tag{15}$$

We expect to see this relation in the continuum, infinite-volume limit for our lattice.

Computation of renormalized mass and binding energy I

Here is what we do to compute the binding energy:

1. Compute the propagator (L_{xy}^{-1}) for different bare masses ($m_0 = 0.001$ to $m_0 = 0.050$).
2. Choose multiple (1, 5, 20 or 60) sources in each configuration to calculate G and $G^{(2)}$
3. Fit the G and $F = G^{(2)}/G^2$ to a logarithmic form: $f(r) = Xr + Y + Z\log(r)$

The parameter X should give us the renormalized mass when we fit G, and binding energy when we fit F.

$$L_{xy} = (D_x + m_0^2) - A_{xy}$$

$$G(r) = \left\langle \frac{\sum_{x,y} L_{xy}^{-1} \delta_{|x-y|,r}}{\sum_{x,y} \delta_{|x-y|,r}} \right\rangle \text{ and } G^{(2)}(r) = \left\langle \frac{\sum_{x,y} (L_{xy}^{-1})^2 \delta_{|x-y|,r}}{\sum_{x,y} \delta_{|x-y|,r}} \right\rangle$$

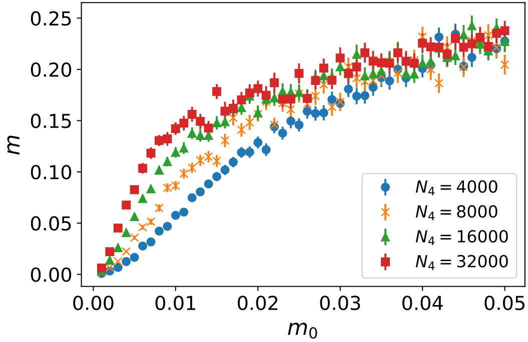

Figure 2: Example of renormalized mass vs bare mass. ($\beta = 0$)

The renormalized mass approaches zero as the bare mass approaches zero. This is in agreement with the requirement by **shift symmetry**, that the mass must be only multiplicatively renormalized.

Mass dependence of the binding energy

We fit the binding energy and renormalized mass to $E(m) = Am^{\alpha}$. $A = G^2/4$ and $\alpha = 5$ is expected in the non-relativistic regime and the continuum, infinite-volume limit.

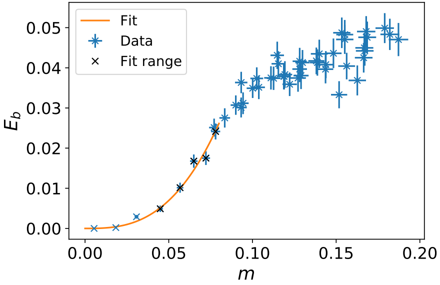

Figure 3: The power-law fit to the binding energy plotted against the renormalized mass for the $N_4 = 16,000$, $\beta = -0.776$ ensemble. The fit range is shown in black, and the solid line is the fit to the data. The fit corresponds to a $\chi^2/\text{d.o.f.} = 0.59$, with a p-value of 0.62.

With G and α from multiple ensembles, we are able to do the continuum, infinite-volume extrapolation.

Overview

We choose to fit the simplest ansatz suggested by finite-size scaling and discretization dependence suggested by symmetries of the theory.

$$\alpha = \frac{H_\alpha}{V} + I_\alpha \ell_{\text{rel}}^2 + \frac{J_\alpha}{V^2} + K_\alpha \ell_{\text{rel}}^4 + L_\alpha \tag{16}$$

and

$$G = \frac{H_G}{V} + I_G \ell_{\text{rel}}^2 + \frac{J_G}{V^2} + K_G \ell_{\text{rel}}^4 + L_G, \tag{17}$$

where H_i, I_i, J_i, K_i, and L_i are fit parameters for their respective quantities.

In addition, we also perform fits dropping the $\sim \ell_{\text{rel}}^4$ term, which we are able to do when we also drop the two coarsest lattice spacing.

Continuum, infinite volume extrapolation II

From the fit, we get a α value of 4.6 ± 0.9.

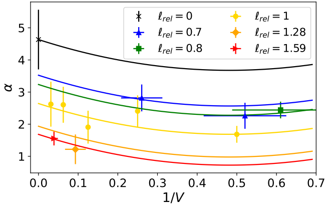

Figure 4: For this fit we find $\chi^2/\text{d.o.f.} = 0.56$ corresponding to a p-value of 0.73, and the continuum, infinite volume value is $\alpha = 4.6(9)$. The physical volume is measured by 1000 4-simplices. For example, an ensemble with 4000 4-simplices has V = 4.

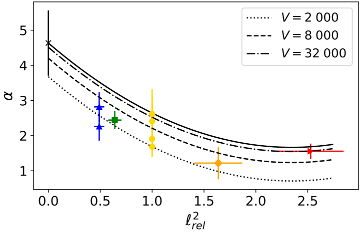

Figure 5: Same data and fit from Figure 4

In $1+1$ dimensions, $E_1 \propto m$. In $2+1$ dimensions, $E_1 \propto m^2$. Taking a simple quadratic fit, we get

$$\alpha = d^2 - 4d + 5$$

Such an α value indicates a dimension between 3.6 and 4.1.

For the gravitational constant on our lattice, we get:

$$G = 15 \pm 5$$

in units of our fiducial lattice spacing.

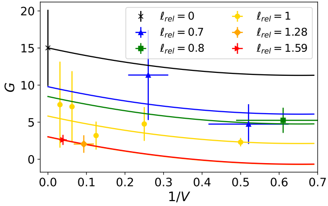

Figure 6: For this fit we find χ^2/d.o.f. = 0.37 corresponding to a p-value of 0.87, and the continuum, infinite volume value is $G = 15(5)$. The physical volume is measured by 1000 4-simplices. For example, an ensemble with 4000 4-simplices has V = 4.

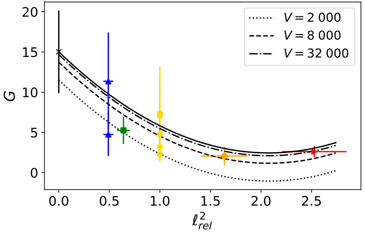

Figure 7: Same data and fit from Figure 6

We can then determine our fiducial lattice spacing ℓ_{fid}, for the first time in EDT studies, to be

$$\sqrt{G} = \ell_{Pl} = (3.9 \pm 0.7)\ell_{\text{fid}} \quad (18)$$

Conclusion

We show numerically that the renormalized scalar mass approaches zero as bare mass approaches zero. And that the relation between binding energy and renormalized mass can be well-described by Newton's potential in the appropriate non-relativistic, classical limit.

We verified that the Newtonian binding on our EDT lattice in the continuum, infinite-volume limit matches Newtonian gravity in the non-relativistic, weak-coupling limit, with $\alpha = 4.6 \pm 0.9$, indicating a dimension between 3.6 and 4.1.

We obtain a value of $G = 15 \pm 5$ (in units of our fiducial lattice spacing) so that we are able to relate our lattice spacing to the Planck length. In particular, our fiducial lattice spacing is related to the Planck length as:

$$\sqrt{G} = \ell_{Pl} = (3.9 \pm 0.7)\ell_{\text{fid}} \tag{19}$$

References

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