The de Sitter Instanton from Euclidean Dynamical Triangulations

Lattice 2021

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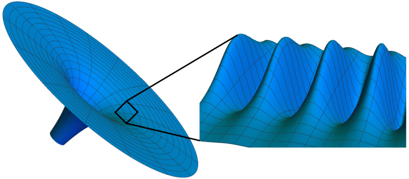
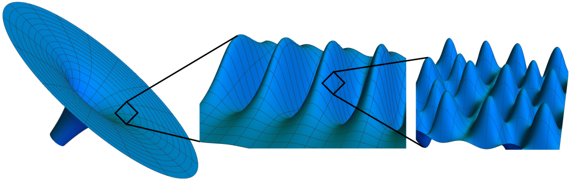
In collaboration with S. Bassler, J. Laiho, and J. Unmuth-Yockey,

based on **Phys.Rev.D 103 (2021) 114504**.

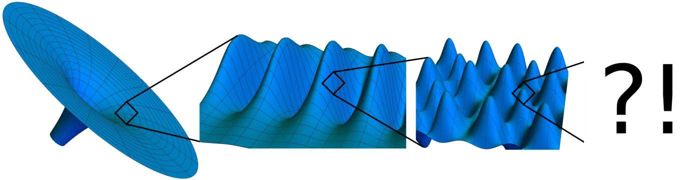
Asymptotically Safe Quantum Gravity





- Perturbative quantum gravity:

Asymptotically Safe Quantum Gravity



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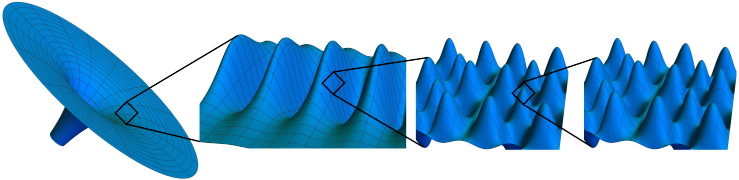

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  Quantum realization of
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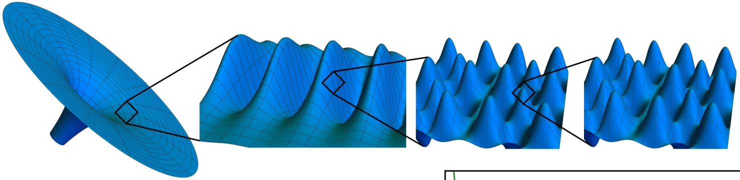
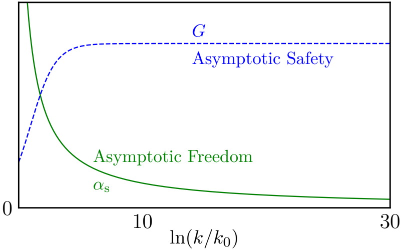


- Perturbative quantum gravity:
  loss of predictivity
- Key idea of asymptotic safety:
  Quantum realization of scale symmetry
  - imposes infinitely many conditions on theory space
  - relevant directions: need measurement
  - irrelevant directions: predictions of theory



$$k\partial_k \alpha_s = -\frac{11}{2\pi}\alpha_s^2 + \mathcal{O}(\alpha_s^4)$$

$$k\partial_k G = \epsilon G - \frac{50}{3}G^2 + \mathcal{O}(G^3)$$

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with Euclidean Einstein-Regge action

$$S_{\mathrm{E}} = -\kappa_2 N_2 + \kappa_4 N_4$$

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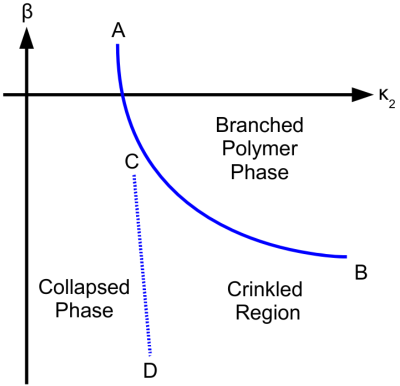

[Ambjorn, Glaser, Goerlich, Jurkiewicz, 2013]
[Coumbe, Laiho, 2014]

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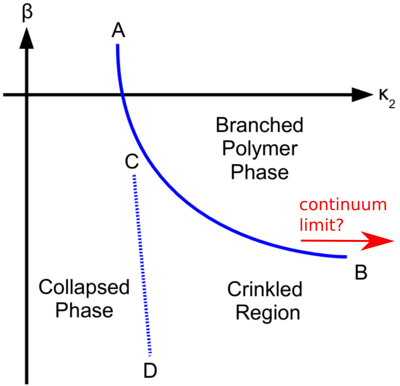

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- Power counting:

$$\frac{1}{16\pi G} \int \mathrm{d}^4 x\, R \sim \frac{\sqrt{V}}{G}\,,$$

  and therefore

$$f(N_4, \kappa_2) = e^{k(\kappa_2)\sqrt{N_4}}\,.$$

The Saddle point approximation in EDT II

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  [Ambjorn, Goerlich, Jurkiewicz, Loll, 2012]

$$\langle N_4 \rangle \simeq \frac{k^2}{4(\kappa_4 - \kappa_4^c)^2} \Rightarrow k = |\kappa_4 - \kappa_4^c| \sqrt{N_4} \, .$$

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  Assumption: Continuum is dominated by de Sitter instanton [Hawking, Moss, 1987]
  Extract G from lattice data:

$$\frac{G}{\ell_{\text{fid}}^2} \sim \left(\frac{a}{\ell}\right)^2 \frac{\ell_{\text{rel}}^2}{|s|} \,,$$

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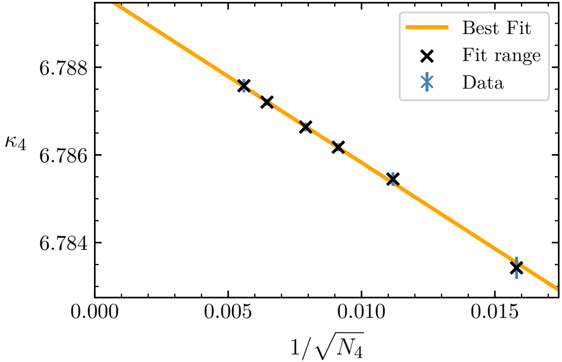
Numerical result: finite volume scaling

Example at $\beta = -0.6, \ \kappa_2 = 2.245$:



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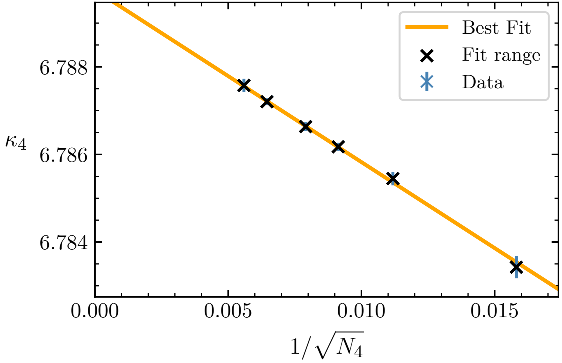

Extract slope s: $\kappa_4(N_4) = A + s\frac{1}{\sqrt{N_4}}$

Fit result: $s = -0.393 \pm 0.022$

$\chi^2/\text{d.o.f} = 0.15$, p-value: 0.96.

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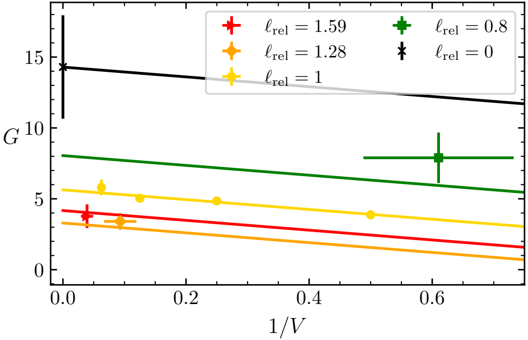

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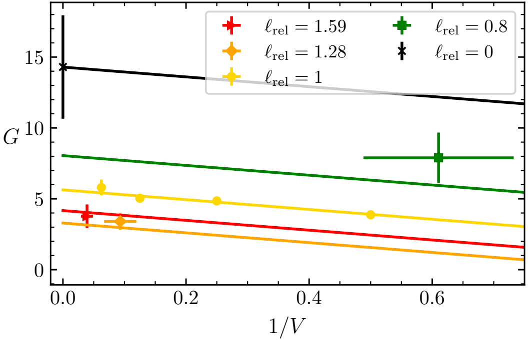

Continuum, infinite volume limit: $G = 14.3 \pm 3.6$

$\chi^2/\text{d.o.f} = 0.87$, p-value: 0.46.

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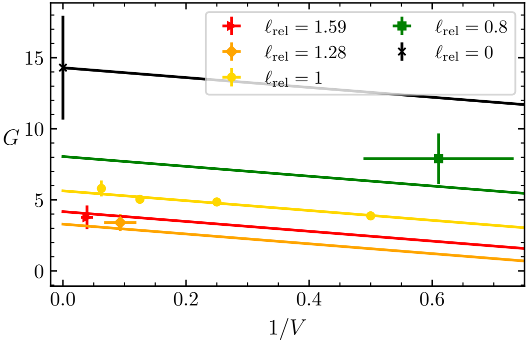

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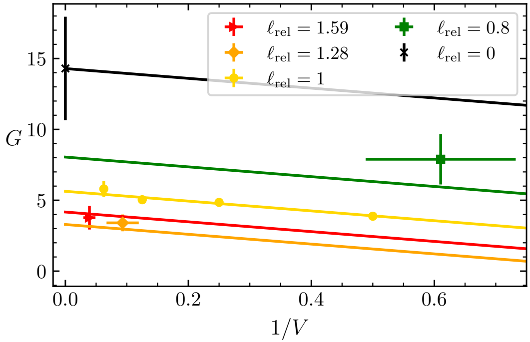

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Thank you for your attention!

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ℓ_{rel}	V	β	κ_2	$\|s\|$	$\chi^2/\mathrm{d.o.f.}$	p-value
1.59(10)	25.6(6.4)	1.5	0.5886	0.724(32)	1.4	0.24
1.28(9)	10.7(3.0)	0.8	1.032	0.6840(55)	0.35	0.79
1	2.0(0)	0	1.605	0.652(14)	0.60	0.62
1	4.0(0)	0	1.669	0.521(11)	1.4	0.24
1	8.0(0)	0	1.7024	0.502(12)	0.43	0.65
1	16.0(0)	0	1.7325	0.436(39)	0.76	0.38
0.80(4)	1.64(32)	-0.6	2.45	0.393(22)	0.15	0.96