Testing dilaton potentials for near conformal gauge theories

Chik Him (Ricky) Wong[1]

on behalf of
Lattice Higgs Collaboration ($\mathrm{L_{at}HC}$)
Zoltan Fodor[1], Kieran Holland[2], Julius Kuti[3]

1: University of Wuppertal
2: University of the Pacific
3: UC, San Diego

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Outline

- Dilaton Effective Field Theory
- Tests on nearly conformal gauge theories
  - SU(3) with $N_f = 2$ fermions in two-index symmetric (Sextet) representation
  - SU(3) with $N_f = 8$ fermions in Fundamental representation
- Conclusion

Dilaton Effective Field Theory

- Near conformal gauge field theories are candidates of beyond standard model with emergent composite Higgs boson (0^{++} scalar)
- QCD χPT cannot work properly in the chiral limit: As conformal window is approached, light 0^{++} scalar adds extra degree of freedom
- Dilaton hypothesis: The scalar acts as a dilaton from scale symmetry breaking
- Lagrangian of Dilaton Effective Field Theory:

$$L = \frac{1}{2}\,\partial_\mu\chi\,\partial_\mu\chi - V(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu\,\Sigma^\dagger\,\partial_\mu\Sigma\right] + \frac{f_\pi^2\,m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left(\Sigma+\Sigma^\dagger\right)$$

- $y = 3-\gamma,\ \ \gamma$: mass anomalous dimension
- $\chi(x) = f_d\, e^{\,\sigma(x)\,/f_d}$, $\sigma(x)$: Dilaton field
- $\Sigma = e^{\,i\,\pi^a\,\tau^a\,/f_\pi}$: τ^a = Pauli matrices, $m_\pi^2 = 2B_\pi m$

- Two representative forms of V are possible
  - V_d: Deformation of CFT parametrically

    [e.g. M. Golterman and Y. Shamir,Phys. Rev. D94 (2016) 054502]
  - V_σ: Linear-σ model inspired potential

    [T. Appelquist et al, JHEP 07 (2017) 035; T. Appelquist et al, JHEP 07 (2018) 039;W. D. Goldberger et al, Phys. Rev. Lett. 100 (2008) 111802]

$$V_d = \frac{m_d^2}{16 f_d^2}\, \chi^4 \left(4 \ln \frac{\chi}{f_d} - 1 \right), \quad V_\sigma = \frac{m_d^2}{8 f_d^2} \left(\chi^2 - f_d^2 \right)^2$$

- Our previous study on the models cannot distinguish the two

  [PoS LATTICE2019 (2020) 246]
- A parameterized version interpolating between the two limits was proposed
  [T. Appelquist et al, Phys. Rev. D 101, 075025 (2020); Z. Chacko & R. K. Mishra, Phys. Rev. D 87, no. 11, 115006 (2013)]

$$V_\Delta(\chi) \equiv \frac{m_d^2\, \chi^4}{4(4-\Delta)\, f_d^2} \left[1 - \frac{4}{\Delta} \left(\frac{\chi}{f_d} \right)^{\Delta-4} \right],$$

$$\lim_{\Delta \to 4} V_\Delta = V_d, \quad \lim_{\Delta \to 2} V_\Delta = V_\sigma \text{ (up to an irrelevant additive constant)}$$

- It is suggested that near conformal models can be described by the EFT with some values of Δ

  [M. Golterman and Y. Shamir, Phys. Rev. D 102, 114507 (2020); T. Appelquist et al, Phys. Rev. D 101, 075025 (2020)]

Tests on near conformal gauge theories

Task: Test the dilaton effective theory against data from two near conformal theories

- **SU(3) with $N_f = 2$ fermions in two-index symmetric (Sextet) representation**
  - "Sextet" dataset:
    - Gauge action: tree-level Symanzik-improved
    - Fermion action: 2-step $\rho = 0.15$ stout-smeared staggered fermion in sextet representation
- **SU(3) with $N_f = 8$ fermions in Fundamental representation**
  - "LSD" dataset:
    [LSD Collaboration, Phys. Rev. D 99, 014509 (2019)]
    - Gauge action: fundamental and adjoint plaquette terms with couplings $\beta_A/\beta_F = -0.25$
    - Fermion action: improved nHYP-smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$
  - "KMI" dataset:
    [LatKMI collaboration, Phys. Rev. D 96, 014508 (2017)]
    - Gauge action: tree-level Symanzik-improved
    - Fermion action: Highly Improved Staggered Quarks (HISQ) action
    - In a higher fermion mass range than "LSD" dataset

Compare with previous similar studies of other groups
[T. Appelquist et al; M. Golterman et al]

Tests on near conformal gauge theories

- Tree level predictions :
  - V-independent scaling relation : $0 = \left(\frac{F_\pi}{f_\pi}\right)^{(\gamma-1)} M_\pi^2 - 2\, B_\pi\, m$
  - Expanding: $\frac{f_\pi^2\, m_\pi^2}{4} \left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left(\Sigma + \Sigma^\dagger\right) = \frac{N_f\, m_\pi^2\, f_\pi^2}{2} \left(\frac{\chi}{f_d}\right)^y + \dots ,$

    One can define:

    $W(\chi) \equiv V_\Delta(\chi) - (N_f\, m_\pi^2 f_\pi^2\, /\, 2)(\chi\, /\, f_d)^y ,$

    requiring $W'(F_d) = 0,\ W''(F_d) = M_d^2$: (Defining $\delta \equiv 4 - \Delta$)

$$\delta \neq 0:\ 0 = \left(\frac{M_\pi}{F_\pi}\right)^2 (3-\gamma)\, N_f\, \delta - 2\left(\frac{m_d}{f_\pi}\frac{f_d}{f_\pi}\right)^2 \left(1 - \left(\frac{f_\pi}{F_\pi}\right)^\delta\right),$$

$$0 = \left(\frac{M_d}{f_\pi}\frac{f_\pi}{m_d}\right)^2 \delta - \left(1 + \gamma + (\delta - 1 - \gamma)\left(\frac{f_\pi}{F_\pi}\right)^\delta\right)$$

$$\delta = 0:\ 0 = F_\pi^{(1+\gamma)} \ln\left(\frac{F_\pi}{f_\pi}\right)\left(\frac{f_d}{f_\pi}\frac{m_d}{f_\pi}\right)^2 - (3-\gamma)\, m\, N_f\, B_\pi\, f_\pi^{(\gamma-1)},$$

$$0 = \left(\frac{F_\pi}{M_\pi}\right)^2 \left(3\, \ln\left(\frac{F_\pi}{f_\pi}\right) + 1\right) 2 \left(\frac{f_d}{f_\pi}\frac{m_d}{f_\pi}\right)^2 - 2\left(\frac{f_d}{f_\pi}\frac{M_d}{M_\pi}\right)^2 - (2-\gamma)\,(3-\gamma)\, N_f$$

$M_\pi(m), F_\pi(m), M_d(m)$: pion masses, pion decay constants and 0^{++} masses at m

- Inputs at each m :
  $X^{\rm data}(m) \equiv \{M_\pi, F_\pi, M_d\}|_{\rm data}(m)$ measured at the largest volumes available (infinite volume limit is taken if possible) with their corresponding variances $\sigma^2(m)$
- Parameters: $\theta \equiv \{f_\pi,\ B_\pi,\ \gamma,\ m_d/f_\pi,\ f_d/f_\pi\}$($\delta$ is hold fixed)
- In our previous studies for the cases at fixed $\delta = 0$ and 2 , the tree-level predictions were treated as constraints of θ and the fits were done by implicit maximum likelihood estimate (IMLE)
- Accidentally, the dimension of X is the same as the number of constraints $\Rightarrow X^\theta(m) \equiv \{M_\pi, F_\pi, M_d\}|_\theta(m)$ can indeed be obtained by direct inversion
- $\Rightarrow$ simplifies to ordinary MLE
- θ is fitted by minimizing the weighted χ^2:

$$\chi^2 \equiv \sum_{i,\ m} \frac{\left(X_i^{\rm data}(m) - X_i^\theta(m)\right)^2}{\sigma_i^2(m)}$$

- Analysis is done by scanning δ between -0.2 and 2. In addition, fits with δ included as the sixth parameter are also performed wherever possible.

- Tree-level predictions on taste breaking pattern :

[M. Golterman et al,Phys.Rev.D 102, 034515 (2020)]

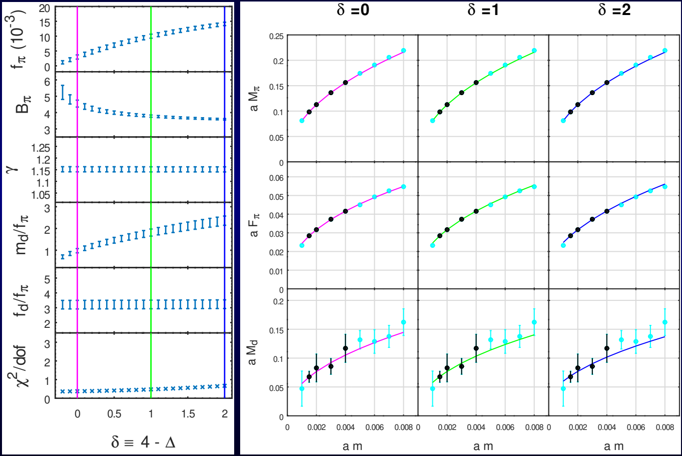
$$\begin{aligned}
\Delta(\Gamma_5) \equiv \Delta_P &= 0 ,\\
\Delta(\Gamma_{\mu 5}) \equiv \Delta_A &= C_1 E(\gamma_1) + 3C_3 E(\gamma_3) + C_4 E(\gamma_4) + 3C_6 E(\gamma_6),\\
\Delta(\Gamma_{\mu\nu}) \equiv \Delta_T &= 2C_3 E(\gamma_3) + 2C_4 E(\gamma_4) + 4C_6 E(\gamma_6),\\
\Delta(\Gamma_{\mu}) \equiv \Delta_V &= C_1 E(\gamma_1) + C_3 E(\gamma_3) + 3C_4 E(\gamma_4) + 3C_6 E(\gamma_6),\\
\Delta(\Gamma_I) \equiv \Delta_S &= 4C_3 E(\gamma_3) + 4C_4 E(\gamma_4)
\end{aligned}$$

where $\Delta(\Gamma_i) \equiv a^2(M^2_{\Gamma_i} - M^2_\pi)$, $E(\gamma_i) \equiv \left(\frac{F_\pi}{f_\pi}\right)^{4-\gamma_i}$

- X extends to $X(m) \equiv \{M_\pi, F_\pi, M_d, \Delta_A, \Delta_T, \Delta_V, \Delta_S\}(m)$
- θ extends to
  $\theta \equiv \{f_\pi,\ B_\pi,\ \gamma,\ m_d/f_\pi,\ f_d/f_\pi, \gamma_1, \gamma_3, \gamma_4, \gamma_6, C_1, C_3, C_4, C_6\}$
- Dimension of X is still the same as number of constraints $\Rightarrow$ inversion is still doable

SU(3) with $N_f = 2$ fermions in Sextet representation

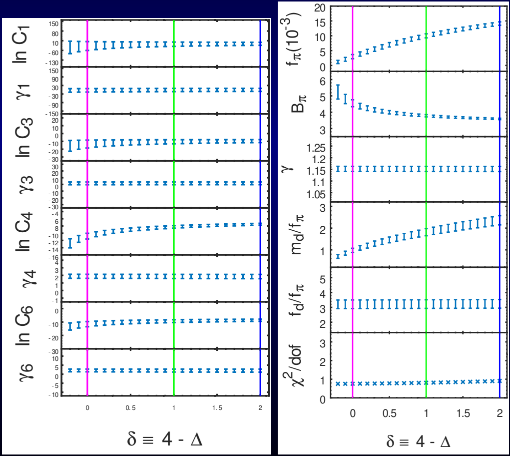
- "Sextet" dataset $\beta = 3.20$



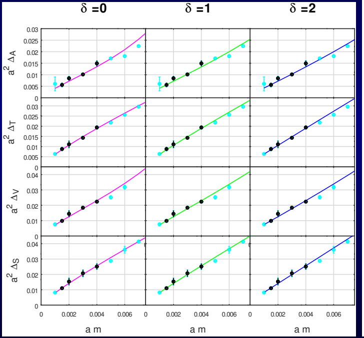
- Consistent with broad range of δ, with wide range of parameter values

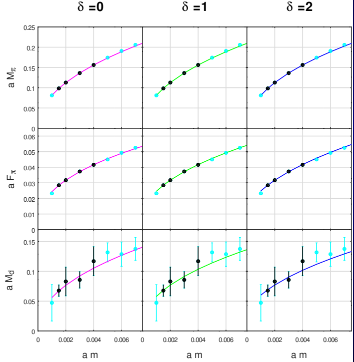
SU(3) with $N_f = 2$ fermions in Sextet representation

- Taste-breaking effects can be taken into account, and the parameter values stay the same

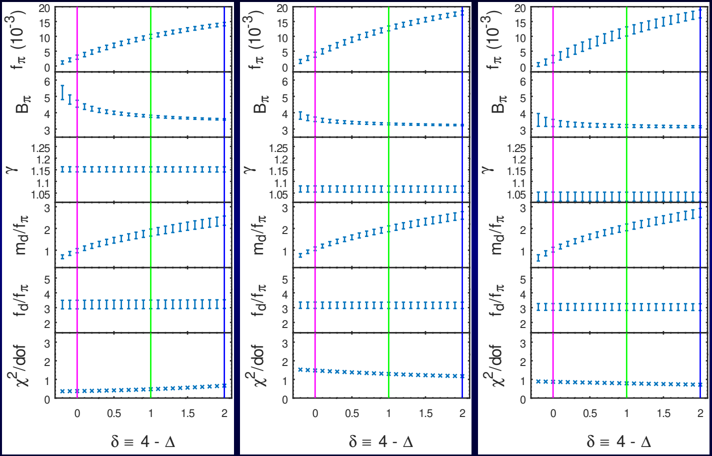


SU(3) with $N_f = 2$ fermions in Sextet representation





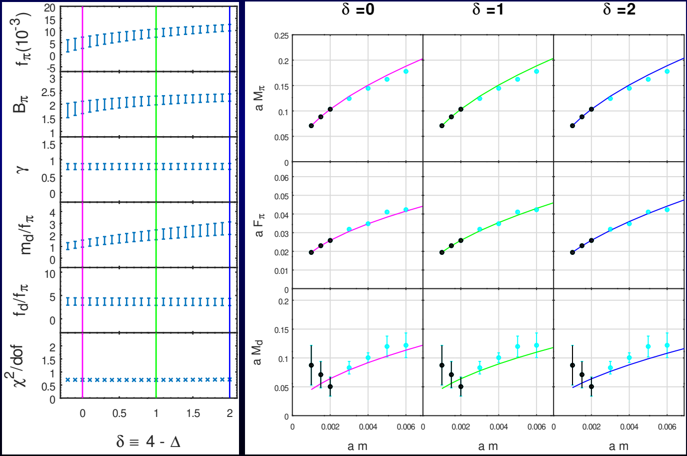
- Similar results in different m-ranges, while γ seems to drift slowly



$m = 0.0015 - 0.004$ $m = 0.003 - 0.007$ $m = 0.004 - 0.007$

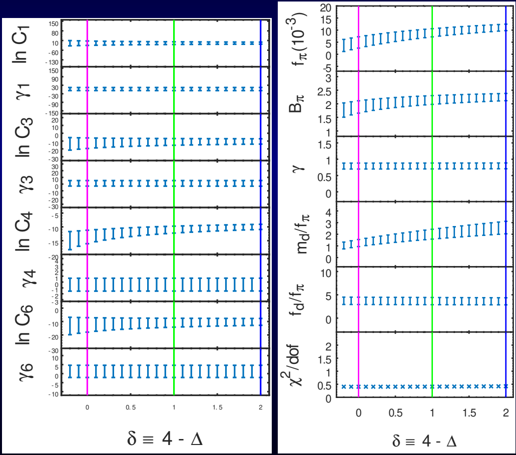
SU(3) with $N_f = 2$ fermions in Sextet representation

- "Sextet" dataset $\beta = 3.25$
- Similar to $\beta = 3.20$

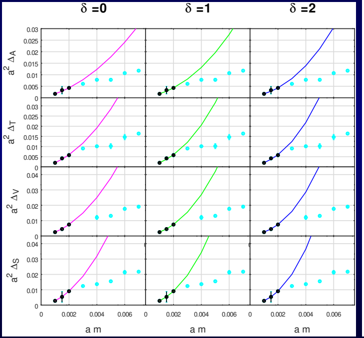


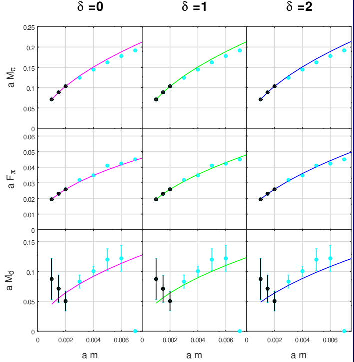
SU(3) with $N_f = 2$ fermions in Sextet representation

- Fits including taste-breaking effects barely works



SU(3) with $N_f = 2$ fermions in Sextet representation





SU(3) with $N_f = 8$ fermions in Fundamental representation

- For comparison with results from other groups, θ is redefined with other combinations of the parameters as follows:

$$d_0 \equiv 2\, B_\pi\, f_\pi^{\gamma-1}$$

$$d_1 \equiv \frac{4(3-\gamma)}{(m_d/f_\pi)^2\ (f_d/f_\pi)^2}$$

$$d_2 \equiv \frac{f_\pi^2}{2\, B_\pi}$$

$$d_3 \equiv (m_d/f_\pi)^2$$

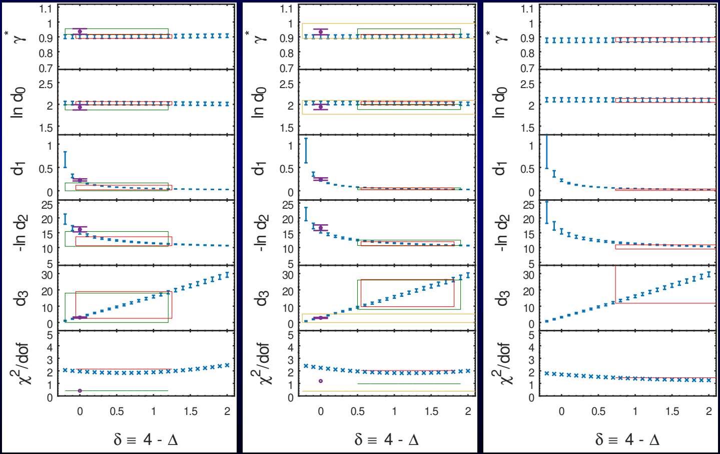
- “LSD” dataset: Consistent with a broad range of δ values
- γ does not change significantly in different m ranges

red: Fit with δ as the sixth parameter

green: M. Golterman and Y. Shamir, Phys. Rev. D 102, 114507 (2020)

orange: T. Appelquist et al, Phys. Rev. D 101, 075025 (2020)

violet: M. Golterman et al,Phys.Rev.D 102, 034515 (2020)

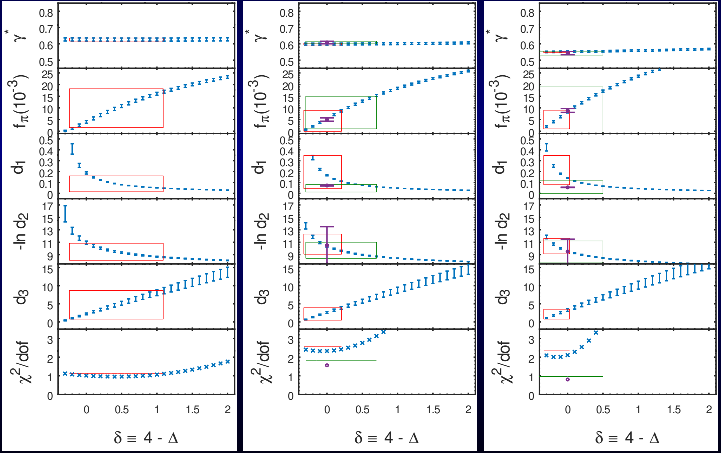


$m = 0.00125 - 0.0075$ $\quad m = 0.00125 - 0.00889$ $\quad m = 0.00222 - 0.00889$

- "KMI" dataset: Consistent with a broad range of δ values
- γ seems to depend on m range, but fits fail for most m ranges except some:

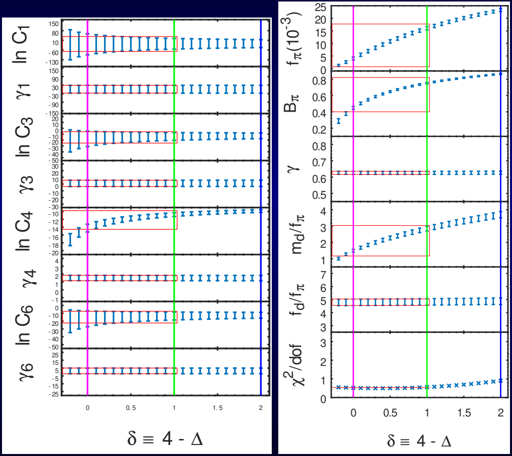
  red: Fit with δ as the sixth parameter

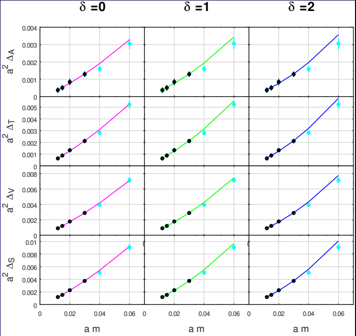
  green,violet: M. Golterman and Y. Shamir, Phys. Rev. D 102, 114507 (2020)

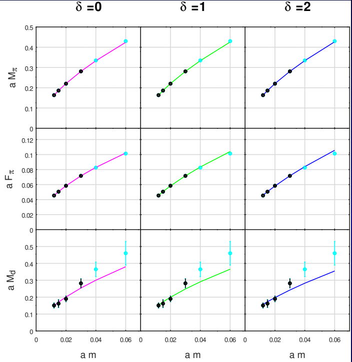


$m = 0.012 - 0.03$ $\quad$ $m = 0.012 - 0.04$ $\quad$ $m = 0.02 - 0.06$

- Taste breaking effects can be included into the fit for $m = 0.012 - 0.03$
  red: Fit with δ as the sixth parameter

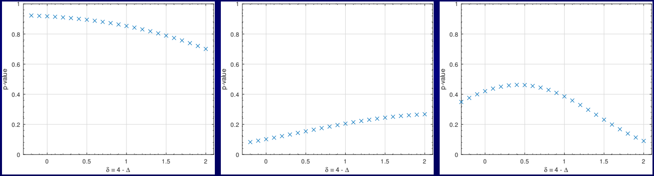



δ =0
δ =1
δ =2
$a^2 \Delta_A$
$a^2 \Delta_T$
$a^2 \Delta_V$
$a^2 \Delta_S$
a m


δ =0
δ =1
δ =2
a M_π
a F_π
a M_d
a m

Conclusion

- The dilaton EFT is tested against two near conformal theories using three datasets:
  - SU(3) gauge theory with $N_f = 2$ in sextet representation
  - SU(3) gauge theory with $N_f = 8$ in fundamental representation
- All datasets are consistent with a broad range of δ values. More data is needed in order to narrow it down.



"Sextet" "LSD" "KMI"

- Possible improvement:

  [M. Golterman et al, Phys. Rev. D 102, 114507 (2020), Phys. Rev. D 102,034515 (2020)]

  It is suggested that due to hyperscaling, γ becomes m dependent as m increases
- There are more ongoing works