A new phase in the Lorentzian type IIB matrix model and the emergence of continuous space-time

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Based on the collaboration with
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superstring theory

- promising candidate for the quantum gravity
- consistent only in 10D space-time

- How to describe our 4D space-time in superstring theory
  - compactification
    - ✳ Size of the extra 6d space is significantly small.

      Low-energy physics in 4D depends on the structure of the extra dimension.
    - ✳ # of perturbatively stable vacua is extremely large.

      It is difficult to choose a unique vacuum which describes our 4D space-time by the perturbative analysis.

Lorentzian type IIB matrix model

- A promising candidates for non-perturbative definition of superstring theory

- Previous works using Monte Carlo simulation
  - SSB: SO(9) → SO(3)

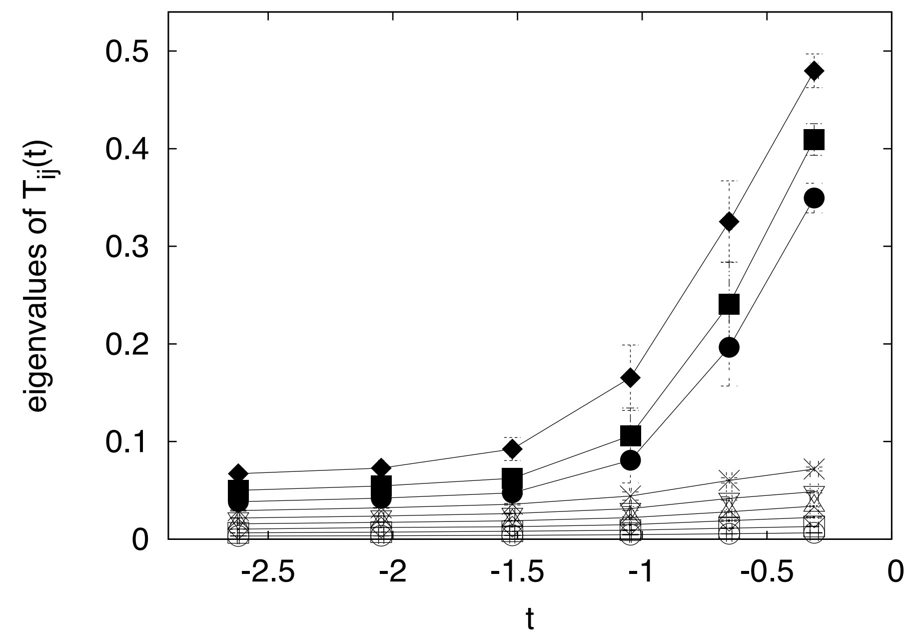
    Kim, Nishimura, Tsuchiya ('12)

  - expansion of the 3d space
    - exponential expansion at early times

      Ito, Kim, Nishimura, Tsuchiya ('13)

    - power law expansion at late times

      Ito, Nishimura, Tsuchiya ('15)



Lorentzian type IIB matrix model

- Previous works using Monte Carlo simulation (cont'd)
  - structure of the 3d space — Aoki, Hirasawa, Ito, Nishimura, Tsuchiya ('19)
    - not continuous — ← due to the approximation?

An approximation for the partition function was used to avoid the sign problem.

Complex Langevin method can be used to overcome the sign problem. Nishimura, Tsuchiya ('19)

In this study, we have found a new phase, which allows us to simulate the Lorentzian case directly.

- Continuous space-time structure appears.
- The SSB is not observed yet.
- We find an equivalence between the Euclidean and Lorentzian models. (Hatakeyama's talk)

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Definition of the Lorentzian type IIB matrix model

- Action

$$S = \underbrace{-\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu]\right)}_{S_b} \underbrace{-\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right)}_{S_f}$$

A_μ : 10d Lorentz vector
Ψ_α : 10d Majorana-Weyl spinor
} $N \times N$ Hermitian matrices

- Partition function

$$Z = \int dA\, e^{iS_\mathrm{b}}\, \mathrm{Pf}\mathcal{M}(A)$$

phase factor

We introduce two cutoffs to make the integral converge.

$$\frac{1}{N}\mathrm{Tr}\,(A_0)^2 = \kappa, \quad \frac{1}{N}\mathrm{Tr}\,(A_i)^2 = 1$$

Kim, Nishimura, Tsuchiya ('11)

We can not apply usual Monte Carlo method due to the sign problem.

→ We use the complex Langevin Method.

Generalization of the model

We focus on the bosonic model ($\mathrm{Pf}\mathscr{M} = 1$).

$$S_{\mathrm{b}} = N\beta \left\{ -\frac{1}{2} \mathrm{Tr} \left(F_{0i} \right)^2 + \frac{1}{4} \mathrm{Tr} \left(F_{ij} \right)^2 \right\}$$

$$Z = \int dA e^{iS_{\mathrm{b}}}$$

$\beta = \frac{1}{g^2 N}$

$$F_{\mu\nu} = i \left[A_\mu, A_\nu \right]$$

↓ Wick rotation

s : a parameter of the Wick rotation on the world sheet

k : a parameter of the Wick rotation in the target space

$$\tilde{S} = -iN\beta \mathrm{e}^{is\pi/2} \left\{ -\frac{1}{2} \mathrm{e}^{-ik\pi} \mathrm{Tr} \left(F_{0i} \right)^2 + \frac{1}{4} \mathrm{Tr} \left(F_{ij} \right)^2 \right\}$$

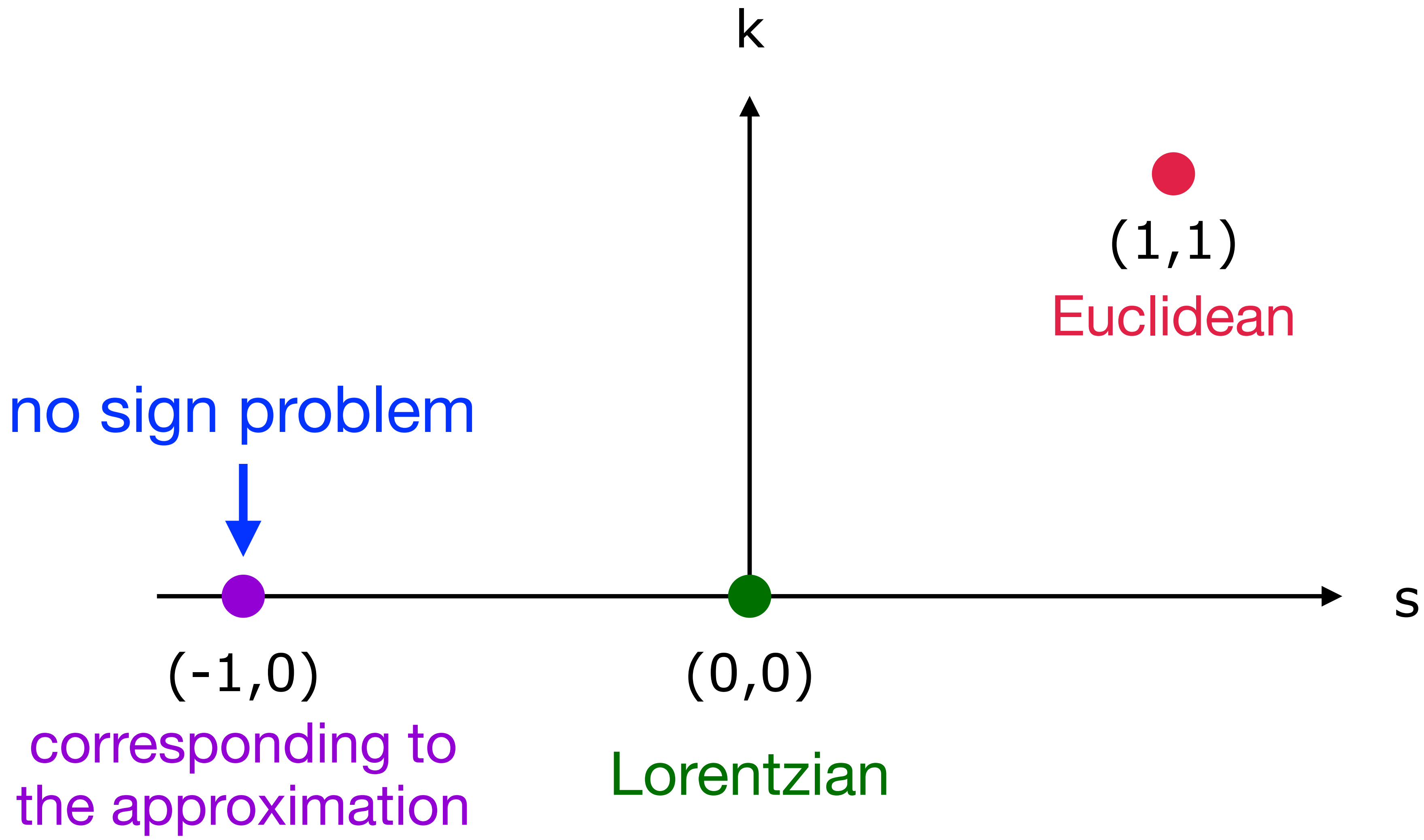
$$Z = \int dA e^{-\tilde{S}}$$

$s = k = 0$ corresponds to the Lorentzian type IIB matrix model.

The approximation which was used in the previous works corresponds to setting $(s, k) = (-1, 0)$.

Aoki, Hirasawa, Ito, Nishimura, Tsuchiya ('19)

parameter for the Wick rotation



How to extract the time-evolution

We choose an SU(N) basis which diagonalizes A_0.

$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, ..., \alpha_N)$$

$$\alpha_1 < \alpha_2 < ... < \alpha_N$$

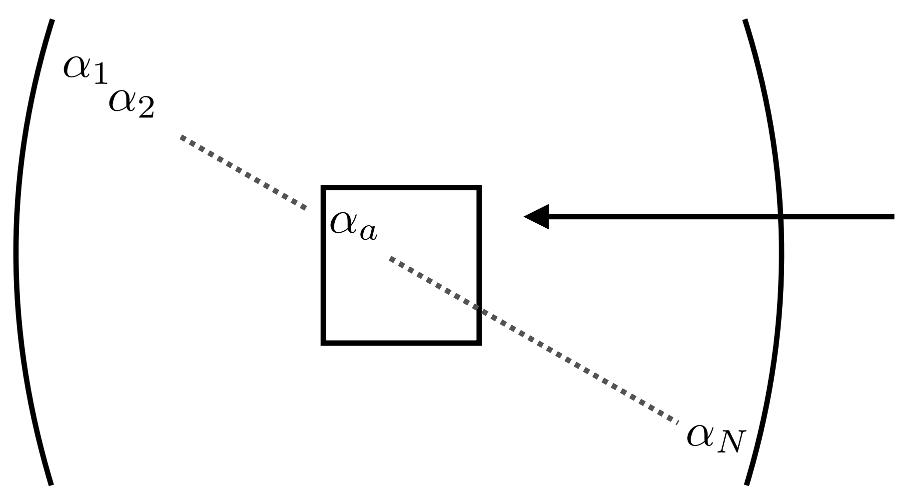
This ordering is realized by the change of variables:

$$\alpha_1 = 0, \quad \alpha_2 = \mathrm{e}^{\tau_1}, \quad \alpha_3 = \mathrm{e}^{\tau_1} + \mathrm{e}^{\tau_2}, \quad \cdots, \quad \alpha_N = \sum_{a=1}^{N-1} \mathrm{e}^{\tau_a}$$

Nishimura, Tsuchiya ('19)

band diagonal structure (dynamical property)

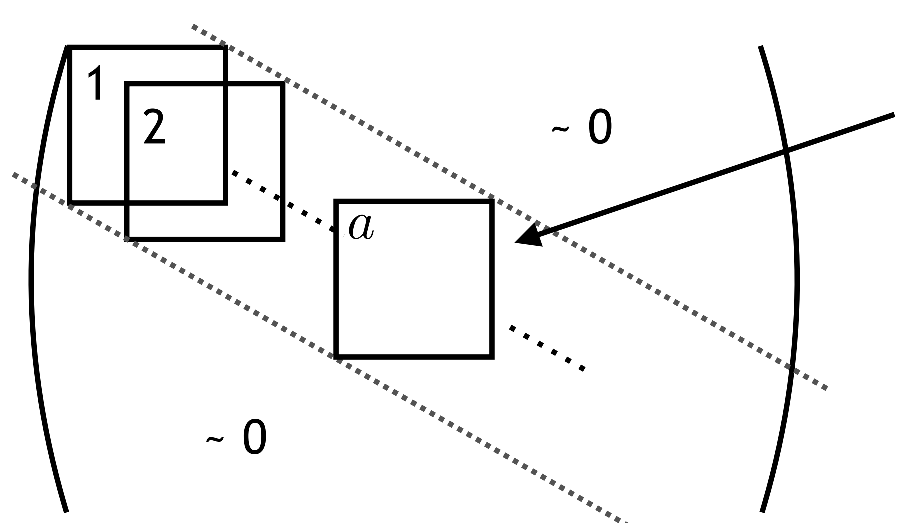
$A_0 =$



We define the time t_a as:

$$t_a \equiv \frac{1}{n} \sum_{i=0}^{n-1} \alpha_{a+i}$$

$A_i =$



$\bar{A}_i(t)$: $n \times n$ matrix

This matrix describes the structure of the space at time t_a .

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Review of complex Langevin method

Parisi ('83), Klauder ('84)

$$Z = \int dx \; \underline{w(x)} \qquad\qquad x \in \mathbb{R}$$

complex-valued function

Usual Monte Carlo method is not applicable due to the sign problem.

- complexify the variables

$$x \in \mathbb{R} \longrightarrow z \in \mathbb{C}$$

complex Langevin equation

t : Langevin time

$$\frac{dz_i}{dt} = \underline{\frac{1}{w}\frac{\partial w}{\partial z_i}} + \eta_i(t)$$

drift term

← Gaussian noise

$$\mathrm{P}\left(\eta_i(t)\right) \propto \exp\left(-\frac{1}{4}\int dt \sum_i \{\eta_i(t)\}^2\right)$$

—A criterion for the correct convergence—

The drift histogram falls off exponentially or faster with the magnitude of the drift term.

Nagata, Nishimura, Shimasaki ('16)

We apply this method to the type IIB matrix model.

contents

- introduction
- definition of the Lorentzian type IIB matrix model
- complex Langevin method
- results
- summary and future work

Result for the bosonic model at $(s, k) = (-1, 0)$

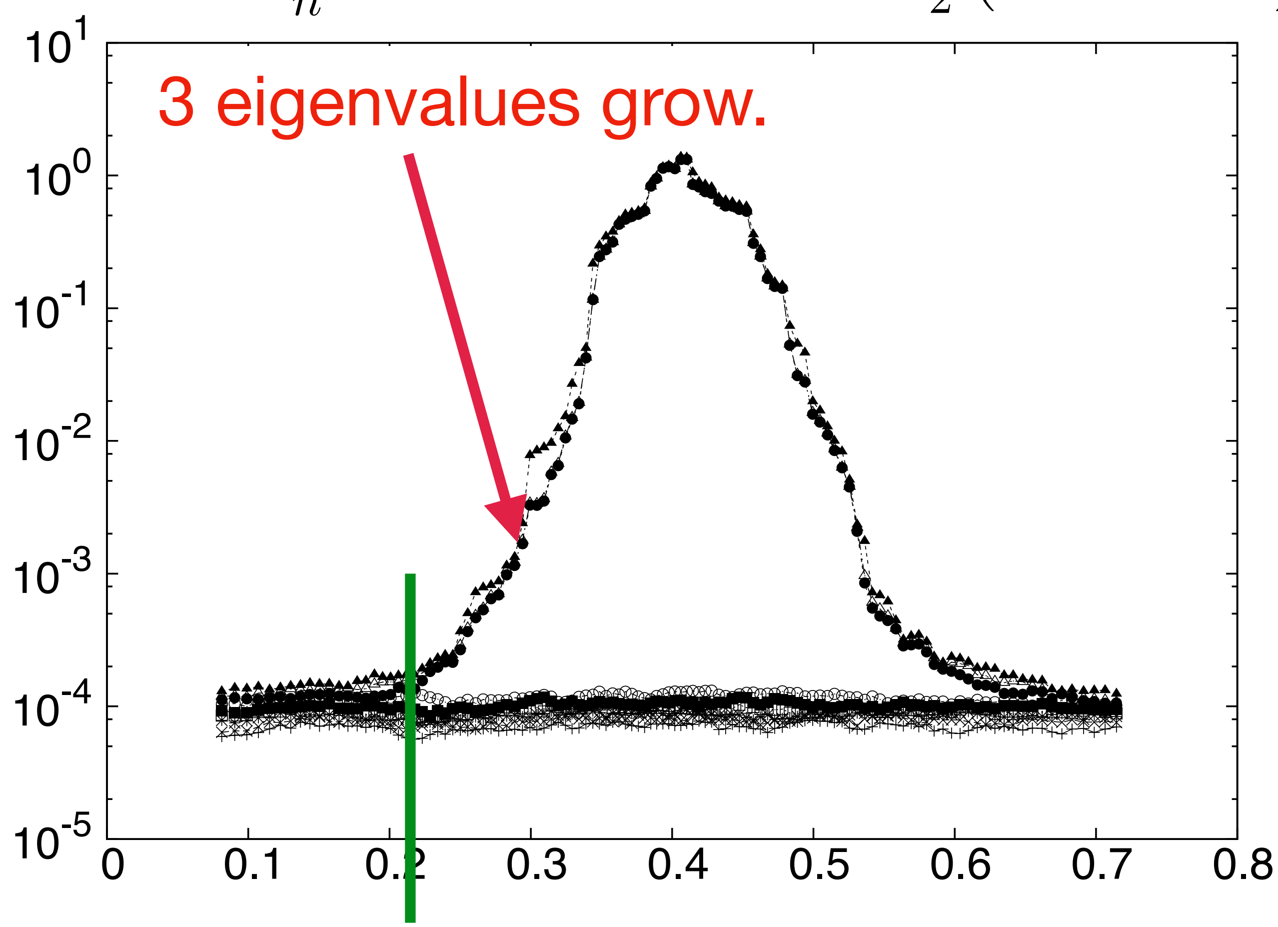
N=128, β=8.0, κ=0.04, n=32

corresponds to the approximation

an order parameter for SSB of SO(9)

The eigenvalues of "moment of inertia tensor"

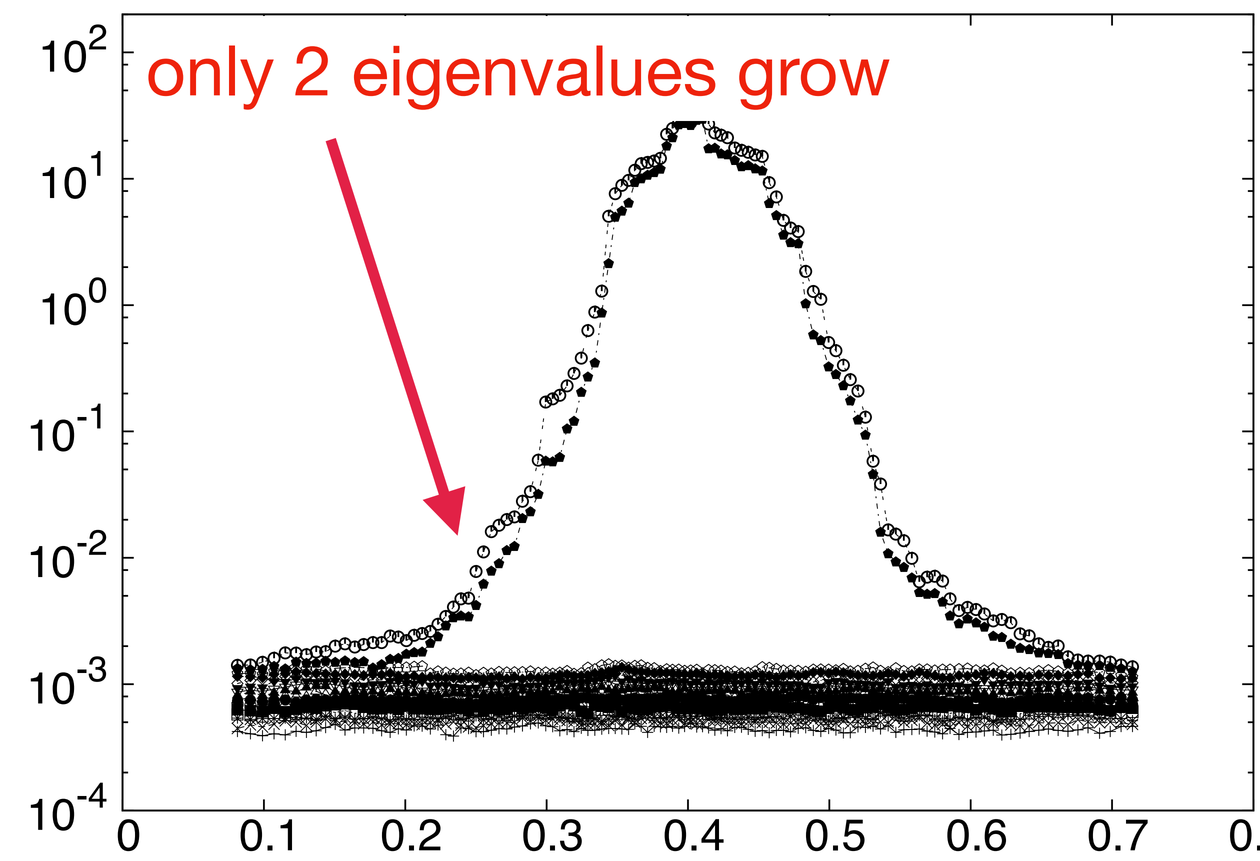
$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\big(X_i(t)X_j(t)\big) \qquad X_i(t) \equiv \frac{1}{2}\left(\bar{A}_i(t) + \bar{A}_i^\dagger(t)\right)$$



expansion of 3d space

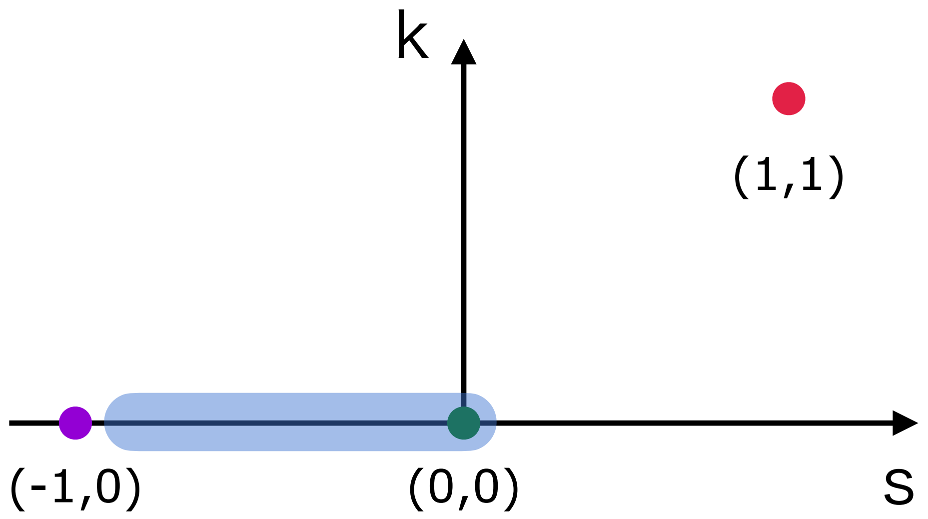
The observables which describe how space spreads in radial direction

eigenvalues of $Q(t) = \sum_{i=1}^{9} (X_i(t))^2$



The space is not continuous.

results for various s

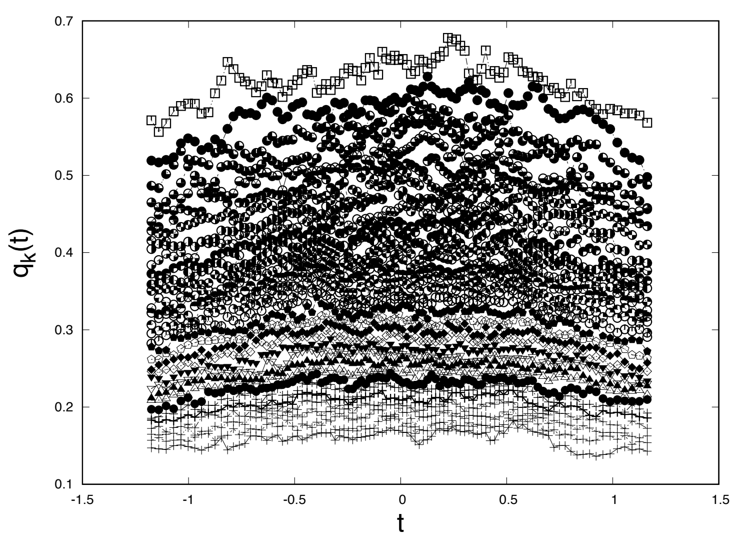


The observables which describe how space spreads in radial direction

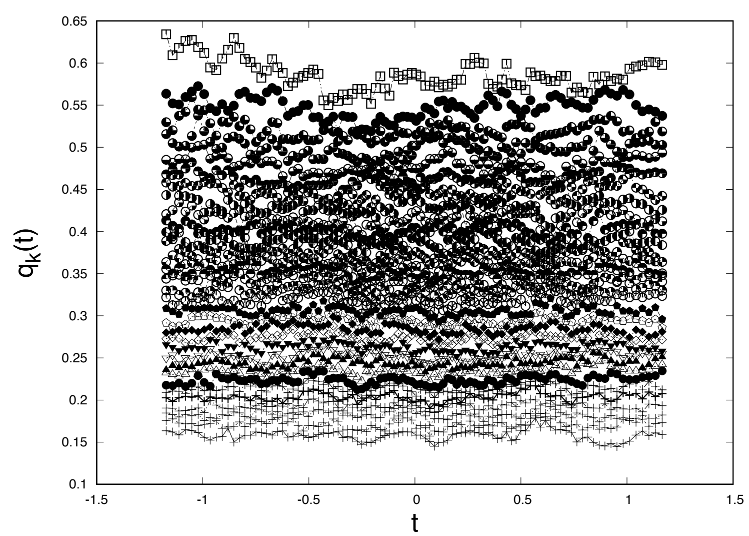
eigenvalues of $Q(t) = \sum_{i=1}^{9} (X_i(t))^2$

N=128, k=0, β=2.5, κ=0.8, n=32

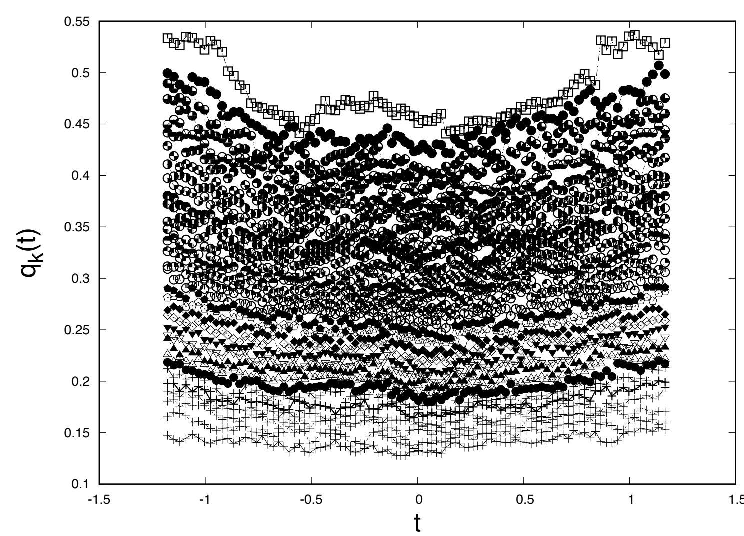
New phase appears.



s=-0.8



s=-0.6



s=0

the Lorentzian model

results for the Lorentzian bosonic model

N=128, s=k=0, β=2.5, κ=0.8, n=32

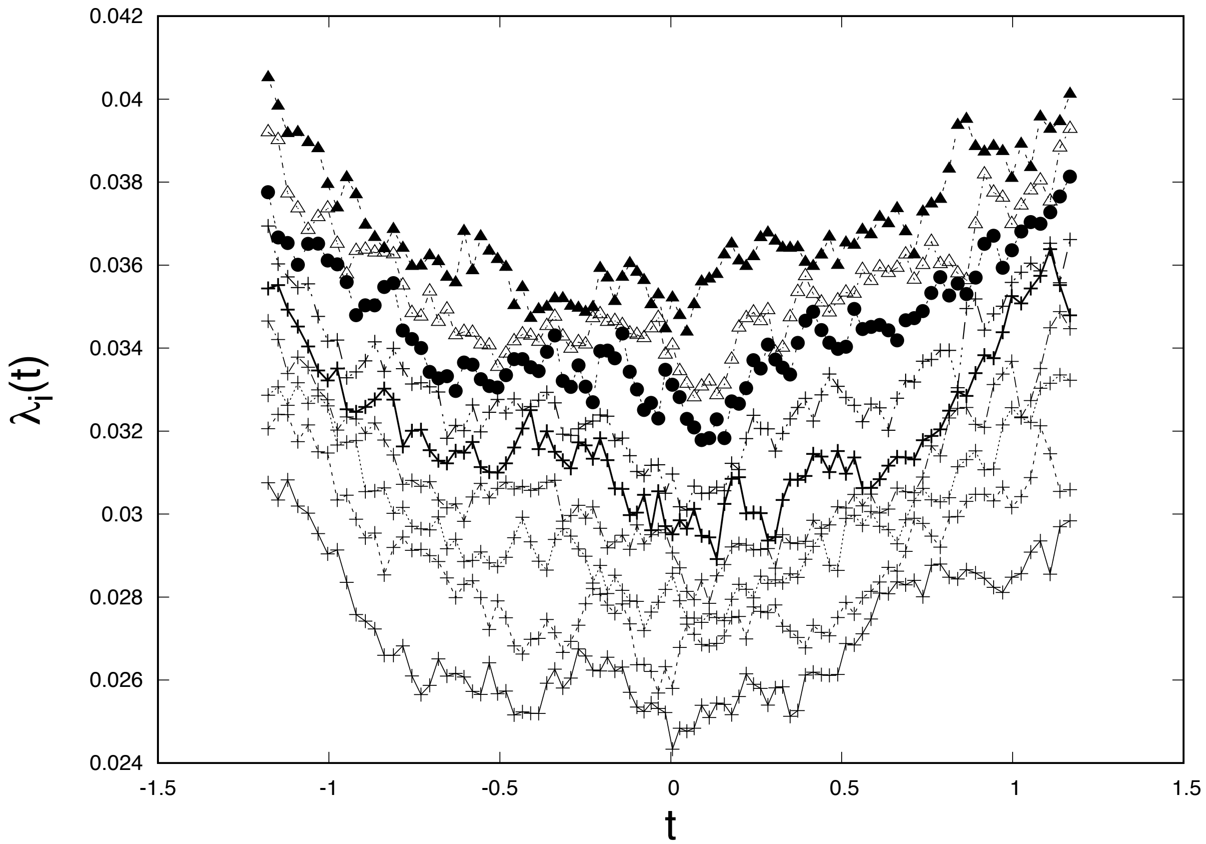
"New phase"

an order parameter for SSB of SO(9)

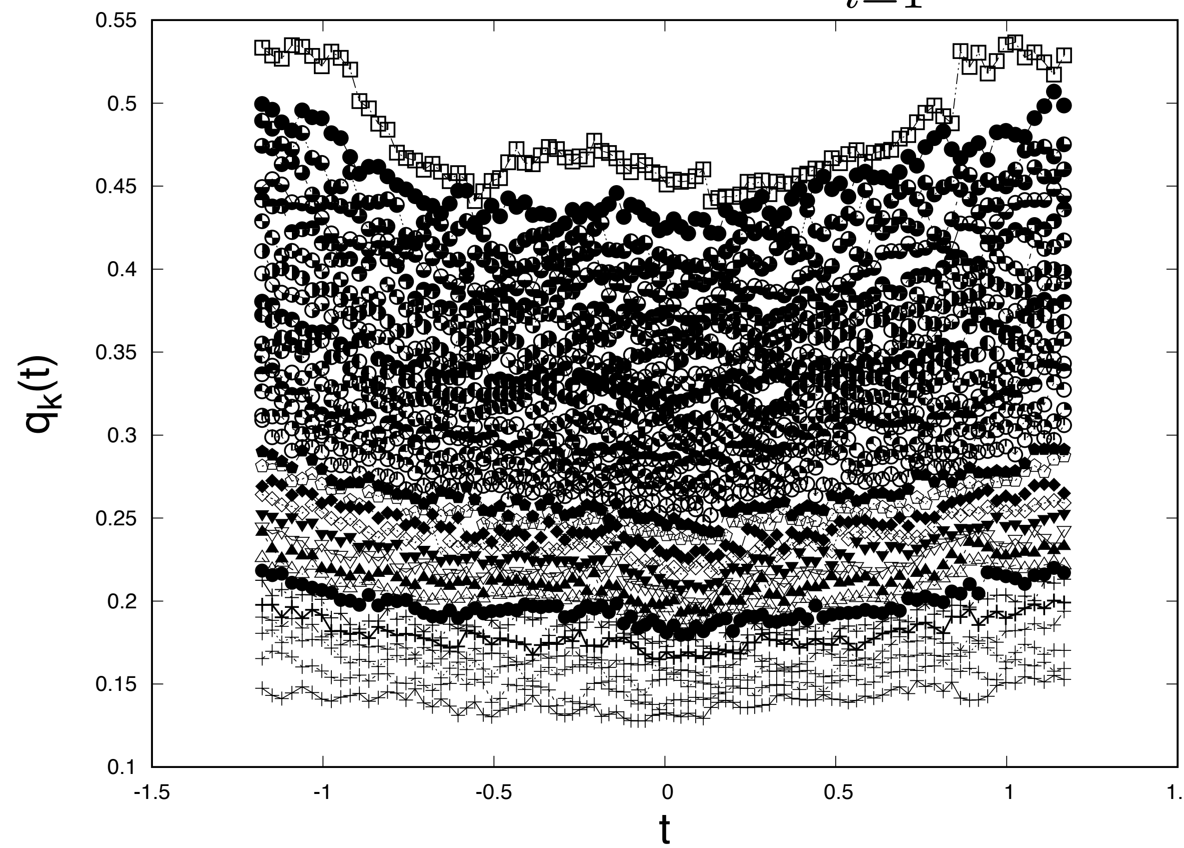
The observables which describe how space spreads in radial direction

eigenvalues of $T_{ij}(t) = \frac{1}{n}\,\mathrm{tr}\,(X_i(t)X_j(t))$

eigenvalues of $Q(t) = \sum_{i=1}^{9} (X_i(t))^2$



SO(9) symmetric



continuous space

These behaviors seem to be the same as in the Euclidean case.

Hotta, Nishimura, Tsuchiya ('98)

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summary and discussions

- **We applied the complex Langevin method to 10d bosonic model and found a new phase.**
  - The CLM works well at the Lorentzian case ($s = k = 0$).
  - Continuous space structure appears. But the SSB of SO(9) symmetry doesn't occur yet in our preliminary studies.
    - These behaviors seem to be the same as in the Euclidean bosonic case.
    - We expect that the fermions are important for the SSB as in the Euclidean case. Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis ('20)
  - The new phase also exists at large N ($N = 512$).

In Hatakeyama's talk (next talk), he will discuss

- the relation between the Lorentzian and Euclidean models
- a possible scenario to describe our 4D space-time structure