Goldstone Boson Scattering with a Light Composite Scalar

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James Ingoldby (ICTP, Trieste)
for the LSD Collaboration

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Outline

Collaborators

Lattice Strong Dynamics

Richard Brower
Claudio Rebbi

David Schaich

Oliver Witzel

Xiaoyong Jin
James Osborn

Anna Hasenfratz
Ethan Neil

Evan Weinberg

Thomas Appelquist
George Fleming
Kimmy Cushman

Andrew Gasbarro

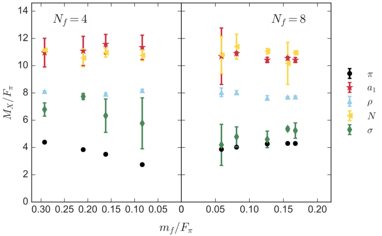
Pavlos Vranas

Enrico Rinaldi

The Light Scalar

Earlier lattice studies indicate that the SU(3) gauge theory with 8 flavors has a light scalar:

[LatKMI PRD96(2017)014508], [LSD PRD 99(2019)014509]



- The σ and π much lighter than the ρ for $N_f = 8$.
- Expect the $\pi\pi$ scattering amplitude to have nearby poles.

SU(3) with $N_f = 8$ can be used to build composite Higgs models, e.g [PRL126(2021)191804]

$\pi\pi$ Scattering

1. We would like to learn more about the scalar besides its mass. $\pi\pi$ scattering observables probe the interaction strength between the light scalar and the PNGBs.
2. Tests low energy EFT descriptions of the 8 flavor theory - we will compare with predictions from dilaton EFT.
3. On the lattice, we measure $k \cot \delta_{\ell=0}^{I=2}$ - in the $I = 2$ isospin, $\ell = 0$ channel in the 8 flavor theory. Makes coupled channel analysis unnecessary.

Lattice Calculation of Scattering Phase Shift

[M. Lüscher NPB354(1991)]

$$k^2 = \frac{1}{4} E_{\pi\pi}^2 - M_\pi^2 \tag{1}$$

$$k \cot \delta(k) = \frac{2\pi}{L} \pi^{-3/2} Z_{00} \left(1, \frac{k^2 L^2}{4\pi^2}\right) \tag{2}$$

- Restrict ourselves to $I = 2$ channel.
- $E_{\pi\pi}$ is the two–PNGB ground state energy.
- Measured at finite volume (L) on the lattice from a fit to a two point correlation function of two PNGB operators. Schematically: $C(t) \sim \langle \mathcal{O}^{I=2}(t) \mathcal{O}^{\dagger\, I=2}(0) \rangle$ where $\mathcal{O}^{I=2} \sim \pi\pi$.

Lattice Calculation of Scattering Phase Shift

The difference between $\frac{1}{4}E_{\pi\pi}^2$ and M_π^2 is *very* small compared to the sum.

Need very high precision on measurements of $\frac{1}{4}E_{\pi\pi}^2$ and M_π^2, to get reasonable precision on k^2.

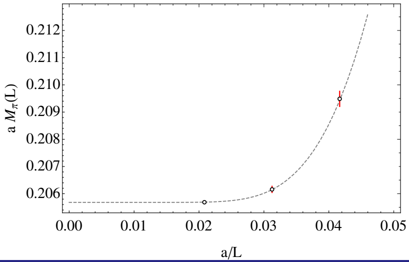
- More statistics for M_π than in earlier LSD $N_f = 8$ studies.
- Use AIC procedure to control fit range systematic errors.
  See lattice conference presentation by E. Neil

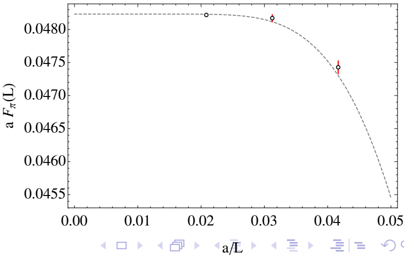
Finite Volume

$$M_\pi(L) = M_\pi \left(1 + \alpha \frac{M_\pi^2}{F_\pi^2} \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}\right),$$

$$F_\pi(L) = F_\pi \left(1 - \beta \frac{M_\pi^2}{F_\pi^2} \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}\right).$$

- Use crude model to extrapolate M_π and F_π to infinite volume limit.
- We use model to conservatively estimate the uncertainties in M_π and F_π.





Review of Previous Leading Order EFT Results

[Golterman, Shamir PRD94(2016)054502], [PRD102(2020)114507] [Golterman, Shamir, Neil PRD102(2020)034515], [Appelquist, Ingoldby, Piai JHEP07(2017)035], [PRD101(2020)07525]

Minimizing potential for dilaton field = solving a transcendental equation to determine pNGB decay constant F_π:

$$\frac{F_\pi^{4-y}}{(4-\Delta)f_\pi^{4-y}}\left[1-\left(\frac{f_\pi}{F_\pi}\right)^{4-\Delta}\right]=\frac{yN_f f_\pi^2 m_\pi^2}{2f_d^2 m_d^2}, \tag{3}$$

Note that $m_\pi^2 \equiv 2B_\pi m$, where m is a small degenerate mass for the 8 flavors.

Review of Previous Leading Order EFT Results

Use result for F_π to find the pNGB mass M_π and singlet scalar mass M_d.

$$\frac{M_\pi^2}{m_\pi^2} = \left(\frac{F_\pi^2}{f_\pi^2}\right)^{\frac{y}{2}-1} . \tag{4}$$

$$\frac{M_d^2}{F_\pi^2} = \frac{m_d^2}{(4-\Delta)f_\pi^2}\left(4 - y + (y-\Delta)\left(\frac{f_\pi}{F_\pi}\right)^{4-\Delta}\right). \tag{5}$$

Scattering Parameters from the EFT

Scattering phase shift can be extracted from scattering amplitude $\mathcal{M}$

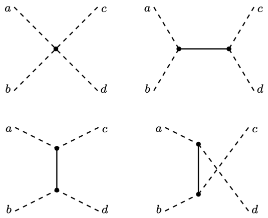
$$\mathcal{M}_\ell^I(s) = \frac{\sqrt{M_\pi^2 + k^2}}{k \cot \delta_\ell^I - ik}. \tag{6}$$

As k^2 is small on the lattice, it is convenient to expand both sides in k^2, using the effective range expansion

$$k \cot \delta^I(k) = \frac{1}{a^I} + \frac{1}{2} r^I M_\pi^2 \left(\frac{k^2}{M_\pi^2}\right) + \mathcal{O}\left(\frac{k^4}{M_\pi^4}\right). \tag{7}$$

We can then calculate scattering length a^I, and effective range r^I in dilaton EFT.

$I = 2$ Scattering Length



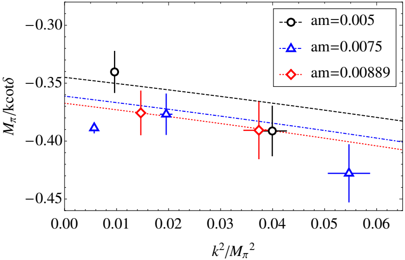
- First diagram, same as χPT. The others only arise for light scalar (dilaton).
- Projecting out the $I = 2$ component removes s-channel diagram.
- Project out $\ell = 0$ component.

$$M_\pi a^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left(1 - (y-2)^2 \frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2}\right). \qquad (8)$$

Simplifies to χPT result when $y \to 2$ or $f_\pi^2/f_d^2 \to 0$.

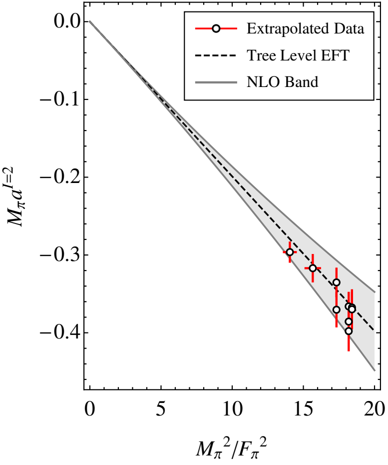
Momentum Dependence

There is mild evidence for momentum dependence in the lattice scattering data, which we can compare with dilaton EFT predictions.



Confirms that effective range expansion approach is applicable.

Qualitative Comparison



- M_π^2/F_π^2 are extrapolated.
- Effective range contribution subtracted from $k\cot\delta$ to get scattering length.
- 9 points total - many with same m but different L.
- Dashed line uses values for y etc taken from global fit.

$$a^{\text{band}} = a^{I=2}\left[1 \pm \frac{M_\pi^2}{(4\pi F_\pi)^2}\right]$$

Points all lie within the band.

Quantitative Comparison

We perform a global fit of LO dilaton EFT to lattice data for M_π, F_π, M_d and $k\cot\delta_{\ell=0}^{I=2}$.

Parameter	Value
y	2.1321*(61)*
aB_π	2.210*(60)*
f_π^2/f_d^2	0.0865*(42)*
Δ	3.11*(20)*
$a^2 f_\pi^2$	5.8*(2.1)* $\times 10^{-5}$
m_d^2/f_d^2	1.28*(26)*
χ^2/N_{dof}	54.5/18=3.03

- χ^2/N shows that LO fit is not a perfect description.
- But earlier studies and previous figure suggest dilaton EFT is a reasonable approximate description.
- Motivates study of NLO effects in dilaton EFT. NLO could change parameters outside quoted uncertainty ranges.

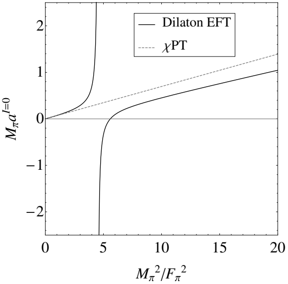
Summary

1. First lattice determination of scattering phase shift in a gauge theory with a confirmed light composite scalar, using Lüscher procedure.
2. Scattering observables provide an independent test of any EFT description of the PNGBs and light scalar.
3. We have calculated the scattering length and effective range in the $I = 2$ channel in LO dilaton EFT.
4. We have fitted the LO dilaton EFT to all the available LSD lattice data. Our results suggest that it is a fair approximate description, but we see discrepancies between the EFT and the very precise lattice data.
5. NLO might help resolve the discrepancies?

$I = 0$ Scattering

We calculate the $I = 0$ scattering length in dilaton EFT. The s-channel Feynman diagram contributes. The dilaton makes an unsuppressed contribution to the scattering length.

$$M_\pi a^{I=0} = \frac{7M_\pi^2}{32\pi F_\pi^2} + \frac{f_\pi^2}{f_d^2} \frac{M_\pi^4 \left(M_d^2(5y^2+4y+20) - 8M_\pi^2(y-2)^2\right)}{32\pi F_\pi^2 M_d^2 \left(M_d^2 - 4M_\pi^2\right)}$$



$I = 2$ Interpolating Operators

$$\pi^+(t) = \sum_{\vec{x}} \bar{\chi}_2(x)\epsilon(x)\chi_1(x), \text{ where } \epsilon(x) = (-1)^{x+y+z+t} \tag{9}$$

$$\mathcal{O}_{I=2}(t) = \pi^+(t)\pi^+(t+1) \tag{10}$$

$$\begin{aligned} C_{I=2}(t, t_0) &= \langle \mathcal{O}_{I=2}(t)\mathcal{O}_{I=2}(t_0)^\dagger \rangle \\ &= \sum_{\vec{x}_1,\cdots,\vec{x}_4} \langle \pi^+(t_4, \vec{x}_4)\pi^+(t_3, \vec{x}_3)\pi^+(t_2, \vec{x}_2)^\dagger \pi^+(t_1, \vec{x}_1)^\dagger \rangle \end{aligned} \tag{11}$$

Wall sources used - moving wall method.

Lattice Action

- Our numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$. [LSD PRD 99(2019)014509]
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%. [LSD PRD 99(2019)014509]