Searching for Continuous Phase Transitions in 5D SU(2) Lattice Gauge Theory

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Our Goal

Find examples of **non-trivial fixed points** in **non-renormalizable** quantum field theories in 5d. We used **Monte Carlo** methods to study the discretized version of **pure SU(2) Yang-Mills** in 5 dimensions. This search was motivated by perturbative results for the β-**function** in **4+**ε dimensions.

Adrien Florio, João M. Viana P. Lopes, **José Matos**, João Penedones. *Searching for continuous phase transitions in 5D SU(2) lattice gauge theory. https://arxiv.org/abs/2103.15242*

Sketch of the Problem

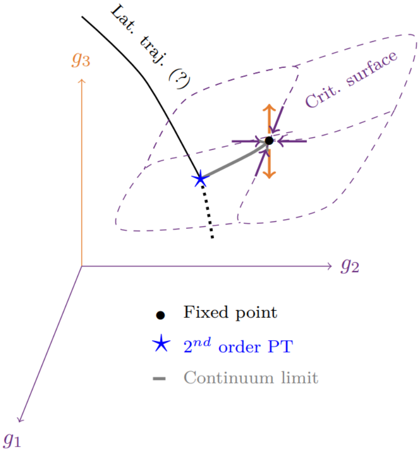


Figure: Renormalization flow sketch. arXiv:2103.15242

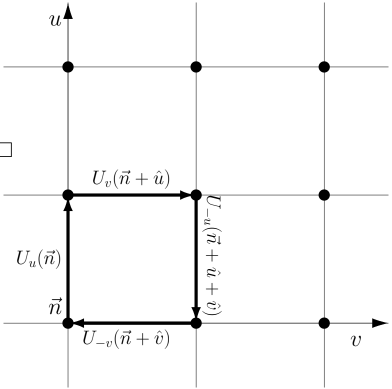
Discrete Pure Yang-Mills Action - Wilson Action

The continuum action is transformed into a discrete action

$$-\frac{1}{4g_{\mathrm{YM}}^2}\int d^5 x F^a_{\mu\nu} F^{a\nu\mu} \to \beta \sum_{\Box} S_{\Box}$$

where the sum is over **plaquettes**.

$$S_{\Box} = 1 - \frac{\mathrm{Re\ Tr}\left(\Box^R_{\mu\nu}(n)\right)}{d\,(R)}$$



$$\Box_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\hat{\mu}) U_{-\mu}(n+\hat{\mu}+\hat{\nu}) U_{-\nu}(n+\hat{\nu})$$

Extended Actions

There is freedom in choosing the lattice action (extending the Wilson action). We focused on two extensions

$$S = \overbrace{\sum_{\square} \frac{\beta_f}{2} \operatorname{Tr}\left(1 - \square^f_{1\times 1}\right)}^{\text{Wilson Action}} + \sum_{\square} \begin{cases} 1 - \frac{\beta_a}{3} \operatorname{Tr}\left(\square^a_{1\times 1}\right) & \text{Adjoint Action} \\ \frac{\beta_2}{2} \operatorname{Tr}\left(1 - \square^f_{2\times 2}\right) & \text{2 Loops Action} \end{cases}$$

The first result

The simplest model

$$S = \sum_{\square} \frac{\beta_f}{2} \operatorname{Tr}\left(1 - \square^{f}_{1\times 1}\right) \tag{1.1}$$

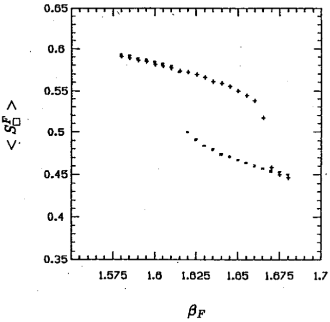


Figure: Thermal cycles for 8^5 [Creutz '79, Kawai et. al. Progress of Theoretical Physics 88(2):341-350 (1992)]

Extended Actions

For the adjoint extension

$$S = \sum_{\square} \frac{\beta_f}{2} \mathrm{Tr}\left(1 - \square^f_{1\times1}\right) + 1 - \frac{\beta_a}{3} \mathrm{Tr}\left(\square^a_{1\times1}\right) \quad (1.2)$$

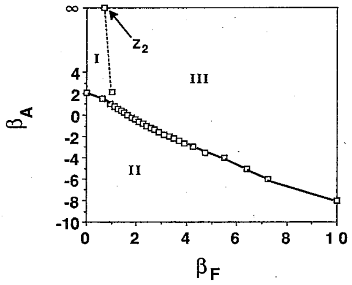


Figure: Lines of first order phase transitions [Kawai et. al. Progress of Theoretical Physics 88(2):341-350 (1992)]

Canonical Ensemble

The ensemble weight, in the canonical ensemble, is

$$P\left[\Sigma\right] \propto e^{-S[\Sigma]}, \tag{2.1}$$

which can be projected into

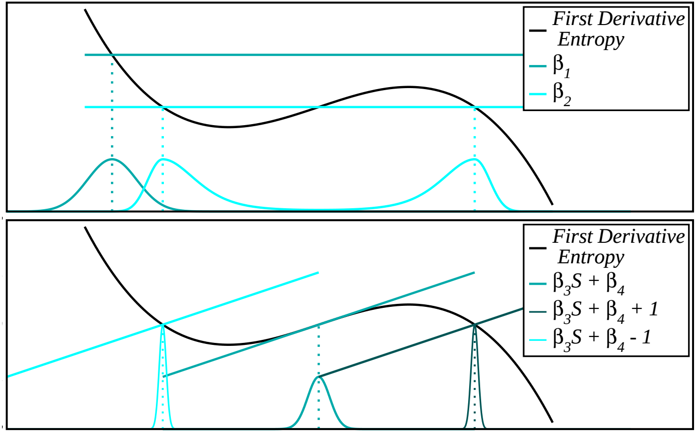
$$W^f_{1\times 1} = \frac{1}{N_{\text{Loops}}} \sum_{\square} \frac{\text{ReTr}\left(1 - \square^f_{1\times 1}\right)}{N}$$
$$\Longrightarrow P\left(W^f_{1\times 1}\right) \propto e^{\mathbb{S}\left(W^f_{1\times 1}\right) - \beta N_{\text{Loops}} W^f_{1\times 1}}.$$

Only the values around the maximum will contribute for the partition function

$$\frac{d\mathbb{S}\left(W^f_{1\times 1}\right)}{dW^f_{1\times 1}} = \beta. \tag{2.2}$$

Generalized Ensemble

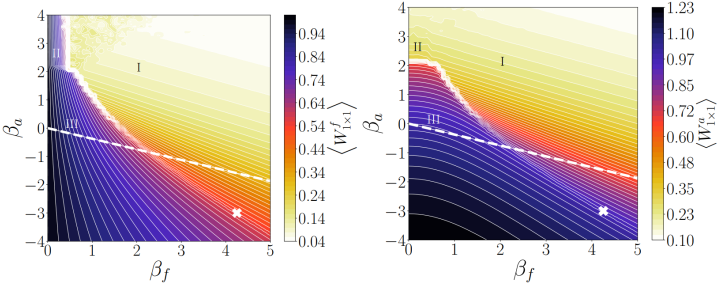
$$P\left(W^f_{1\times1}\right) \propto e^{\mathbb{S}\left(W^f_{1\times1}\right) - \beta N_{\text{Loops}} W^f_{1\times1}}$$



$$P\left(W^f_{1\times1}\right) \propto e^{\mathbb{S}\left(W^f_{1\times1}\right) - N_{\text{Loops}} W^f_{1\times1}\left(\beta_2 W^f_{1\times1} + \beta_1\right)}$$

Adjoint action: Parameter Space Survey

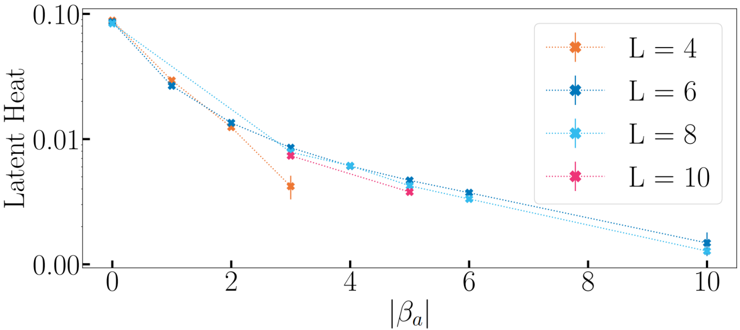
$$S = \sum_{\square} \frac{\beta_f}{2} \mathrm{Tr}\left(1 - \square^f_{1\times1}\right) + 1 - \frac{\beta_a}{3} \mathrm{Tr}\left(\square^a_{1\times1}\right) \qquad (3.1)$$



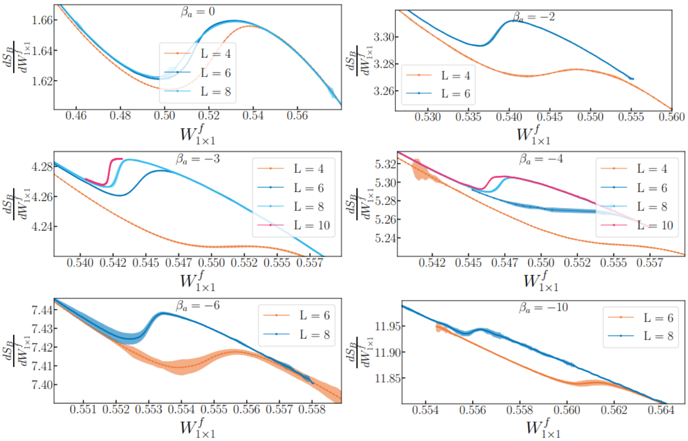
Adjoint action: Latent Heat

Latent heat: Discontinuity in the value of the action in a **first** order phase transition.

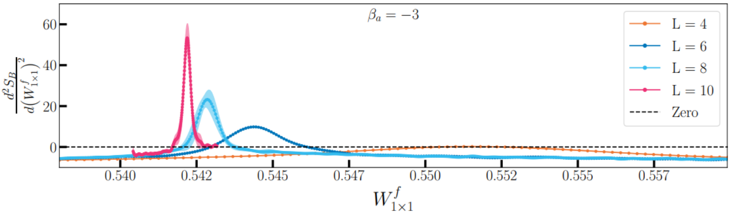
$$S = \sum_{\square} \frac{\beta_f}{2} \mathrm{Tr}\left(1 - \square^f_{1\times1}\right) + 1 - \frac{\beta_a}{3} \mathrm{Tr}\left(\square^a_{1\times1}\right) \qquad (3.2)$$



Adjoint Action: First Derivative of the Entropy

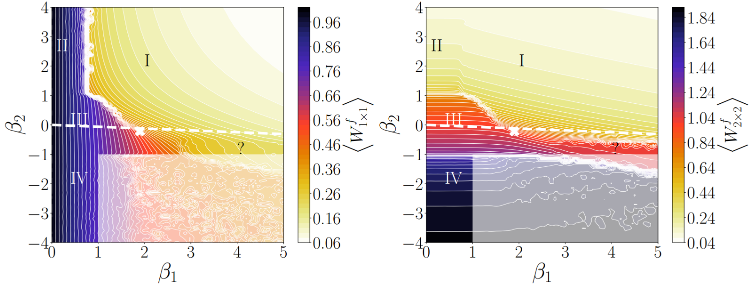


Adjoint Action: Second Derivative of the Entropy



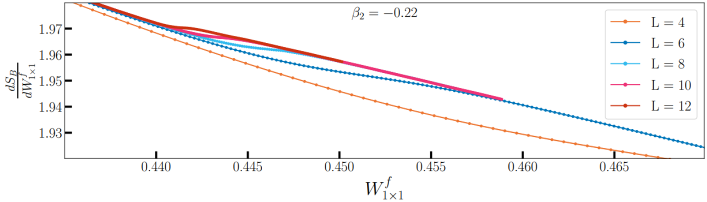
2 Loops Action: Parameter Space Survey

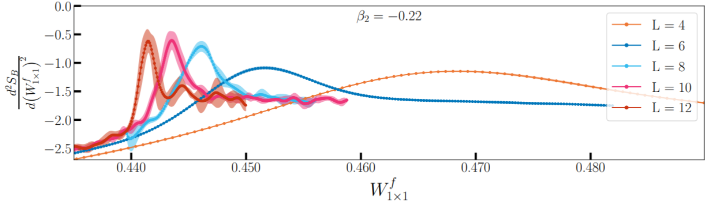
$$S = \sum_{\square} \frac{\beta_1}{2} \operatorname{Tr}\left(1 - \square^f_{1\times1}\right) + \frac{\beta_2}{2} \operatorname{Tr}\left(1 - \square^f_{2\times2}\right) \qquad (3.3)$$



2 Loops Action: First and Second Derivatives of the Entropy

$$S = \sum_{\square} \frac{\beta_1}{2} \mathrm{Tr}\left(1 - \square^f_{1\times 1}\right) + \frac{\beta_2}{2} \mathrm{Tr}\left(1 - \square^f_{2\times 2}\right) \qquad (3.4)$$





2 Loops Action: Infinite Volume Limit

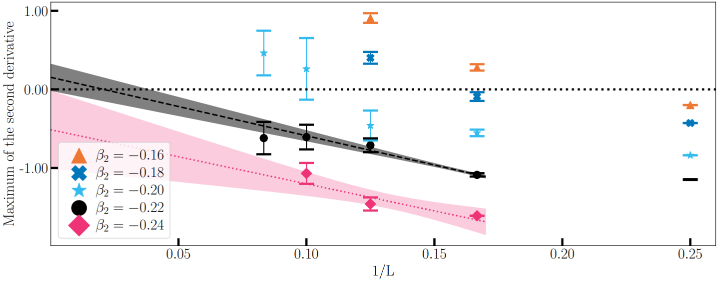


Figure: Maximum of the Second Derivative.

End

THANK YOU!