Searching for Lee-Yang zeros in $O(N)$ models

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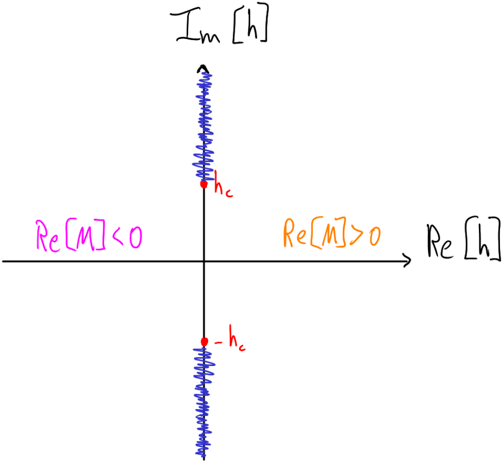
Motivation

Why $O(N)$ models?

- Chiral phase transition of QCD expected to be in the $3d$ $O(4)$ class
- Magnetisation in the $O(4)$ plays the role of quark condensate
- Magnetic equation of state of $O(N)$ has a branch cut at purely imaginary external field h
- In the symmetric phase the branch cut ends at $\pm ih_c$, the *edge singularities*, which are second order critical points
- Radius of convergence of a Taylor expansion is determined by the closest singularity

Introduction

Why $O(N)$ models?



Relevant talks

Searches for Lee-Yang zeros on the lattice

- Gökçe Başar - Monday @ 21:00 EDT
- Simran Singh - Tuesday @ 6:30 EDT
- Guido Nocotra - Tuesday @ 6:45 EDT

Introduction

Formulation

- Euclidean action:

$$S = \int d^d x \left[\frac{1}{2} \partial_\mu \varphi_i \partial_\mu \varphi_i + \frac{m_0^2}{2} \varphi_i \varphi_i + \frac{\lambda_0}{4!} (\varphi_i \varphi_i)^2 + h_0 \varphi_1 \right]$$

  $1 \leq \mu \leq d$, $1 \leq i \leq N$, $m_0^2 \in \mathbb{R}$, $\lambda_0 \in \mathbb{R}$, $h_0 \in \mathbb{C}$.
- Objects of interest:
  - magnetisation

$$M = \frac{1}{V} \frac{\partial}{\partial h_0} \ln Z = \frac{1}{V} \langle \Phi_1 \rangle \ , \quad \Phi_1 = \int d^d x \, \varphi_1(x)$$

  - magnetic susceptibility

$$\chi = \frac{1}{V} \frac{\partial^2}{\partial h_0^2} \ln Z = \frac{1}{V} \left[\left\langle \Phi_1^2 \right\rangle - \langle \Phi_1 \rangle^2 \right]$$

Introduction

Formulation

- Euclidean action:

$$S = \int d^d x \left[\frac{1}{2} \partial_\mu \varphi_i \partial_\mu \varphi_i + \frac{m_0^2}{2} \varphi_i \varphi_i + \frac{\lambda_0}{4!} (\varphi_i \varphi_i)^2 + h_0 \varphi_1 \right]$$

- Because of the complex h_0 we use complex Langevin

$$\frac{d\varphi_i(x,\tau)}{d\tau} = -\frac{\delta S[\varphi_i(x,\tau)]}{\delta \varphi_i} + \eta_i(x,\tau)$$

- where η is a white noise field

$$\langle \eta \rangle = 0\,, \quad \langle \eta_i(x,\tau) \eta_j(y,\tau') \rangle = 2\delta_{ij} \delta(x-y) \delta(\tau - \tau')$$

- expectation values are computed as Langevin time averages (after thermalisation)

$$\langle \mathcal{O} \rangle = \frac{1}{\tau_{\max} - \tau_{\mathrm{term}}} \int_{\tau_{\mathrm{term}}}^{\tau_{\max}} d\tau \, \langle \mathcal{O}(\tau) \rangle$$

Our approach

Different methods used for consistency checks

- Single site model with $\mathrm{O}(2)$ symmetry
  - exact formula (in terms of Bessel functions)
  - computed with CL
- $\mathrm{O}(4)$ model in $d = 3$ dimensions
  - computed with CL
  - *computed with FRG (Polchinsky equation)* ***(currently underway!)***

Single site toy model

- action:

$$S = \frac{m_0^2}{2}\left(x^2 + y^2\right) + \frac{\lambda_0}{4}\left(x^2 + y^2\right)^2 - i\beta x$$

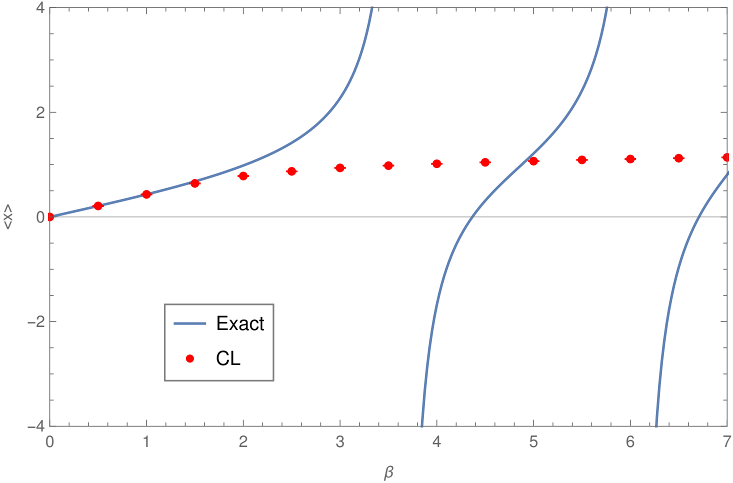
- partition function:

$$Z = 2\pi \int_0^\infty dr\, r J_0(\beta r) \exp\left[-\frac{m_0^2 r^2}{2} - \frac{\lambda_0 r^4}{4}\right]$$

- magnetisation:

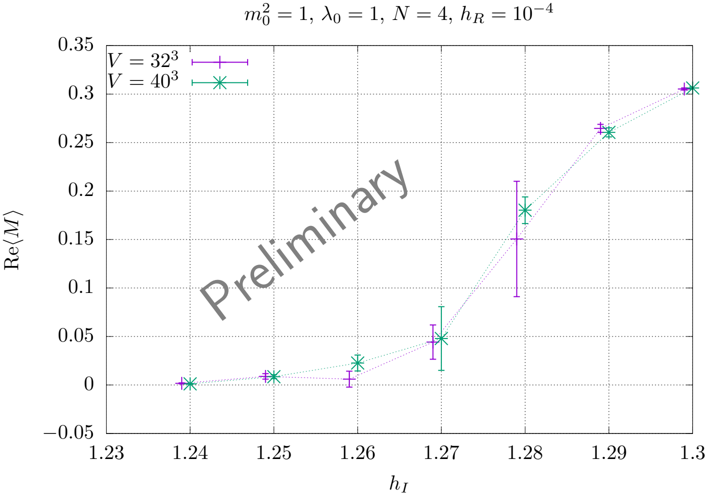
$$M = \langle x \rangle = \frac{2\pi i}{Z} \int_0^\infty dr\, r^2 J_1(\beta r) \exp\left[-\frac{m_0^2 r^2}{2} - \frac{\lambda_0 r^4}{4}\right]$$

Simulation results (single site case)



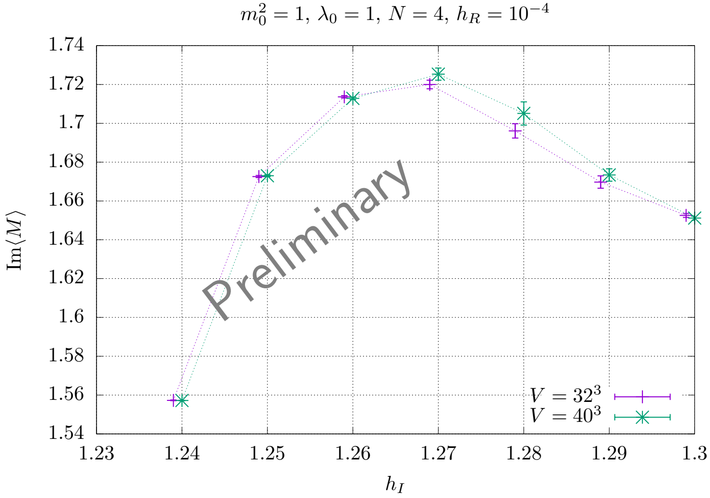
(imaginary part of) magnetisation, $m_0^2 = 1$, $\lambda_0 = 1$, O(2) model

Simulation results ($d = 3$ case)



Points have been shifted sideways to aid visibility.

Simulation results ($d = 3$ case)



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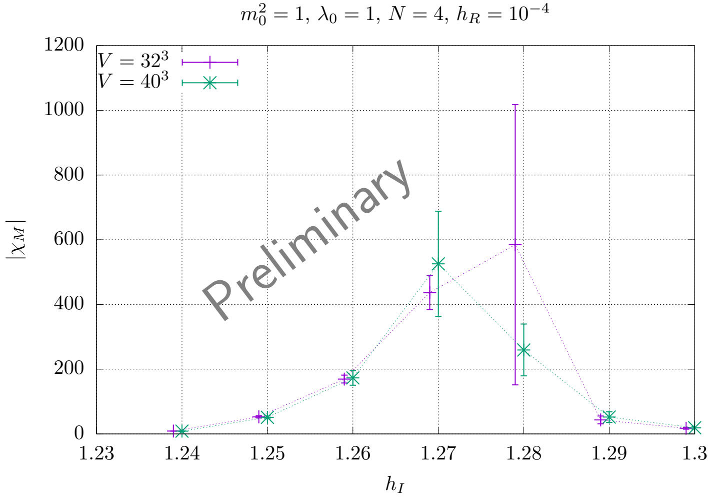
Simulation results



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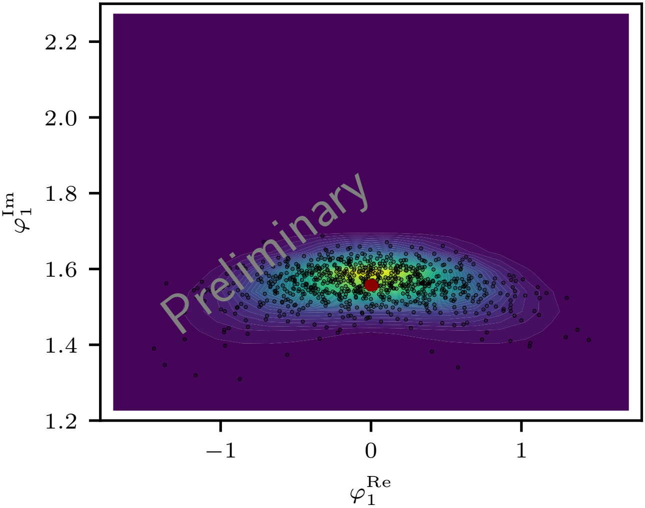
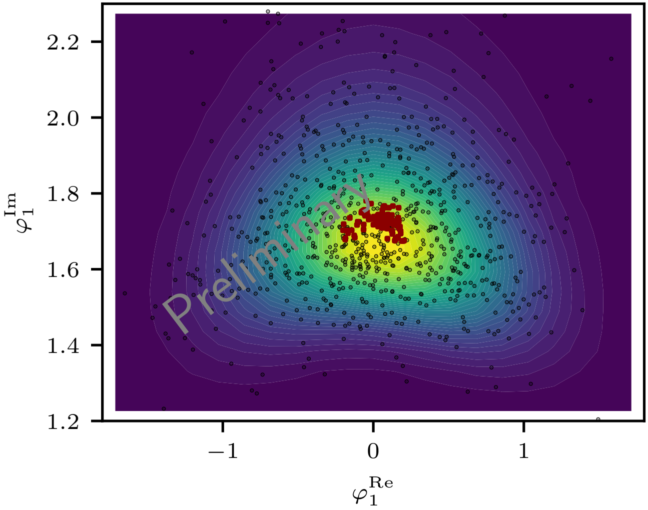
Magnetisation and field values



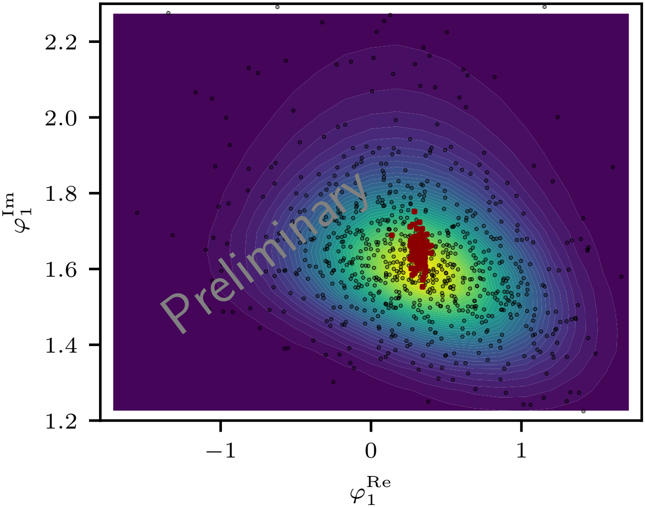
$m_0^2 = 1,\ \lambda_0 = 1,\ N = 4,\ h_R = 10^{-4},\ h_I = 1.24\ (h_I < h_c)$

Magnetisation and field values



$m_0^2 = 1,\ \lambda_0 = 1,\ N = 4,\ h_R = 10^{-4},\ h_I = 1.27\ (h_I \approx h_c)$

Magnetisation and field values



$m_0^2 = 1,\ \lambda_0 = 1,\ N = 4,\ h_R = 10^{-4},\ h_I = 1.30\ (h_I > h_c)$

Notes

- FRG calculations already exist [Connelly, Johnson, Rennecke, Skokov (2020)], but we need to perform finite size scaling in order to compare with them
- Corrections from boundary terms have been computed [Scherzer, Seiler, Sexty, Stamatescu (2020)] and found to be much smaller than the statistical errors

Summary

- Puzzling situation in the site model – tension between CL and exact methods
- On $d = 3$ complex Langevin looks more promising, with what looks like a 2nd order transition for Re$\langle M \rangle$

Outlook

- Interesting questions to be answered:
  - Why does CL see no transition in the site model, but does in $d > 0$?
  - Are the $d = 3$ results reliable?
- Volume scaling analysis to find h_c