Restoration of chiral symmetry in cold and dense Nambu–Jona-Lasinio model with tensor renormalization group

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Plan of Talk

- Introduction
- Collaborators
- TRG Approaches to Quantum Field Theories
- Application of TRG method to
  4D Nambu–Jona-Lasinio (NJL) Model at Finite Density
- Summary

Tensor Network Scheme

What is Tensor Network (TN) Scheme?

Theoretical and numerical methods for high precision analyses of many body problems with tensor network formalism

Advantages of Tensor Renormalization Group (TRG)

Free from sign problem and complex action problem in Monte Carlo method
Computational cost for L^D system size $\propto D \times \log(L)$
Direct treatment of Grassmann numbers
Direct evaluation of partition function Z itself

↓

Applications in particle physics :
Finite density QCD, QFTs w/ θ-term, Lattice SUSY etc.
Also, in condensed matter physics
Hubbard model (Mott transition, High temp. superconductivity) etc.

Collaborators

Y. Kuramashi, Y. Yoshimura S. Akiyama	U. Tsukuba
Y. Nakamura, (Y. Shimizu)	R-CCS
S. Takeda, R. Sakai(→U. Iowa)	Kanazawa U.
D. Kadoh	Doshisha U.

Collaborations are dynamically changed depending on the research topics

TRG Approaches to QFTs (1)

2D models

Real ϕ^4 theory :

Shimizu, Mod.Phys.Lett.A27(2012)1250035,

Kadoh-YK-Nakamura-Sakai-Takeda-Yoshimura, JHEP05(2019)184

Complex ϕ^4 theory at finite density :

Kadoh-YK-Nakamura-Sakai-Takeda-Yoshimura, JHEP02(2020)161

U(1) gauge theory+θ :

YK-Yoshimura, JHEP04(2020)089

Schwinger, Schwinger+θ :

Shimizu-YK, PRD90(2014)014508, PRD90(2014)074503,
PRD97(2018)034502

Gross-Neveu model at finite density :

Takeda-Yoshimura, PTEP2015(2015)043B01

N=1 Wess-Zumino model :

Kadoh-YK-Nakamura-Sakai-Takeda-Yoshimura, JHEP03(2018)141

- Free from sign and complex action problems
- Development of numerical algorithms for scalar, fermion, gauge theories

TRG Approaches to QFTs (2)

3D models

Free Wilson fermion :

Sakai-Takeda-Yoshimura, PTEP2017(2017)063B07,

Yoshimura-YK-Nakamura-Takeda-Sakai, PRD97(2018)054511

Z_2 gauge theory at finite temperature :

YK-Yoshimura, JHEP1908(2019)023

4D models

Ising : Akiyama-YK-Yamashita-Yoshimura, PRD100(2019)054510

Complex ϕ^4 theory at finite density :

Akiyama-Kadoh-YK-Yamashita-Yoshimura, JHEP09(2020)177

NJL model at finite density :

Akiyama-YK-Yamashita-Yoshimura, JHEP01(2021)121

Real ϕ^4 theory :

Akiyama-YK-Yoshimura, arXiv:2101.06953

⇒ Now our research theme is moving from 2D models to 4D ones

NJL Model at Finite Density

Akiyama+, JHEP01(2021)121

NJL model at zero density in the continuum

$$\mathcal{L} = \bar{\psi}(x)\gamma_\nu\partial_\nu\psi(x) - g_0\left\{(\bar{\psi}(x)\psi(x))^2 + (\bar{\psi}(x)\mathrm{i}\gamma_5\psi(x))^2\right\}$$

NJL model at finite density on the lattice w/ Kogut-Susskind fermion

$$S = \frac{1}{2}a^3\sum_{n\in\Lambda}\sum_{\nu=1}^{4}\eta_\nu(n)\left[\mathrm{e}^{\mu a\delta_{\nu,4}}\bar{\chi}(n)\chi(n+\hat{\nu}) - \mathrm{e}^{-\mu a\delta_{\nu,4}}\bar{\chi}(n+\hat{\nu})\chi(n)\right]$$

$$+ ma^4\sum_{n\in\Lambda}\bar{\chi}(n)\chi(n) - g_0a^4\sum_{n\in\Lambda}\sum_{\nu=1}^{4}\bar{\chi}(n)\chi(n)\bar{\chi}(n+\hat{\nu})\chi(n+\hat{\nu})$$

μ : chemical potential

m : fermion mass

g_0 : coupling constant of 4 fermi interaction

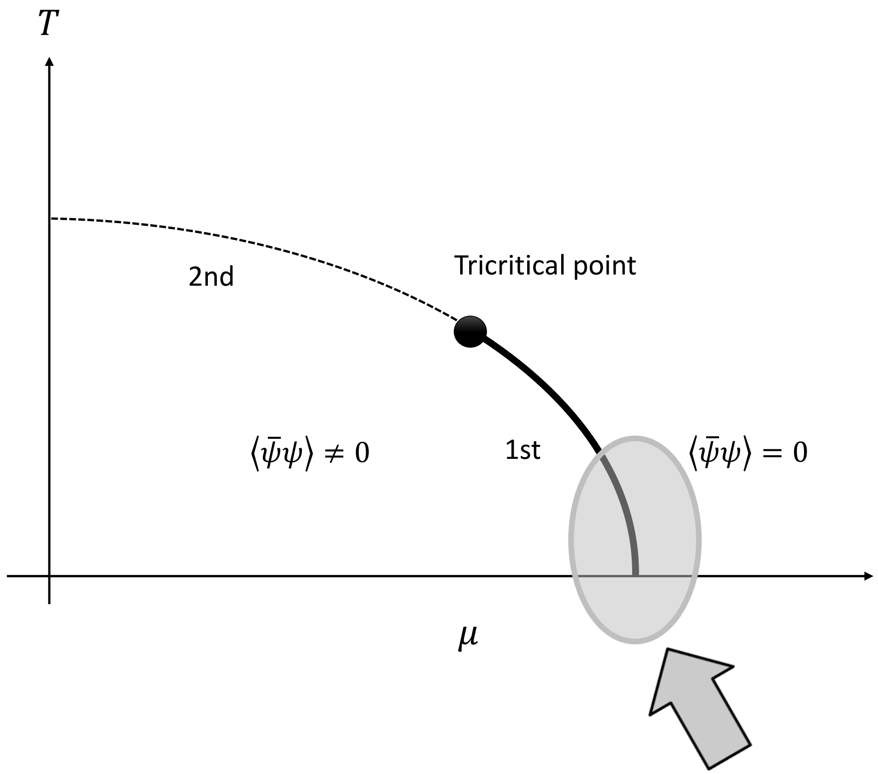
a : lattice spacing

Phase Diagram of NJL Model at Finite Density

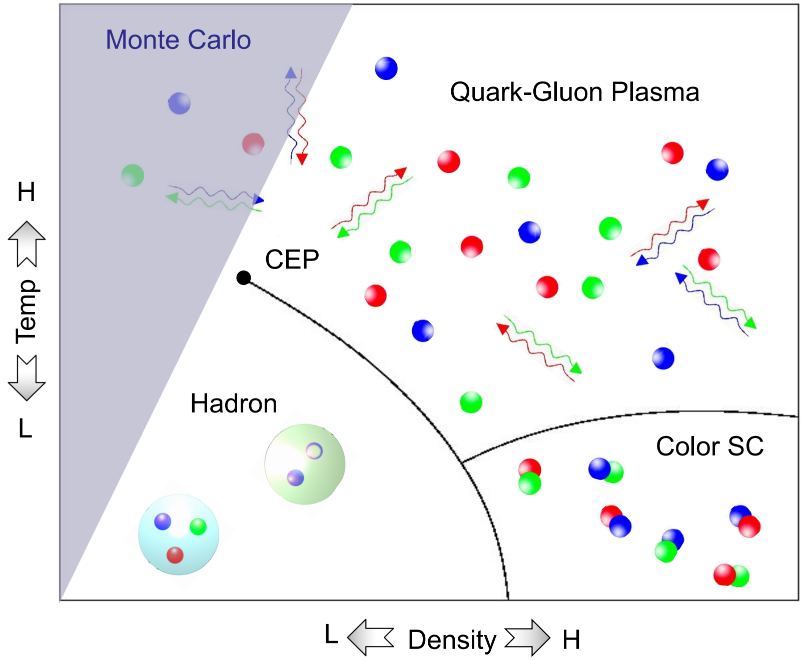
Akiyama+, JHEP01(2021)121

NJL model as a prototype of QCD

Expected phase diagram for NJL model



Expected phase diagram for QCD



First step is to check the first-order phase transition

at cold and dense region

Algorithm and Parameters

Akiyama+, JHEP01(2021)121

TN representation

$$\mathcal{T}_{n;i_4(n)i_1(n)i_2(n)i_3(n)i_4(n-\hat{4})i_1(n-\hat{1})i_2(n-\hat{2})i_3(n-\hat{3})}$$

$$= \int \mathrm{d}\chi \mathrm{d}\bar{\chi}\ \mathrm{e}^{-m\bar{\chi}\chi} \prod_{\nu=1}^{4} \left(\frac{\mathrm{e}^{\frac{\mu}{2}\delta_{\nu,4}}}{\sqrt{2}} \eta_\nu(n)\bar{\chi}\mathrm{d}\Phi_\nu(n) \right)^{i_{\nu,1}(n)} \left(\frac{\mathrm{e}^{\frac{\mu}{2}\delta_{\nu,4}}}{\sqrt{2}} \chi \mathrm{d}\bar{\Phi}_\nu(n) \right)^{i_{\nu,1}(n-\hat{\nu})}$$

$$\times \left(\frac{\mathrm{e}^{-\frac{\mu}{2}\delta_{\nu,4}}}{\sqrt{2}} \eta_\nu(n)\chi\mathrm{d}\Psi_\nu(n) \right)^{i_{\nu,2}(n)} \left(\frac{\mathrm{e}^{-\frac{\mu}{2}\delta_{\nu,4}}}{\sqrt{2}} \bar{\chi}\mathrm{d}\bar{\Psi}_\nu(n) \right)^{i_{\nu,2}(n-\hat{\nu})} (\sqrt{g_0}\bar{\chi}\chi)^{i_{\nu,3}(n)}$$

$$\times (\sqrt{g_0}\bar{\chi}\chi)^{i_{\nu,3}(n-\hat{\nu})} \left(\bar{\Phi}_\nu(n+\hat{\nu})\Phi_\nu(n) \right)^{i_{\nu,1}(n)} \left(\bar{\Psi}_\nu(n+\hat{\nu})\Psi_\nu(n) \right)^{i_{\nu,2}(n)} .$$

$$Z = \sum_{\{t,x,y,z\}} \int \prod_{n\in\Lambda} \mathcal{T}_{n;txyzt'x'y'z'} \qquad (x = i_1, y = i_2, z = i_3, t = i_4)$$

Coarse-graining procedure: GATRG w/ D_{cut}=55

Parameters

$$V = L\times\beta = (aN_\sigma)\times(aN_\tau) = 2^4, \cdots, (1024)^4 \qquad \left(\beta = \frac{1}{T}\right)$$

Periodic BC for spatial direction and anti-periodic BC for temporal direction

a is fixed at finite value (a=1)

$g_0 = 32$ for coupling constant of four-fermi interaction

Heavy Dense Limit as a Benchmark

Akiyama+, JHEP01(2021)121

Heavy dense limit: $m, \mu \to \infty$ while e^{μ}/m kept fixed

Chiral condensate and number density in the heavy dense limit

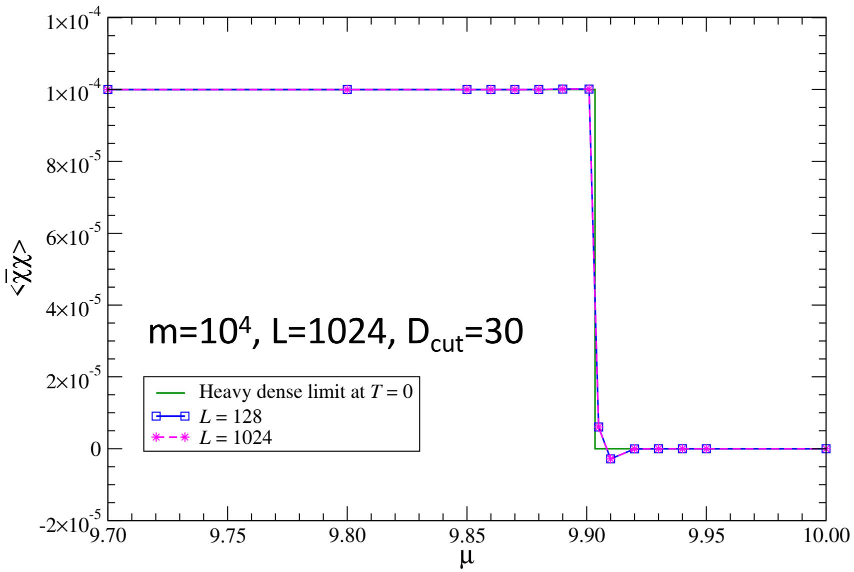
$$\langle \bar{\chi}(n)\chi(n)\rangle = \frac{1}{m}\Theta(\mu_c - \mu) \qquad \langle n \rangle = \Theta(\mu - \mu_c)$$

Pawlowski-Zielinski, PRD87(2013)094509

Analytical solutions are step function at $\mu_c = \ln(2m)$

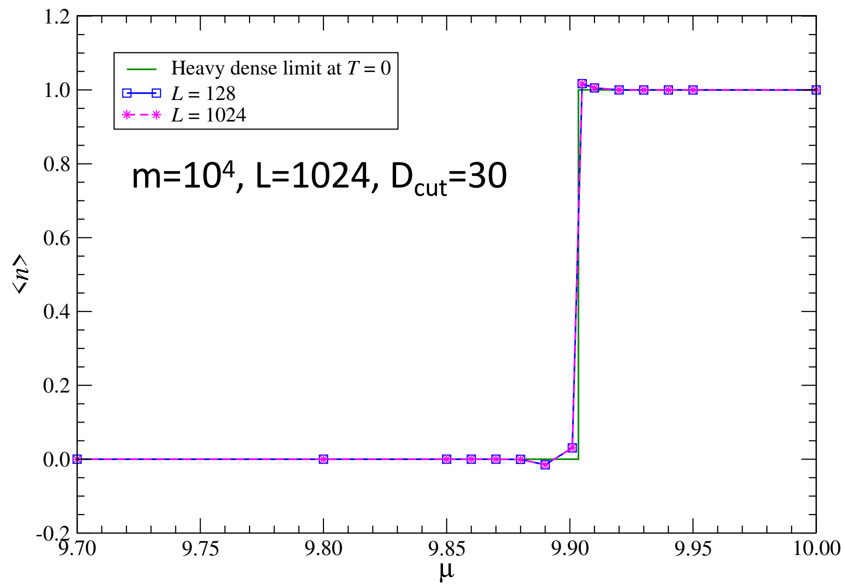
Chiral condensate

$$\langle \bar{\chi}(n)\chi(n)\rangle|_{m=10^4} = \frac{1}{V}\frac{\ln Z(m+\Delta m) - \ln Z(m)}{\Delta m}\bigg|_{m=10^4}$$



Number density

$$\langle n \rangle = \frac{1}{V}\frac{\partial \ln Z(\mu)}{\partial \mu} \approx \frac{1}{V}\frac{\ln Z(\mu+\Delta\mu) - \ln Z(\mu)}{\Delta\mu}$$



Good consistency with analytical solutions

Convergence Property of Free Energy

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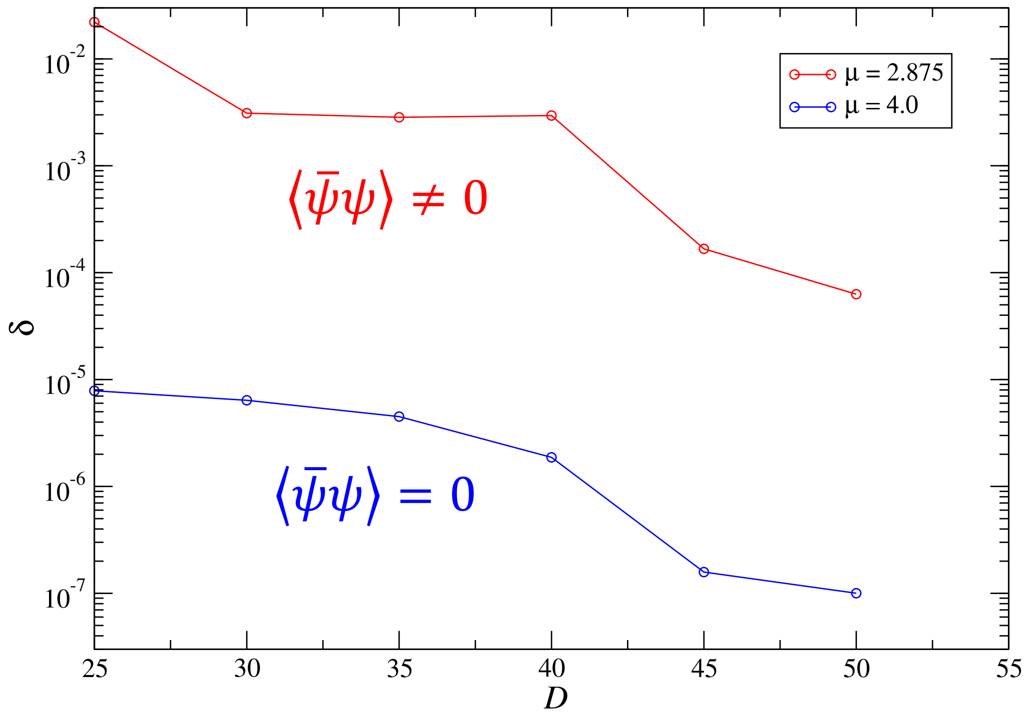
D_{cut} dependence of relative error on $V=(1024)^4$

$$\delta = \left| \frac{\ln Z(D) - \ln Z(D=55)}{\ln Z(D=55)} \right|$$



δ becomes less than $O(10^{-4})$ at μ=2.875 and $O(10^{-7})$ at μ=4.0

μ Dependence of Chiral Condensate

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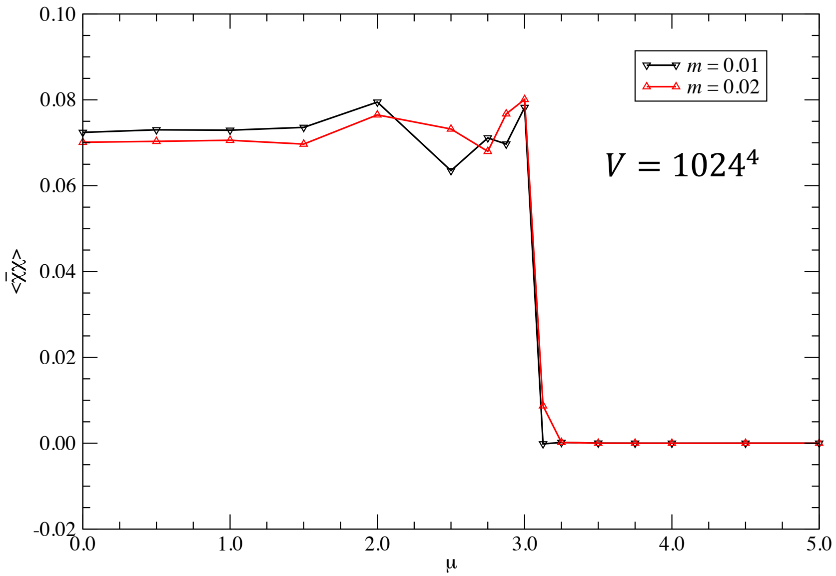
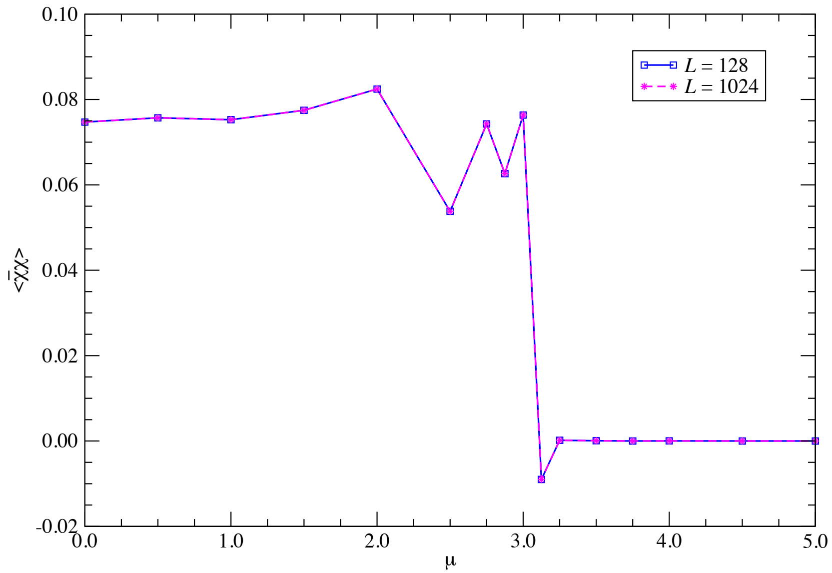
Order parameter of chiral phase transition

$$\langle \bar{\chi}(n)\chi(n)\rangle = \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\frac{\partial}{\partial m}\ln Z$$

Mass dependence@L=1024



Volume dependence@m=0



Jump around $\mu \approx 3.0 \Rightarrow$ First-order phase transition

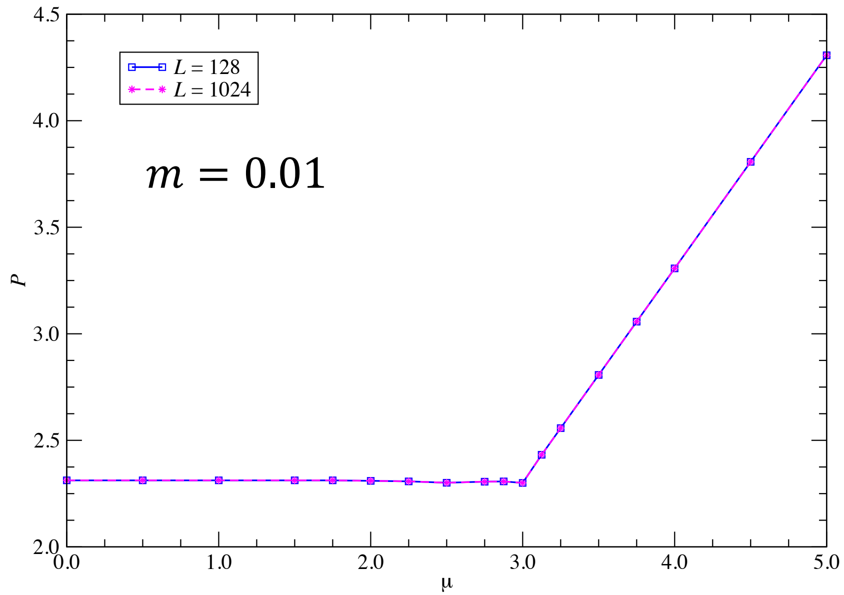
μ Dependence of Number Density

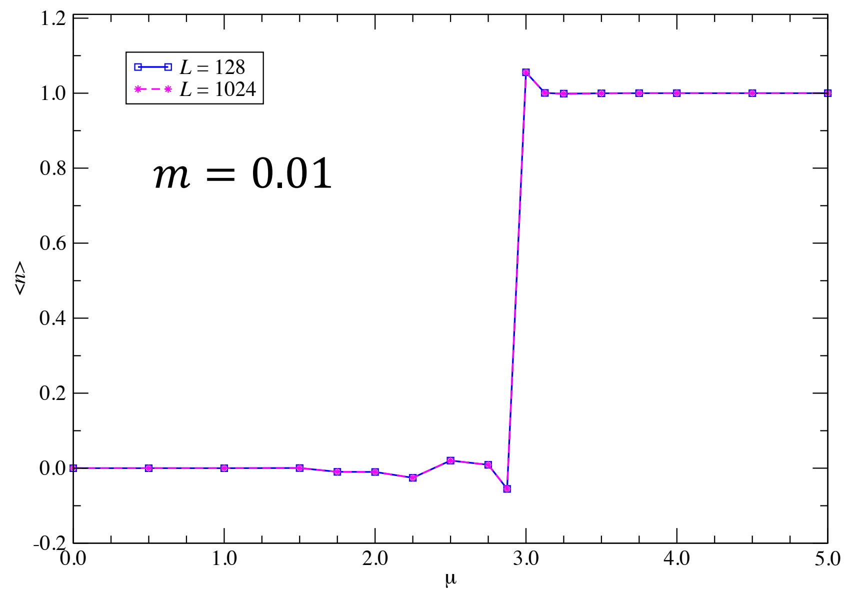
Akiyama+, JHEP01(2021)121

Pressure and number density (EOS)

$$P = \frac{\ln Z}{V}$$

$$\langle n(\mu) \rangle = \frac{\partial P(\mu)}{\partial \mu} \approx \frac{P(\mu + \Delta\mu) - P(\mu)}{\Delta\mu}$$





Jump around $\mu \approx 3.0$ ⇒ Another evidence of first-order phase transition

Summary

<u>What we have achieved so far</u>

- Studies of various 2D models

− Show that the TRG method is free from sign problems

− Development of algorithms for scalar, fermion gauge theories

- Studies of 4D models

− Ising model

− Complex ϕ^4 theory at finite density

− NJL model at finite density

− Real ϕ^4 theory

<u>Current status</u>

- Research theme is moving from 2D models to 4D ones
- A new research direction: Hubbard model in condensed matter physics

Akiyama's talk

Akiyama-YK, PRD104(2021)014504