Introducing Fermionic Quantum Link Models

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Outline

Introduction: What is new here?

From $d = 2$ to $d = 3$ spatial dimensions

Results and (exciting) outlook

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The plaquette in abelian $U(1)$ LGT

- Wilson-Kogut-Susskind: Quantum rotors.

  Wilson(PRD, 1974), Kogut-Susskind (PRD, 1975).

  $U = L^+; \ U^\dagger = L^-; \ E = L^z$

  $[E, U] = U; \ [E, U^\dagger] = -U^\dagger; \ [U, U^\dagger] = 0$

- $H = \frac{e^2}{2} \sum_{x,i} E^2_{x,x+i} - \frac{1}{2e^2} \sum_\Box (U_\Box + U^\dagger_\Box)$.

- Gauge Invariance: $[G_x, H] = 0$.

- Quantum links: Quantum spin-S

  $U = S^+; \ U^\dagger = S^-; \ E = S^z$

  Horn (PLB, 1981), Orland-Rohrlich (NPB, 1990),

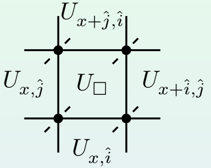
  Wiese-Chandrasekharan (NPB, 1997).

- Identical Gauss' Law, Hamiltonian.

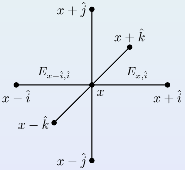
  Acts on different Hilbert space.

- Gauge invariance unaffected by

  $[U, U^\dagger] = 2E$.



$$U_\Box = U_{x,\hat{i}} U_{x+\hat{i},\hat{j}} U^\dagger_{x+\hat{j},\hat{i}} U^\dagger_{x,\hat{j}}$$



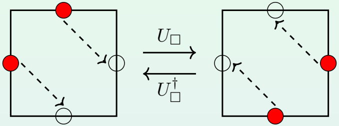
$$G_x = \sum_i (E_{x,\hat{i}} - E_{x-\hat{i},\hat{i}})$$

The fermionic link

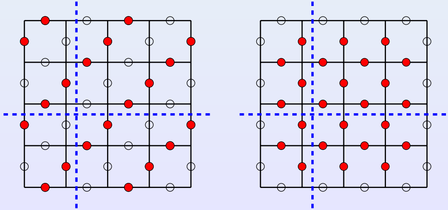
- New physics with quantum links. For $S = 1/2$:
  novel phases J. Stat. Mech.(2013), PRB(2014), PRB(2016).
  anomalous thermalization PRL(2019), PRL(2021).
  quantum simulation Ann. Phys.(2014), PRX(2020), arXiv:2107.01283.
- Realize the 2-d local Hilbert space with fermions.
  $U = c^\dagger; \quad U^\dagger = c; \quad E = n - \frac{1}{2} = c^\dagger c - \frac{1}{2}$
- Plaquette: $U_\square = U_{x,\hat{i}} U_{x+\hat{i},\hat{j}} U^\dagger_{x+\hat{j},\hat{i}} U^\dagger_{x,\hat{j}} = c^\dagger_{x,\hat{i}} c^\dagger_{x+\hat{i},\hat{j}} c_{x+\hat{j},\hat{i}} c_{x,\hat{j}}$
- Gauss' Law: $G_x = \sum_i \left(n_{x,\hat{i}} - n_{x-\hat{i},\hat{i}} \right)$
- Commutation relations work as in the case of quantum spins:
  $[n, c^\dagger] = c^\dagger; \quad [n, c] = -c; \quad [c^\dagger, c] = 2(c^\dagger c - \frac{1}{2})$
- Fermions in different links anti-commute
  $\{c_{x,\hat{i}}, c_{y,\hat{j}}\} = 0 = \{c^\dagger_{x,\hat{i}}, c^\dagger_{y,\hat{j}}\}; \quad \{c^\dagger_{x,\hat{i}}, c_{y,\hat{j}}\} = \delta_{xy}\delta_{ij}.$

The plaquette with fermi links

- Gauge invariant pure gauge model constructed with fermions.
- Plaquette: simultaneous hop of two fermions into empty sites.



- Preserves charge conjugation, parity, and point group symmetries.
- Integer valued winding numbers arising from $U(1)^d$ global symmetry. (left): $(W_x, W_y) = (0, 0)$ and (right): $(W_x, W_y) = (1, 1)$



$$W_x = \frac{1}{L_x} \sum_x \left(n_{r=(x, y_0), \hat{j}} - \frac{1}{2} \right)$$

$$W_y = \frac{1}{L_y} \sum_y \left(n_{r=(x_0, y), \hat{i}} - \frac{1}{2} \right)$$

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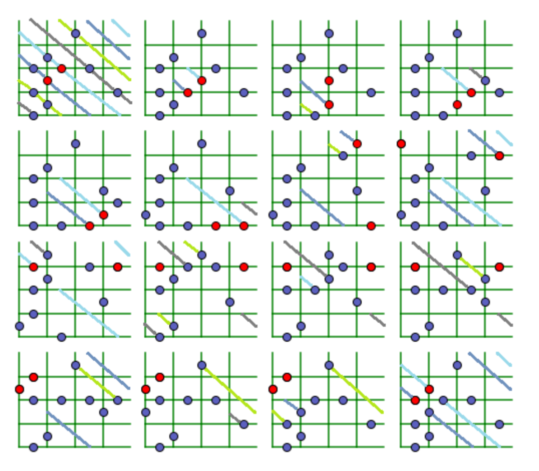
Equivalence with the QLM in $d = 2$

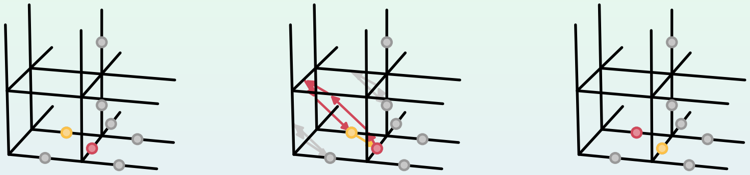


Fermions constrained to move along 1d non-overlapping chains.

Physics is identical to the spin-$\frac{1}{2}$ quantum link model.

A new model in $d = 3$

In 3d, the fermionic and bosonic models behave differently.



The model built with fermions is qualitatively different.

Explored with exact diagonalization upto 72 links.

$\mathcal{H}_{\mathrm{F}} = -J \sum_{\square}(U_{\square} + U_{\square}^{\dagger}) + \lambda \sum_{\square}(U_{\square} + U_{\square}^{\dagger})^2.$

Large negative λ prevents fermions from hopping.

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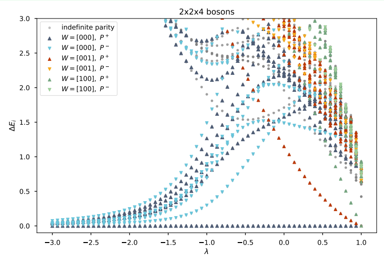
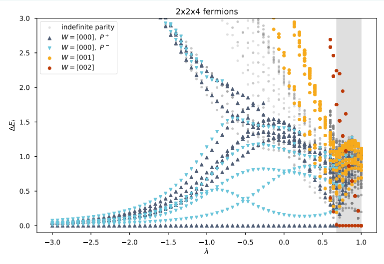
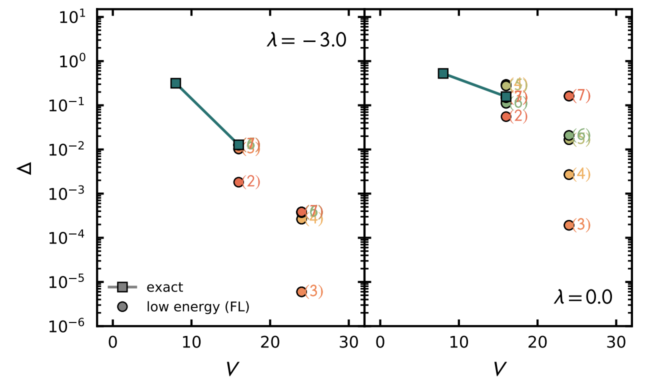
Low-energy spectra

- Large Hilbert spaces in the zero-winding sector.
  $2 \times 2 \times 4$ lattice $\equiv 48$ links
  $\longrightarrow$ 1552024 states.
  $2 \times 2 \times 6$ lattice $\equiv 72$ links
  $\longrightarrow 10^{10}$ states.
- For $\lambda \longrightarrow -\infty$, both the spin and the fermionic models have same (rotation) symmetry breaking.
- Phase transitions occur in both the models for $-1 < \lambda < 0$.
- Fermionic model seems to condense strings in a phase $\lambda < 1$.





Scaling of the spectral gap of the fermionic model



(Rotational) symmetry breaking goes away when λ is decreased.
Consistent with the expectation of a Coulomb phase (better known as quantum spin liquid in condensed matter physics).

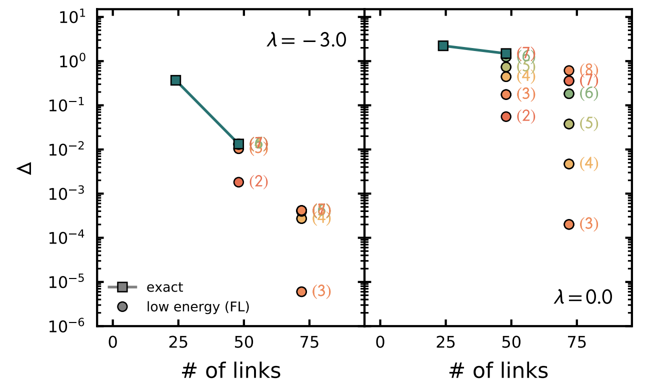
Outlook

- Phase diagram of both the bosonic and the fermionic $U(1)$ link models. Investigate the properties of the phases.
- Investigate dimensional reduction in link models.
- Design the plaquette interaction in cold-atom simulators.
- Dynamical signatures via quantum scars?
- The Wilson-Kogut-Susskind limit?
- The low-energy effective theory of the Coulomb phase with fermionic gauge fields?

Stay tuned! Thank you for your attention!

Schematic phase diagram

Scaling of the spectral gap of the bosonic model



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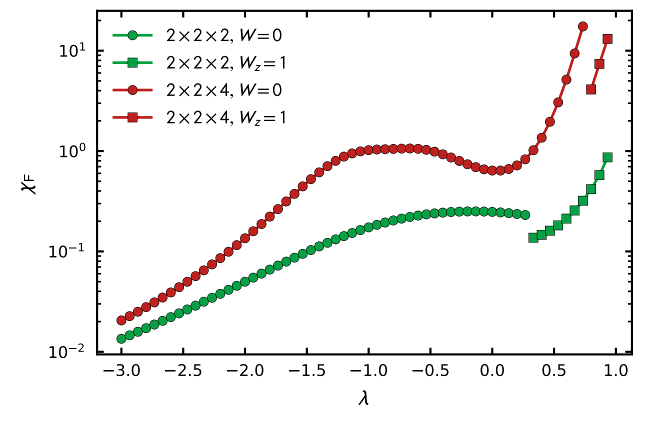
Fidelity susceptibility



$$F(\lambda,\epsilon) = |\langle\psi(\lambda)|\psi(\lambda+\epsilon)\rangle|; \quad \chi_F \equiv -\frac{\partial^2 \log F}{\partial\epsilon^2}\bigg|_{\epsilon=0}.$$