Critical exponents at the spin-charge flip symmetric fixed point

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Collaborators:
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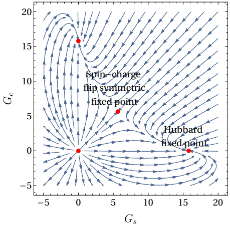
Ribhu Kaul
Hanqing Liu

Motivation

Spin-charge flip symmetric fixed point discussed in previous talk (Hanqing Liu).



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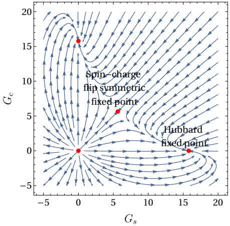

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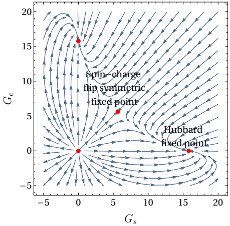

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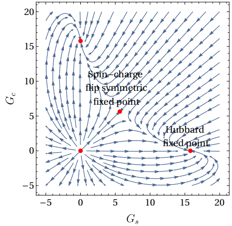

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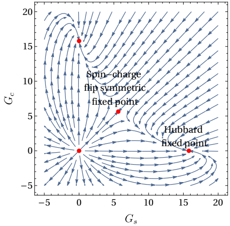

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- But the QMC methods themselves can be difficult due to sign problems! (and efficiency)
- We will be exploring this model in the Hamiltonian picture with a new method, the "Fermion Bag" approach.
- *Universality* → one model can be interesting for multiple fields. **RG→ Gell-Mann, Low, Kadanoff, Wilson, etc.**

Fermion Bags, Goals

- Fermion bag algorithms–first developed in Lagrangian picture, then in Hamiltonian picture. **Chandrasekharan, Eur Phys. (2013) ,EH, Chandrasekharan PRB (2017)**

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  - Models must have some concept of locality, $\hat{H} = -\sum_{x,d} \hat{H}_{x,d}$.
  - Models must be able to be written as sums of exponentiated fermionic bilinear terms, $\hat{H}_{x,d} = \omega_{x,d} e^{\alpha_{x,d} \sum a=1^{N_f} (\hat{c}^\dagger_{x,a} \hat{c}_{x+d,a} + \hat{c}^\dagger_{x+d,a} \hat{c}_{x,a})}$.

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- Qs: Can we take full advantage of localities in Hamiltonian models? Can we compute the critical exponents of the spin-charge flip symmetric model efficiently?

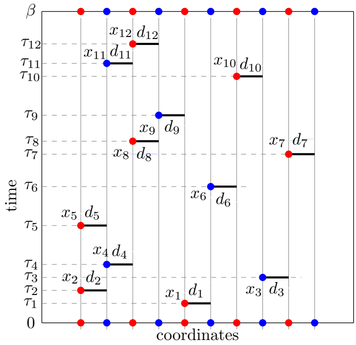
Partition Function

Wang, Liu, Troyer, PRB 93

- **Continuous time:**

$$Z = \mathrm{Tr}\left(e^{-\beta\hat{H}}\right) = \mathrm{Tr}\left((1-\epsilon\hat{H})^N\right)$$

$$= \int [d\tau] \sum_{k,[\langle x,d\rangle]} \mathrm{Tr}\left(\hat{H}_{x_k,d_k}(\tau_k)\ldots\hat{H}_{x_2,d_2}(\tau_2)\hat{H}_{x_1,d_1}(\tau_1)\right),$$



Computing the partition function

- We can evaluate the partition function as:

$$Z = \sum_{[\langle \tau, x, d \rangle]} \int \Omega([\langle \tau, x, d \rangle]),$$

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$$\Big\downarrow \text{BSS formula}$$

$$\Omega([\langle \tau, x, d\rangle]) \; = \; \left(\prod_{x_k,d_k} (d\tau \omega_{x_k,d_k}) \right) \det\left(\mathbb{1} + h_{x_k,d_k} \ldots h_{x_2,d_2} h_{x_1,d_1}\right).$$

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$$h_{x,d} = \begin{pmatrix} 1 & \ldots & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & \mathcal{H}_{x,d} & \ldots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \ldots & 1 \end{pmatrix}, \qquad \mathcal{H}_{x,d} = \begin{pmatrix} \cosh \alpha_{x,d} & \sinh \alpha_{x,d} \\ \sinh \alpha_{x,d} & \cosh \alpha_{x,d} \end{pmatrix}$$

Fermion Bags

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$$\left[\hat{H}_{x,d}, \hat{H}_{x',d'} \right] = 0 \quad \rightarrow \quad \left[h_{x,d}, h_{x',d'} \right] = 0,$$

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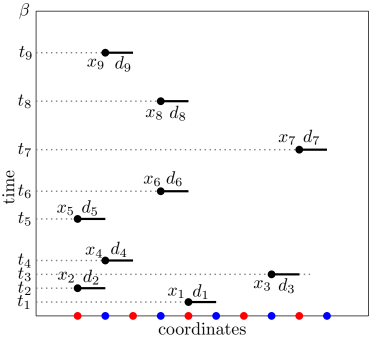

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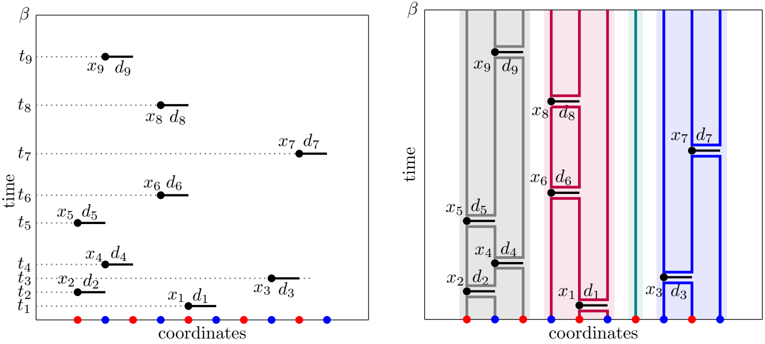
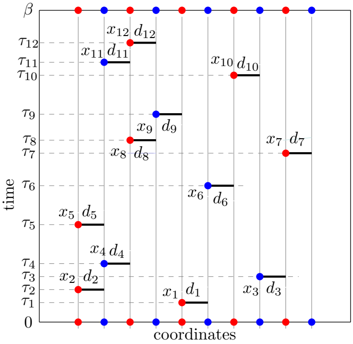


Fermion Bags

- But revisiting our original configuration example, we see everything would be in the same fermion bag–what do we do?



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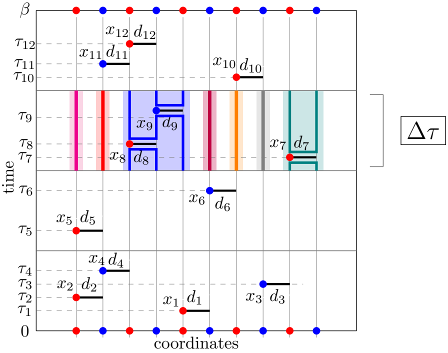

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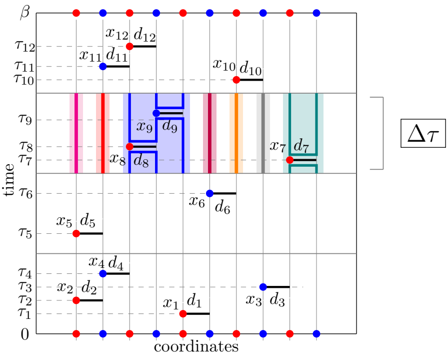

- Very effective in computing critical exponents for t-V model ($4,096 - 10,000$ sites versus 1152 sites in previous calculations). **EH, Chandrasekharan, PRD (2020)**

The Hamiltonian Model

$$\hat{H}_{\mathrm{SC}} = -\sum_{\langle x,d\rangle}\prod_{\sigma=1,2}\left[-t\eta_{x,d}\left(\hat{c}^{\dagger}_{x\sigma}\hat{c}_{x+d\sigma}+\hat{c}^{\dagger}_{x+d\sigma}\hat{c}_{x\sigma}\right)\right.$$
$$\left.+V\left(\hat{n}_{x\sigma}-\frac{1}{2}\right)\left(\hat{n}_{x+d\sigma}-\frac{1}{2}\right)-\frac{t^2}{V}\right]$$
$$= -\delta\sum_{\langle x,d\rangle} e^{\alpha\eta_{x,d}\sum_{\sigma}\left(\hat{c}^{\dagger}_{x\sigma}\hat{c}_{x+d\sigma}+\hat{c}^{\dagger}_{x+d\sigma}\hat{c}_{x\sigma}\right)}$$

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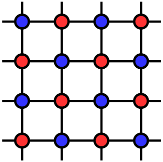
$$= -\delta \sum_{\langle x,d\rangle} e^{\alpha\eta_{x,d}\sum_{\sigma}\left(\hat{c}^{\dagger}_{x\sigma}\hat{c}_{x+d\sigma}+\hat{c}^{\dagger}_{x+d\sigma}\hat{c}_{x\sigma}\right)}$$



A. Sandvik

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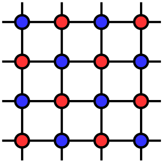

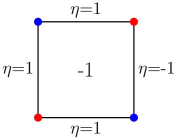

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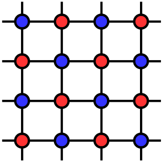

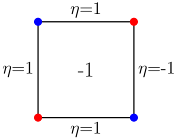

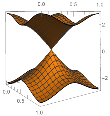

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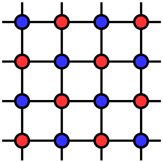

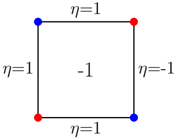

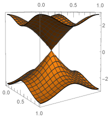

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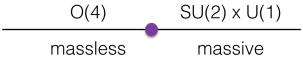

anti-ferromagnetism
or
superconductivity!

Results

- Choosing $\beta = L$ we use the equal time density-density correlation function as an order parameter (z-component of the antiferromagnetic order parameter):

$$\left\langle \hat{C}(L) \right\rangle = (-1)^{L/2} \left\langle \left(\hat{n}_{(0,0),\uparrow} - \hat{n}_{(0,0),\downarrow}\right) \left(\hat{n}_{(L/2,0),\uparrow} - \hat{n}_{(L/2,0),\downarrow}\right) \right\rangle$$

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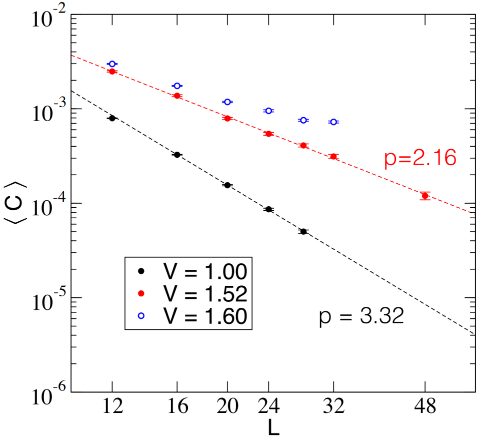
- critical coupling: $\left\langle \hat{C}(L) \right\rangle \sim \frac{1}{L^{1+\eta}}$

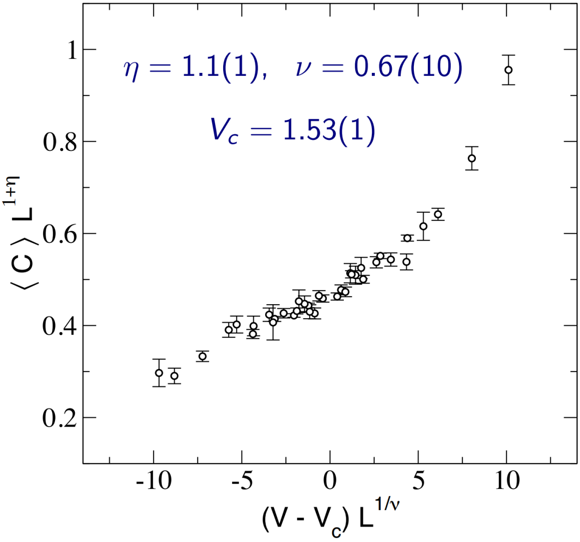
Evidence for two phases and a transition

$$\langle C \rangle \sim \frac{1}{L^p}$$



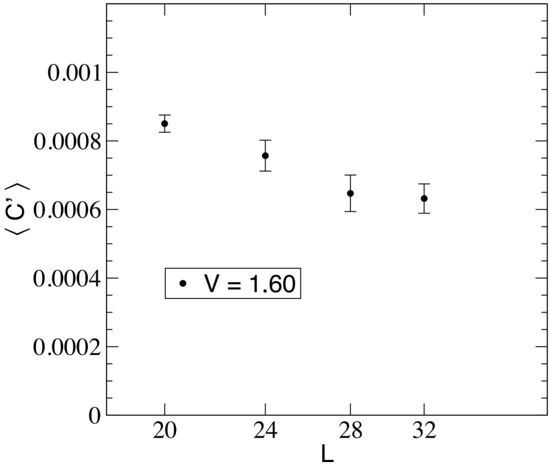
$$V_c \approx 1.52?$$

Critical Scaling (preliminary)



Computing time: 400,000 core-hours

Evidence for SSB of spin-charge flip symmetry



$$\hat{C} = \left(\hat{n}_{(0,0),\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{(0,0),\downarrow} - \frac{1}{2}\right)\left(\hat{n}_{(L/2,0),\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{(L/2,0),\downarrow} - \frac{1}{2}\right)$$

Conclusions

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- The Hamiltonian lattice model with spin-charge flip symmetry is suitable for fermion bag calculations and leads to new fixed points (different from those from the Hubbard interaction).
- The phase transition is between a semi-metal and an antiferromagnet/superconductor, which is accompanied by the breaking of the spin-charge flip symmetry.
- An analysis up to $L_x = L_y = 48, \beta = 48.0$ lattices suggests $\eta = 1.1(1)$, $\nu = 0.67(10)$ (preliminary).