A Spin-charge Flip Symmetric Fixed Point in 2+1d with Dirac Fermions

Hanqing Liu (Duke University)

In collaboration with

Shailesh Chandrasekharan, Emilie Huffman and Ribhu Kaul

July 26, 2021

LATTICE 21

JULY 26-30 2021, ZOOM/GATHER@MIT

Motivations

- Relativistic fermions in $2+1$d with four-fermion couplings are interesting
  - Rich class of fixed points, allowing us to study critical behaviors near quantum critical points.
  - Graphene: mass generation by strongly coupled fermions
  - Deconfined quantum critical points and emergent symmetries
  - Interesting as field theories by themselves
- Study relativistic fermion models non-perturbatively on the lattice using Monte Carlo method
  - The sign problem
  - Efficiency
- Goal: construct interacting Hamiltonians with interesting physics, in the meanwhile,
  - Free from the sign problem
  - Efficient algorithms: fermion bag approach (see Emilie Huffman's talk after this one)

Overview

This talk will focus on the continuum analysis of our lattice model

- Identify all the symmetries of our lattice Hamiltonian
- Map the lattice symmetries to the continuum Lagrangian
- Construct all the four-fermion couplings in the continuum respecting those lattice symmetries
- Identify the relevant interactions which give correct physics in the broken phase, confirmed by the effective potential
- Calculate the β function, confirm the existence of a spin-charge flip symmetric fixed point and calculate the critical exponents in $4-\varepsilon$ dimension and large N_f limit

The lattice Hamiltonian

2-flavor fermions on square lattice with π-flux

$$H = -\delta \sum_{\langle ij \rangle} \exp(\alpha \eta_{ij} \sum_{\sigma=1,2} (c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma}))$$

Equivalently,

$$H = -\sum_{\langle ij \rangle} \prod_{\sigma} [- t\eta_{ij}(c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma}) + V(n_{i\sigma} - \frac{1}{2})(n_{j\sigma} - \frac{1}{2}) - \frac{t^2}{V}]$$

Symmetries of the lattice Hamiltonian

According to Wilson RG, all interactions respecting the symmetries of the lattice Hamiltonian can be generated in the continuum.

Since each bond in this Hamiltonian is exponential of the free hopping term, it has all the explicit space-time and internal symmetries of the free Hamiltonian:

- Translations by one unit $T_a^{1,2}$
- $\mathbb{Z}_4$ rotation symmetry R
- Parity P
- Time-reversal Θ
- Charge conjugation C
- $\mathrm{SU}(2)_s$ spin symmetry
- $\mathrm{SU}(2)_c$ charge symmetry

spin-charge flip symmetry F

Free lattice Hamiltonian in the continuum

Linearize the dispersion relation near the Fermi points, we get the following Euclidean Dirac Lagrangian density in the continuum,

$$\mathcal{L}_0 = \bar{\psi}_\alpha \gamma^\mu \partial_\mu \psi^\alpha$$

- ψ^α: 2-flavor 4-component Dirac fermions
- $\bar{\psi}_\alpha := \psi^\dagger_\alpha \gamma^3$
- $\mu = 1, 2$ (spatial) and 3 (temporal)
- $\gamma^1, \cdots, \gamma^5$: five 4×4 hermitian matrices satisfying the Clifford algebra $\{\gamma^i, \gamma^j\} = 2\delta^{ij}\mathbb{1}_4$. It turns out that we can choose $\gamma^{1,2}$ to be imaginary, and $\gamma^{3,4,5}$ to be real.

Lattice space-time symmetries in the continuum

Space-time transformations on the lattice mix Dirac components,

- $T_a^{1,2}$: $\psi \mapsto \mathrm{i}\gamma^{4,5}\psi$
- R: $\psi \mapsto \mathrm{e}^{\mathrm{i}\frac{\pi}{4}(\mathrm{i}\gamma^1\gamma^2+\mathrm{sgn}(x)\mathrm{i}\gamma^4\gamma^5)}\,\psi$
- P: $\psi \mapsto \mathrm{i}\gamma^5\gamma^1\psi$
- Θ: $\psi \mapsto \gamma^3 K\psi$

where $\mathrm{sgn}(x) = \pm 1$ depends on whether it comes from an even site or an odd one, and K is the complex conjugation operator.

- $T_a^{1,2}, P, \Theta$ act as $\mathbb{Z}_2$ symmetries on fermion bilinears
- R acts as a $\mathbb{Z}_4$ symmetry, and will be enhanced to an $\mathrm{SO}(2)_R$ symmetry in the continuum

Lattice internal symmetries in the continuum

When analyzing the internal symmetries, especially the charge symmetry, it is more convenient to use Majorana representation

$$\mathcal{L}_0 = \xi_\alpha^T \gamma^3 \gamma^\mu \partial_\mu \xi^\alpha$$

where $\psi^1 = \xi^1 - \mathrm{i}\xi^2$, $\psi^2 = \xi^3 - \mathrm{i}\xi^4$, and now $\alpha = 1, 2, 3, 4$.

In $2+1$d four-component fermions are reducible representation of the Lorentz group, this Lagrangian has an $\mathrm{O}(8)$ symmetry.

- The spin and charge symmetries, together with lattice rotations R, are embedded inside a subgroup $\mathrm{SO}(3)_s \times \mathrm{SO}(3)_c \times \mathrm{SO}(2)_R \subset \mathrm{O}(8)$
- The $\mathbb{Z}_2$ spin-charge flip symmetry F flips $\mathrm{SO}(3)_s$ with $\mathrm{SO}(3)_c$

Decompose fermion bilinears under $\mathrm{SO}(3)_s \times \mathrm{SO}(3)_c \times \mathrm{SO}(2)_R$

Allowed four-fermion interactions must be singlet under $\mathrm{SO}(3)_s \times \mathrm{SO}(3)_c \times \mathrm{SO}(2)_R$, and they can be constructed from fermion bilinears. They form reducible representation of the symmetry group, and decompose into irreducible representations (irreps) as

36 mass terms (6 irreps)

$$\mathbf{36} = (\mathbf{3} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{3}) \otimes \mathbf{1} + (\mathbf{3} \otimes \mathbf{3} + \mathbf{1} \otimes \mathbf{1}) \otimes (\mathbf{2} + \mathbf{1})$$

28 currents (6 irreps)

$$\mathbf{28} = (\mathbf{3} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{3}) \otimes (\mathbf{2} + \mathbf{1}) + (\mathbf{3} \otimes \mathbf{3} + \mathbf{1} \otimes \mathbf{1}) \otimes \mathbf{1}$$

Allowed four-fermion couplings

If we also take into account the $\mathbb{Z}_2$ space-time symmetries, no mass terms are invariant under all of them, and therefore our continuum theory is massless. Building singlets from those bilinear irreps, we get 6 Gross-Neveu couplings and 6 Thirring couplings. However, due to Fierz identity, only 4 of them are independent, and remarkably, they can all be chosen to be Gross-Neveu couplings,

$$\mathcal{L}_s = \frac{G_s}{2}|\vec{M}_s|^2, \mathcal{L}_c = \frac{G_c}{2}|\vec{M}_c|^2, \mathcal{L}_R = \frac{G_R}{2}|\vec{M}_R|^2, \mathcal{L}_{\text{singlet}} = \frac{G_{\text{singlet}}}{2}M_{\text{singlet}}^2$$

The continuum model

From numerical studies (see Emilie Huffman's talk following this) :

Massless phase Massive phase

$$\mathrm{SO}(3)_s \times \mathrm{SO}(3)_c \times \mathrm{SO}(2)_R \times {\color{red}\mathbb{Z}_2} \quad V_c \quad \mathrm{SO}(3) \times \mathrm{SO}(2) \times \mathrm{SO}(2)_R$$

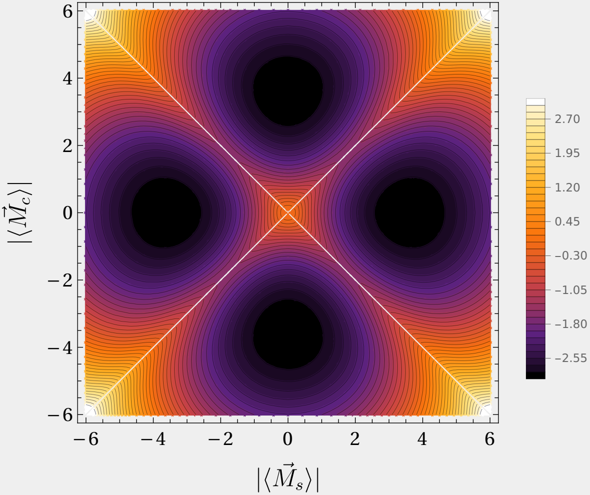
Difference from the Hubbard model!

$$\implies |\langle \vec{M}_s \rangle| \neq 0, \text{ or } |\langle \vec{M}_c \rangle| \neq 0, \text{ but } |\langle \vec{M}_R \rangle| = 0$$

Therefore it is reasonable to analyze the model with spin and charge couplings,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_s + \mathcal{L}_c$$

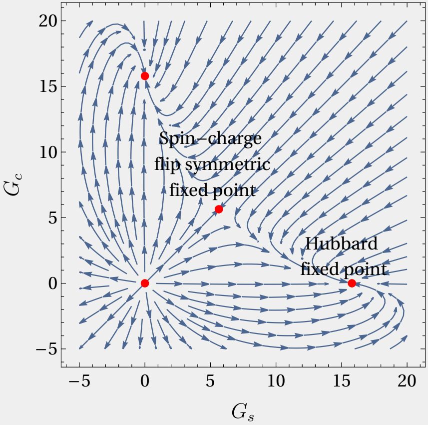
The effective potential



Either spin order or charge order, but not both, and therefore the $\mathbb{Z}_2$ spin-charge flip symmetry is also spontaneously broken.

Effective potential at $g_s = g_c$ in the broken phase

RG analysis and critical exponents



RG flow of the Gross-Neveu couplings in $4-\varepsilon$ expansion

- Critical exponents using $4-\varepsilon$ expansion: $\eta = \frac{2\varepsilon}{7}$, $1/\nu = 2 - \frac{6\varepsilon}{7}$
- Critical exponents using $4-\varepsilon$ expansion at large N_f: $\eta = \varepsilon - \frac{5\varepsilon}{N_f+5}$, $1/\nu = 2 - \varepsilon - \frac{3\varepsilon}{N_f+5}$
- Also agree with our large N_f expansion setting $d = 4 - \varepsilon$

Conclusions

- Lattice Hamiltonian solvable using the fermion bag approach has a spin-charge flip symmetry
- This symmetry leads to a new fixed point without fine tuning
- An interesting phase transition featuring spontaneous spin symmetry breaking or charge symmetry breaking, and spin-charge flip symmetry breaking
- Estimate the critical exponents near this fixed point using $4-\varepsilon$ expansion and large N_f expansion.

Thanks for attention!